Math 204
Differential Equations
Definitions and Calssification

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Introduction to Differential Equations (DE)

About the course

Text Books:

1. Differential Equations by Said Mesloub, Mostafa Damlakhi and Khawaja Zafar Elahi.
2. Differential equations with boundary value problems: by Dennis G. Zill and Michael R Cullen (Seventh or sixth edition)

Course Topics:

1. Definition and classification of Differential equations
2. First Order Differential equations with application
3. Higher order Differential equations.
4. Solving systems of Linear Equations by Elimination Method.
5. Series solutions of Linear Equations.
6. Orthogonal Functions and Fourier series.
7. Fourier cosine and sine series.
8. Fourier Integral.

Grading:

- First midterm 25
- Second midterm 25
- Quizzes 10
- Final 40

Definition and types of differential equations

Definition of a Differential Equation (DE)

Definition

A differential equation is an equation containing the derivative of one or more dependent variables with respect to one or more independent variables.

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A differential equation is an equation containing the derivative of one or more dependent variables with respect to one or more independent variables.

Example

$$y = e^{x^2}$$

$$\frac{dy}{dx} = 2xy$$

$$dy = 2xy\ dx$$

Types of Differential Equations

Ordinary Differential Equations (ODE): if the equation has only one independent variable.

Example

$$\frac{dy}{dx} - 5x = 3$$

$$\frac{dx}{dt} - 2\frac{dy}{dt} = t$$

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 + 2y = 0$$

Types of Differential Equations

Partial Differential Equations (PDE): if the equation has more than one independent variable.

Example

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y}$$

$$\frac{x\partial^2 u}{\partial x^2} - 3\frac{\partial^2 u}{\partial t^2} = 0$$

Definition

The order of a differential equation is the order of the highest derivative in the differential equation.

Example

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 + 2y = 0$$

$$(y''')^2 + (y'')^5 = 0$$

$$xdx + ydy = 0$$

Order of Differential Equations

A general nth order ordinary differential equation is

$$F\left(x, y, \frac{dy}{dx}, \dots, \frac{d^n y}{dx^n}\right) = 0$$

Linear Differential Equation

Definition

A differential equation is called *linear* if it can be written in the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x),$$

where $a_0(x), a_1(x), \dots, a_n(x)$ and $g(x)$ are given functions of x, and $a_n(x) \neq 0$.

Example

1. $$\frac{dy}{dx} + 3y = e^x$$

2. $$\frac{dy}{dx} + y^2 = 0$$

3. $$yy' + 3xy = 0$$

4. $$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \sin(x)$$

5. $$\frac{d^2y}{dx^2} + \sin(y) = 0$$

Solution of Differential Equations

Solution of Differential Equations

Example

Prove that $y = e^{2x}$ is a solution of the equation

$$y'' + y' - 6y = 0 \qquad \forall x \in \mathbb{R}$$

Solution of Differential Equations

Example

Show that $y = x^3 e^x$ is a solution of the equation

$$xy'' - 2(x+1)y' + (x+2)y = 0 \qquad \forall x > 0$$

Implicit solution of Differential Equations

Example

Verify that $F(x,y) = x^2 + y^2 - 25 = 0$ defines an implicit solution of the differential equations

$$\frac{dy}{dx} = -\frac{x}{y}, y \neq 0 \qquad \forall x \in (-5, 5)$$

Elimination of arbitrary constant

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant c from the equation

$$y = c\cos x$$

Example

Eliminate the arbitrary constant c from the equation

$$y = cx$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constants c_1 and c_2 from the equation

$$y = c_1 e^{-2x} + c_2 e^{3x}$$

Example

Eliminate the arbitrary constants c_1 and c_2 from the equation

$$y = c_1 e^{-2x} + c_2 e^{3x}$$

Solution

$$y'' - y' - 6y = 0$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant a from the equation

$$(x-a)^2 + y^2 = a^2$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant a from the equation

$$(x-a)^2 + y^2 = a^2$$

Solution

$$2xyy' + x^2 - y^2 = 0$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant b and C from the equation

$$x = C\cos(at + b)$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant b and C from the equation

$$x = C\cos(at + b)$$

Solution

$$\frac{d^2x}{dt^2} + a^2x = 0$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant c from the equation

$$cxy + c^2x + 4 = 0$$

Elimination of arbitrary constant

Example

Eliminate the arbitrary constant c from the equation

$$cxy + c^2x + 4 = 0$$

Solution

$$x^3(y')^2 + x^2yy' + 4 = 0$$

Families of curves

Example

The equation

$$(x-c)^2 + (y-c)^2 = 2c^2$$

$$x^2 + y^2 - 2c(x+y) = 0$$

represents a family of circles.

$$(x^2 + 2xy - y^2)dx - (x^2 - 2xy - y^2)dy = 0$$

Families of curves

Example

Find a differential equation satisfied by the family of parabolas having their vertices at the origin and their foci on the $y-$axis.

Families of curves

Example

Find a differential equation satisfied by the family of parabolas having their vertices at the origin and their foci on the $y-$axis.

Solution

$$y = ax^2$$
$$xy' - 2y = 0$$

Example

Find a differential equation of the family of circles having their centers on the $y-$axis.

Families of curves

Example

Find a differential equation of the family of circles having their centers on the $y-$axis.

Solution

$$x^2 + (y-b)^2 = c^2$$
$$xy'' - (y')^3 - y' = 0$$

Mathematical models

Example

Growth and Decay

$$\frac{dP}{dt} = KP, \qquad P(t_0) = P$$

P: given quantity K: constant of proportion

Mathematical models

Example

Free falling stone

$$\frac{d^2s}{dt^2} = -g$$

s: distance g: acceleration due to gravity