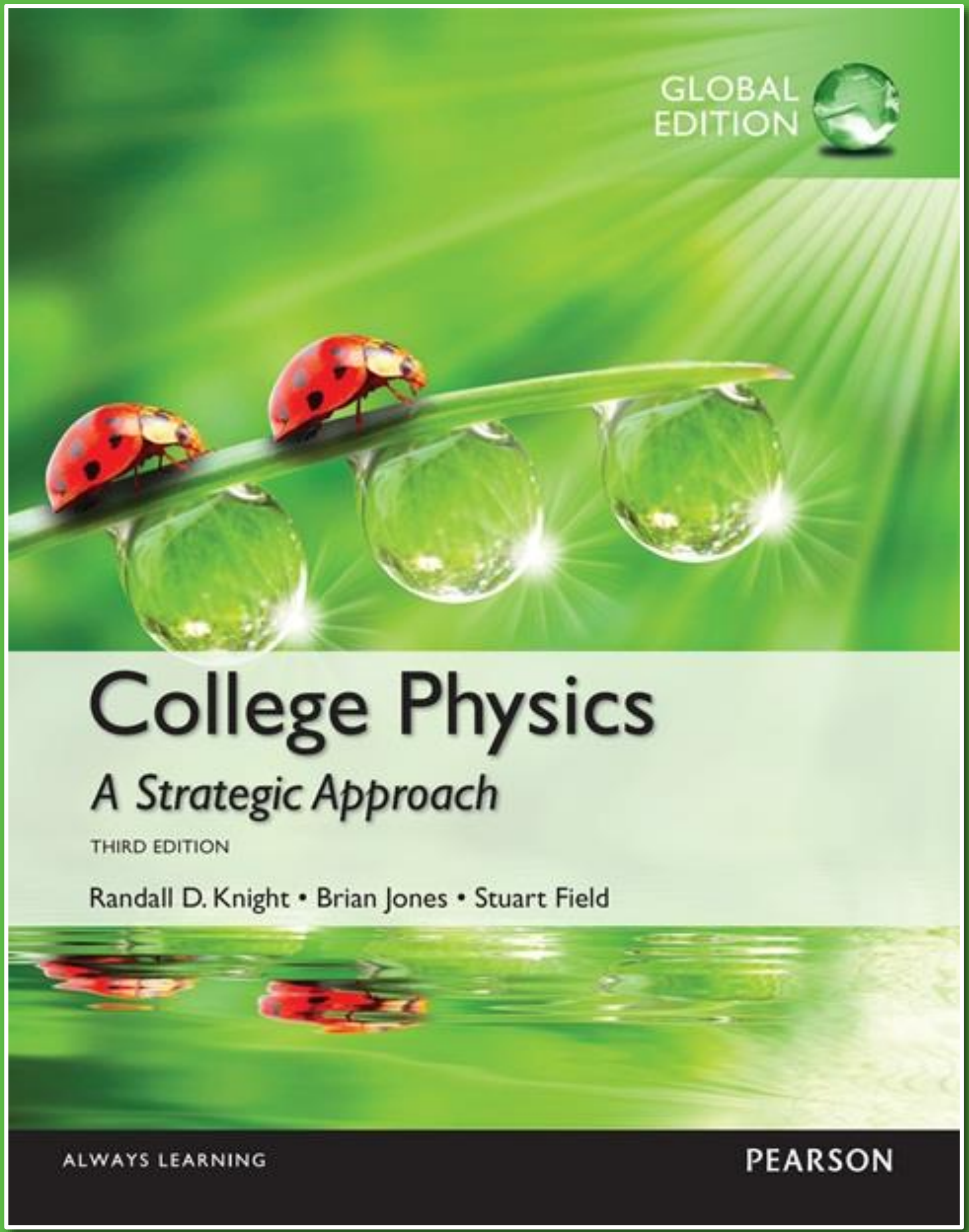


Lecture Presentation

Chapter 9

Impulse and Momentum

Chapter 9 Impulse and Momentum

Section 9.1 Impulse
Section 9.2 Momentum and the Impulse-Momentum Theorem
Section 9.3 Solving Impulse and Momentum Problems
Section 9.4 Conservation of Momentum
Section 9.5 Inelastic Collisions

Chapter 9 Impulse and Momentum

Chapter Goal: To learn about impulse, momentum, and a new problem-solving strategy based on conservation laws.

Chapter 9 Preview
Looking Ahead: Impulse

- This golf club delivers an **impulse** to the ball as the club strikes it.

- You'll learn that a longer-lasting, stronger force delivers a greater impulse to an object.

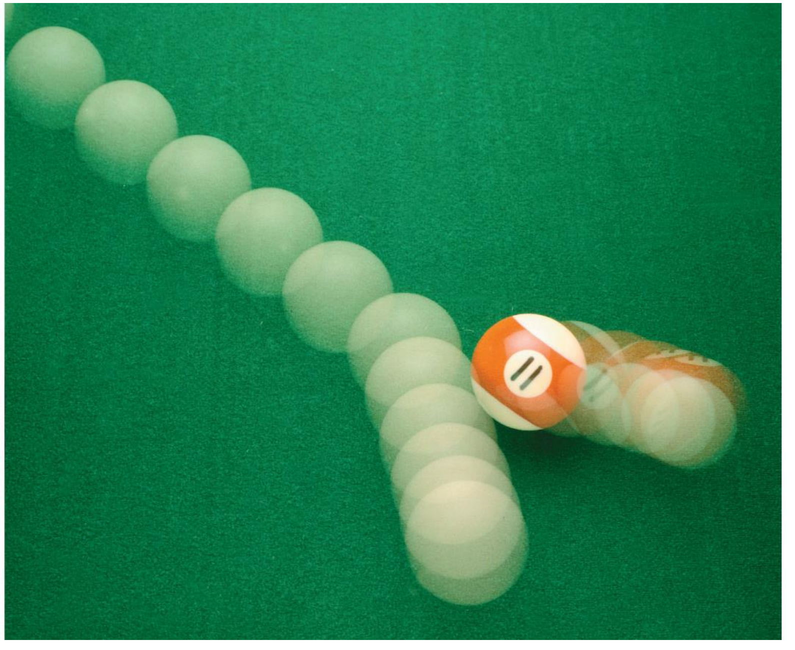
Chapter 9 Preview
Looking Ahead: Momentum and Impulse

- The impulse delivered by the player's head *changes* the ball's **momentum**.

- You'll learn how to calculate this momentum change using the **impulse-momentum theorem**.

Chapter 9 Preview
Looking Ahead: Conservation of Momentum

- The momentum of these pool balls before and after they collide is the *same*—it is **conserved**.



- You'll learn a powerful new *before-and-after* problem-solving strategy using this **law of conservation of momentum**.

Chapter 9 Preview
Looking Ahead

Impulse

This golf club delivers an **impulse** to the ball as the club strikes it.

You'll learn that a longer-lasting, stronger force delivers a greater impulse to an object.

Momentum and Impulse

The impulse delivered by the player's head *changes* the ball's **momentum.**

You'll learn how to calculate this momentum change using the **impulse-momentum theorem**.

Conservation of Momentum

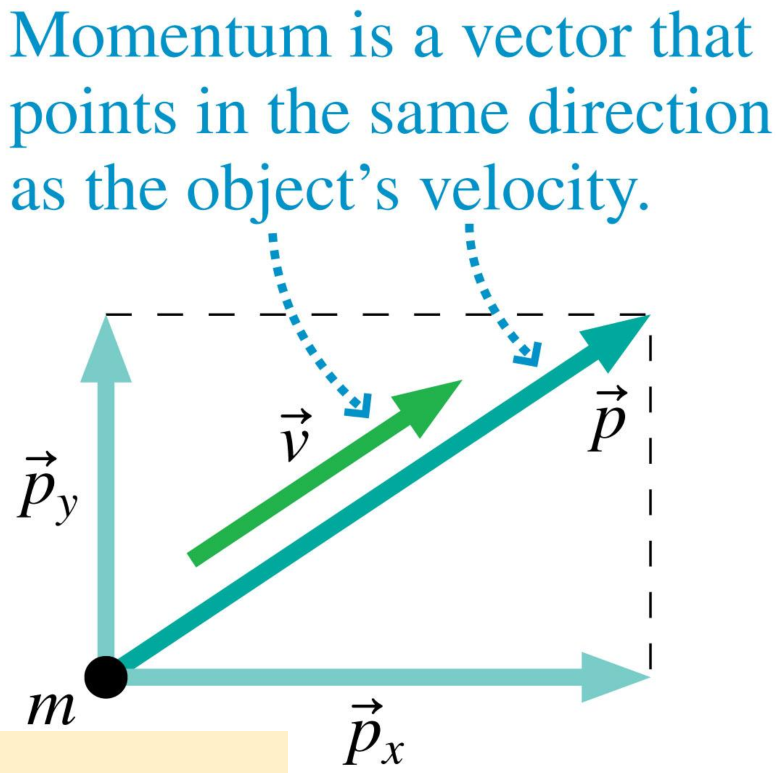
The momentum of these pool balls before and after they collide is the *same*—it is **conserved.**

You'll learn a powerful new *before-and-after* problem-solving strategy using this **law of conservation of momentum.**

Text: p. 288

Momentum

- Momentum is a *vector* quantity that points in the same direction as the velocity vector:
- **Momentum** is the product of the object's mass and velocity. It has units of kg · m/s.
- The *magnitude* of an object's momentum is simply the product of the object's mass and speed.



$$\vec{p} = m\vec{v}$$

Momentum of an object of mass m and velocity $\vec{v}$

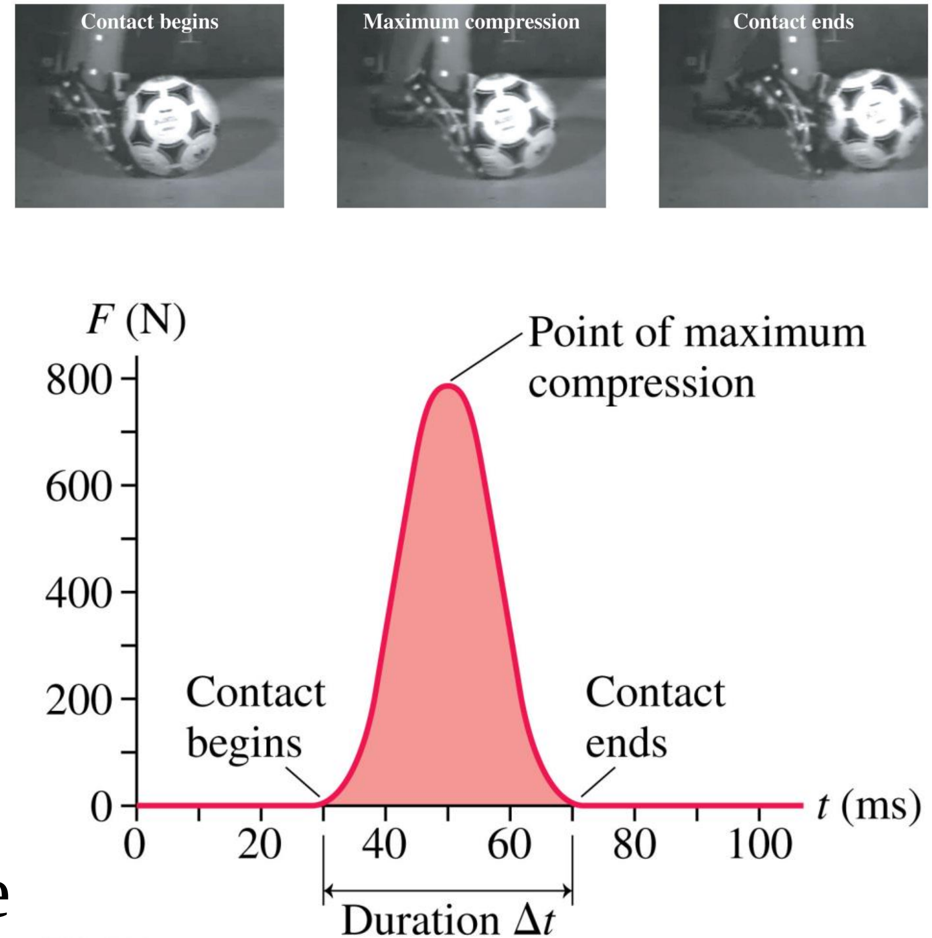
Section 9.1 Impulse

Impulse

- In physics, a collision is an isolated event in which two or more objects exert relatively strong forces on one another for a relatively short time.
- exert relatively strong forces on one another for a relatively short time.
- During a collision, it takes time to compress the object, and it takes time for the object to re-expand.
- The duration of a collision depends on the materials.

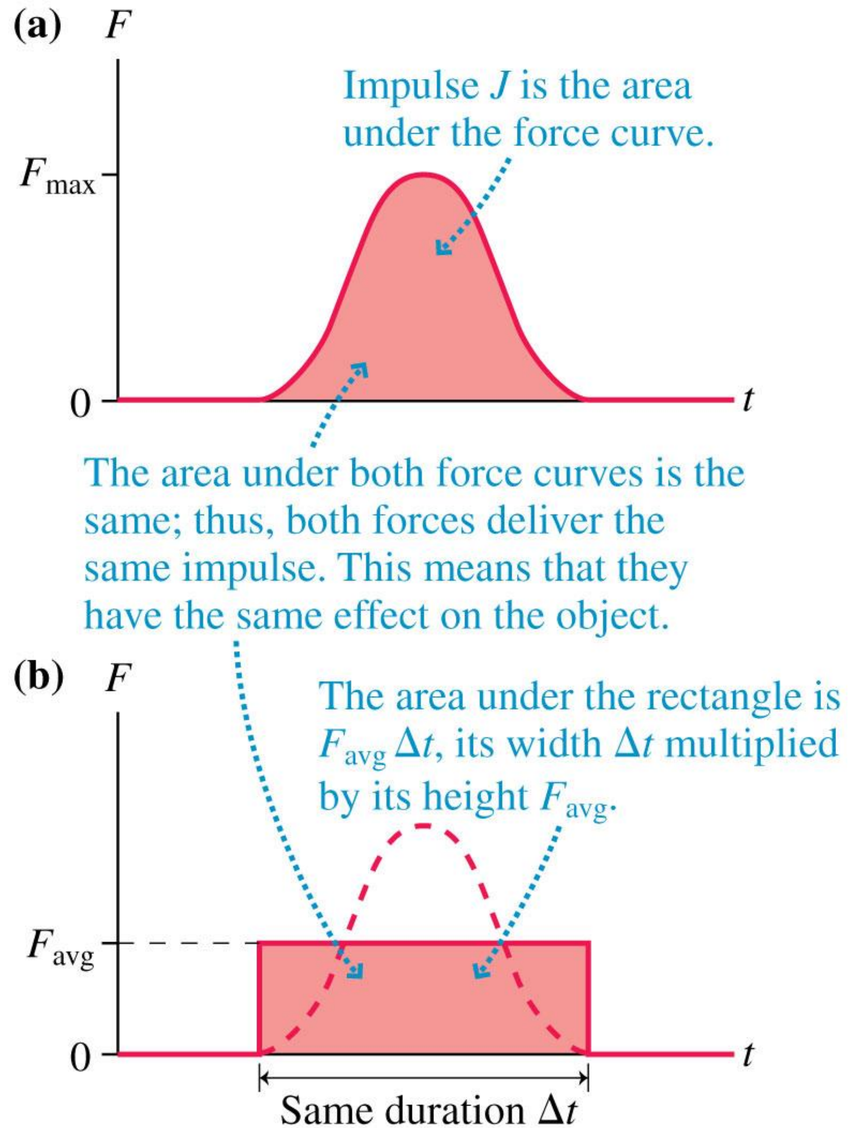
Impulse

- When kicking a soccer ball, the amount by which the ball is compressed is a measure of the magnitude of the force the foot exerts on the ball.
- The force is applied only while the ball is in contact with the foot.
- The **impulse force** is a large force exerted during a short interval of time.



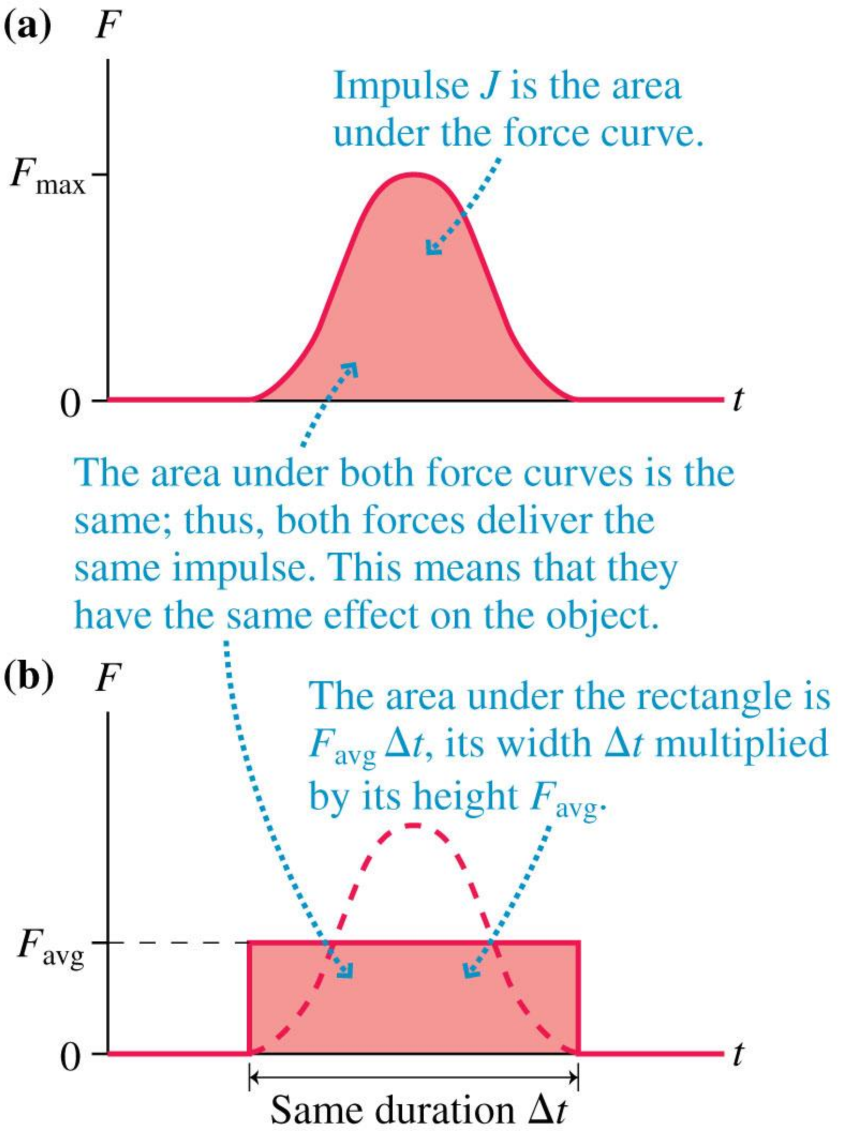
Impulse

- **The effect of an impulsive force is proportional to the area under the force-versus-time curve.**
- The area is called the **impulse** J of the force.



Impulse

- It is useful to think of the collision in terms of an *average* force F_{avg}.
- F_{avg} is defined as the constant force that has the same duration Δt and the same area under the force curve as the real force.



Impulse

impulse J = area under the force curve = $F_{\text{avg}}\,\Delta t$

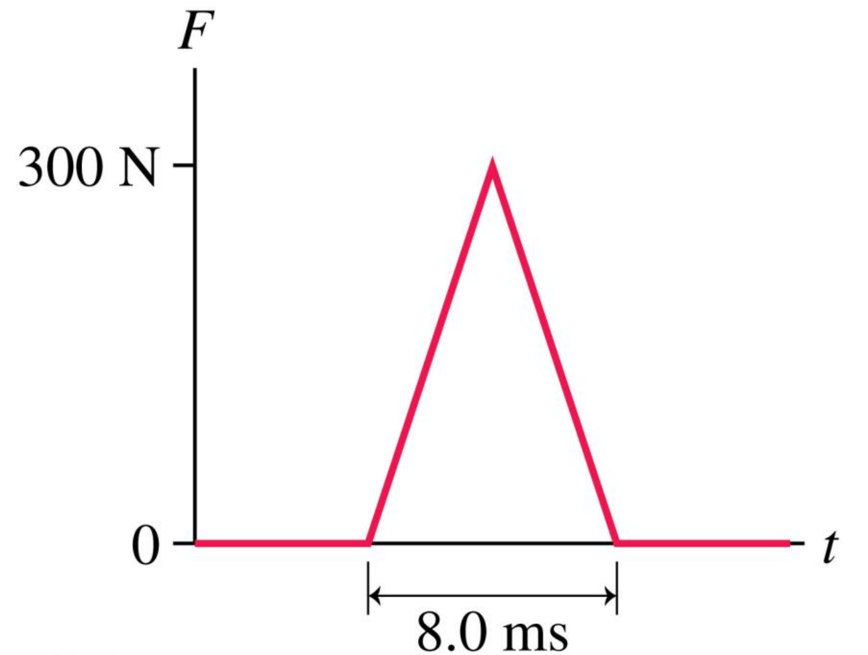
Impulse due to a force acting for a duration Δt

- Impulse has units of N · s same as momentum, but N · s are equivalent to kg · m/s.
- The latter are the preferred units for impulse.
- The impulse is a *vector* quantity, pointing in the direction of the average force vector:

$$\vec{J} = \vec{F}_{\text{avg}}\,\Delta t$$

Example 9.1 Finding the impulse on a bouncing ball

A rubber ball experiences the force shown in the figure as it bounces off the floor.

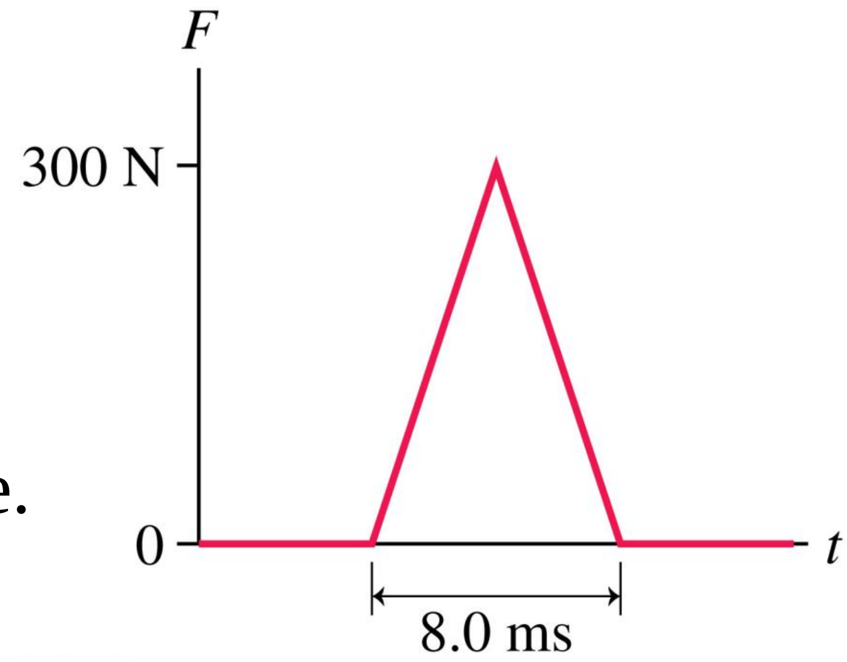


a. What is the impulse on the ball?

b. What is the average force on the ball?

Example 9.1 Finding the impulse on a bouncing ball (cont.)

PREPARE The impulse is the area under the force curve. Here the shape of the graph is triangular, so we'll need to use the fact that the area of a triangle is $\frac{1}{2} \times \text{height} \times \text{base}$.

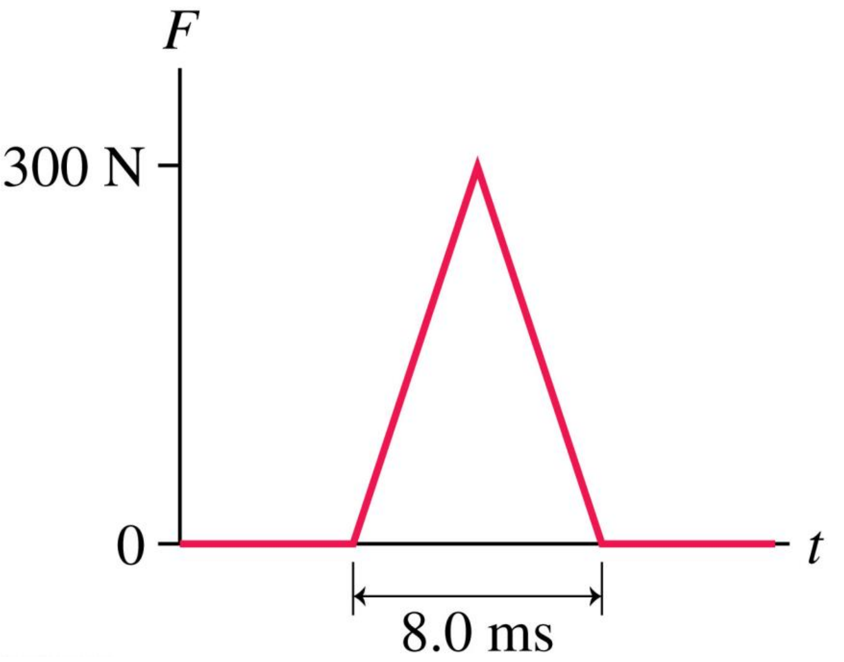


Example 9.1 Finding the impulse on a bouncing ball (cont.)

SOLVE a. The impulse is

$$J = \tfrac{1}{2}(300\text{ N})(0.0080\text{ s}) = 1.2\text{ N}\cdot\text{s} = 1.2\text{ kg}\cdot\text{m/s}$$

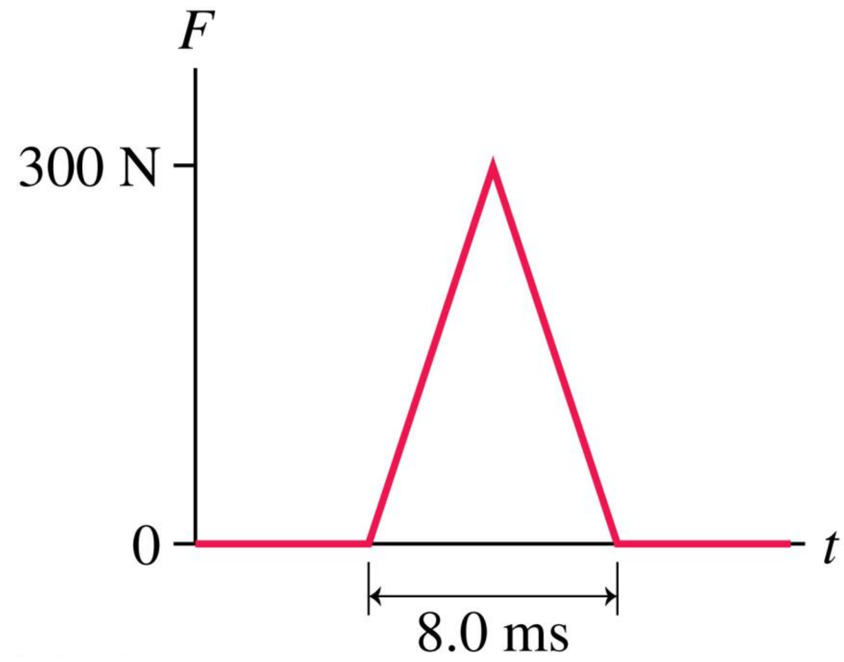
b. We have:
$J = F_{avg}\,\Delta t$, so we can find the average force that would give this same impulse:



$$F_{avg} = \frac{J}{\Delta t} = \frac{1.2\text{ N}\cdot\text{s}}{0.0080\text{ s}} = 150\text{ N}$$

Example 9.1 Finding the impulse on a bouncing ball (cont.)

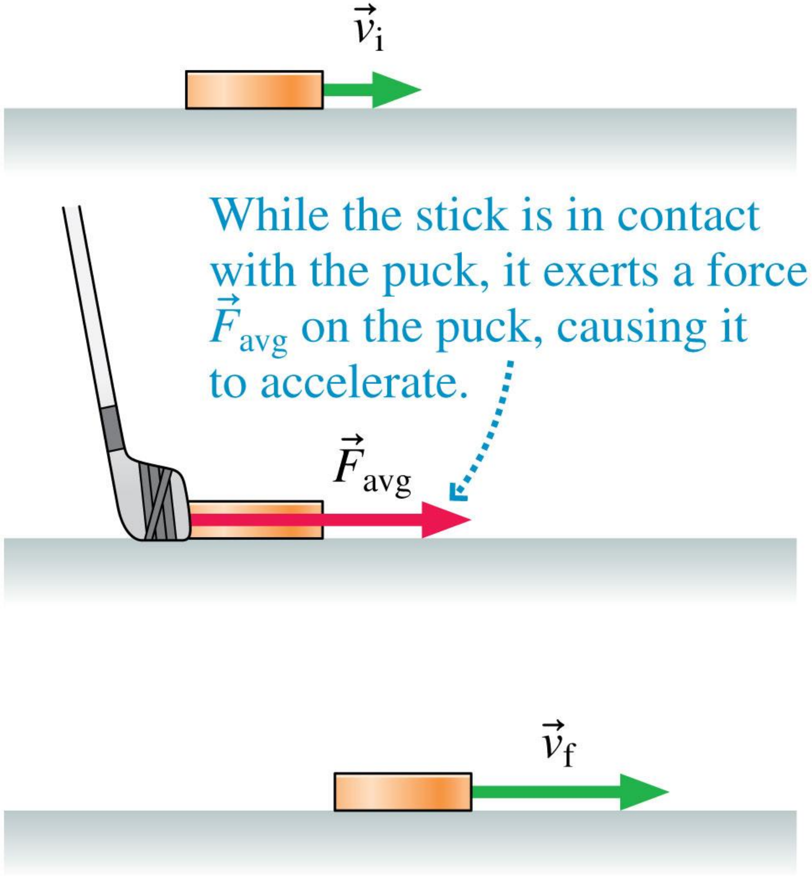
ASSESS In this particular example, the average value of the force is half the maximum value. This is not surprising for a triangular force because the area of a triangle is *half* the base times the height.



Section 9.2 Momentum and the Impulse-Momentum Theorem

Momentum and the Impulse-Momentum Theorem

- We know that giving a kick to a heavy object will change its velocity much less than giving the same kick to a light object.
- We can calculate how the final velocity is related to the initial velocity.



Momentum and the Impulse-Momentum Theorem

- From Newton's second law, the average acceleration of an object during the time the force is being applied is

$$\vec{a}_{\text{avg}} = \frac{\vec{F}_{\text{avg}}}{m}$$

- The average acceleration is related to the change in the velocity by

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}$$

- We combine those two equations to find $\frac{\vec{F}_{\text{avg}}}{m} = \vec{a}_{\text{avg}} = \frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}$

- So

$$\vec{F}_{\text{avg}}\,\Delta t = m\vec{v}_{\text{f}} - m\vec{v}_{\text{i}}$$

The Impulse-Momentum Theorem

- Impulse and momentum are related as:

$$\vec{J} = \vec{p}_{\mathrm{f}} - \vec{p}_{\mathrm{i}} = \Delta\vec{p}$$

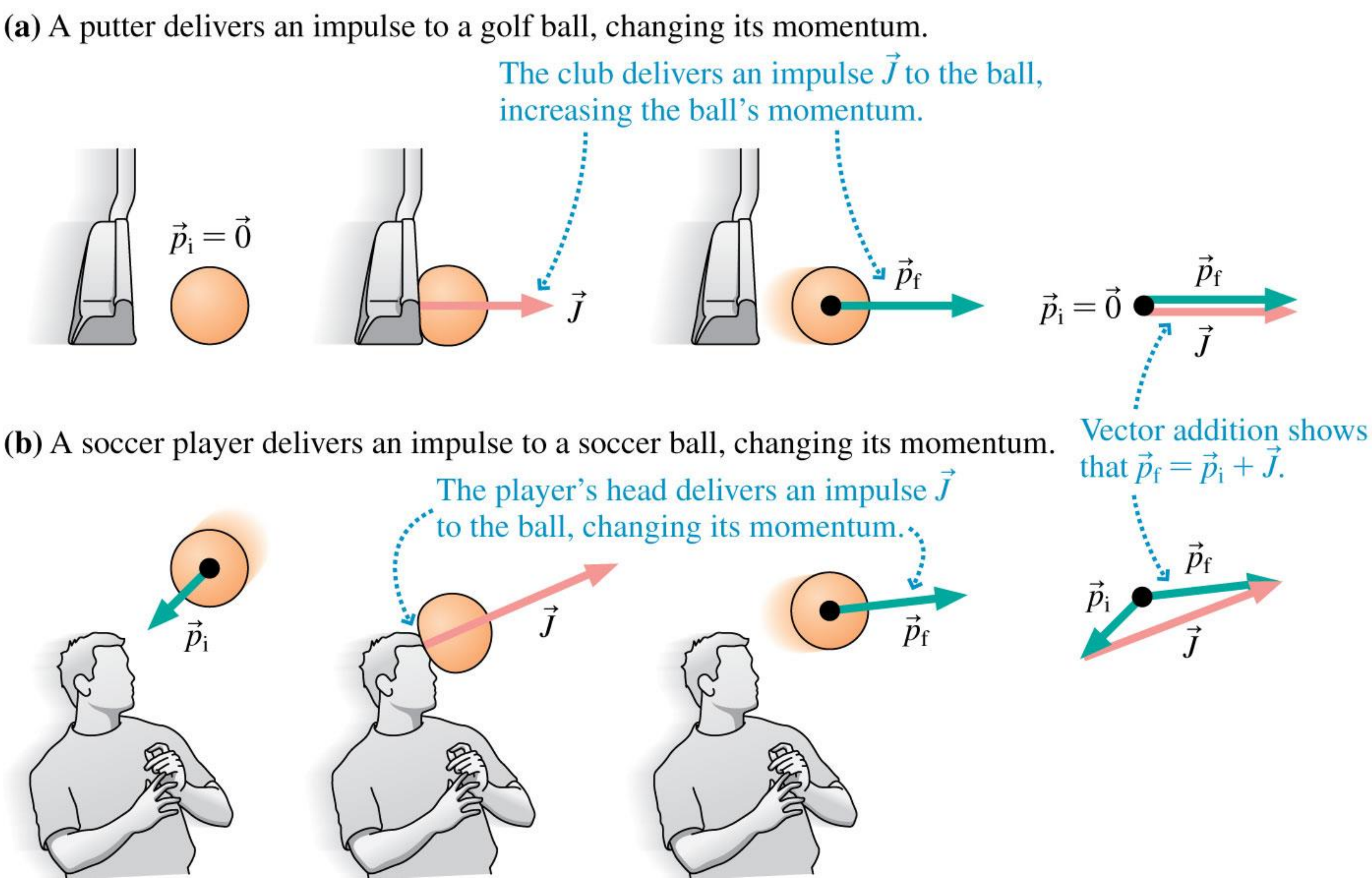
Impulse-momentum theorem

- **The impulse-momentum theorem states that an impulse delivered to an object causes the object's momentum to change.**
- Impulse can be written in terms of its *x*- and *y*-components:

$$J_x = \Delta p_x = (p_x)_{\mathrm{f}} - (p_x)_{\mathrm{i}} = m(v_x)_{\mathrm{f}} - m(v_x)_{\mathrm{i}}$$

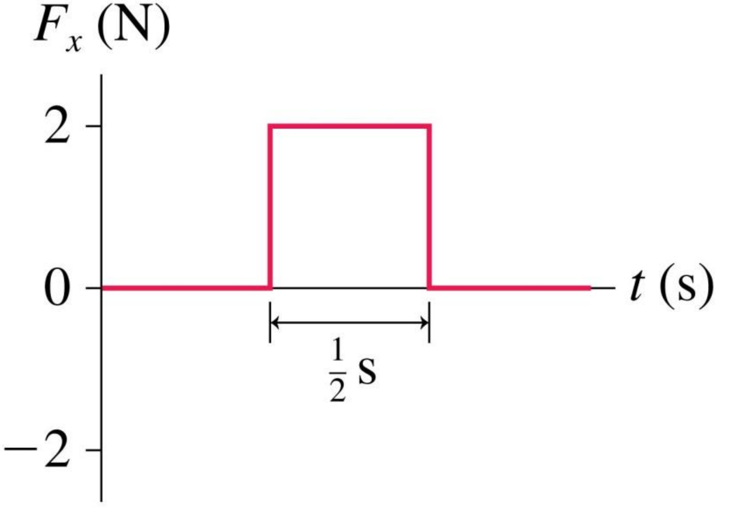
$$J_y = \Delta p_y = (p_y)_{\mathrm{f}} - (p_y)_{\mathrm{i}} = m(v_y)_{\mathrm{f}} - m(v_y)_{\mathrm{i}}$$

The Impulse-Momentum Theorem



QuickCheck 9.2

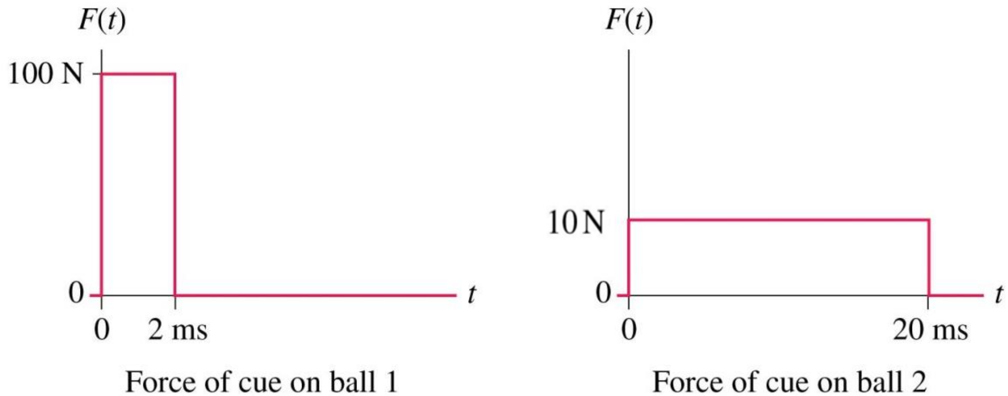
- A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object's speed and direction after the force ends?



A. 0.50 m/s left
B. At rest
C. 0.50 m/s right
D. 1.0 m/s right
E. 2.0 m/s right

QuickCheck 9.6

- Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



A. Ball 1

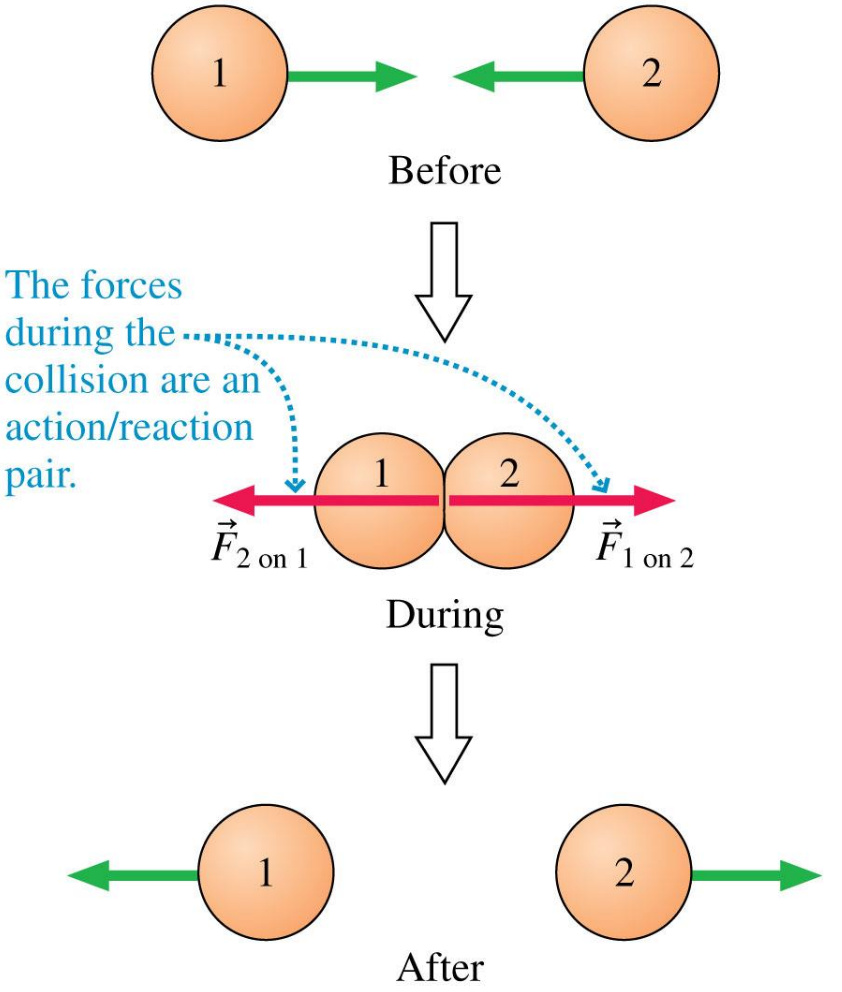
B. Ball 2

C. Both balls have the same final speed.

Section 9.4 Conservation of Momentum

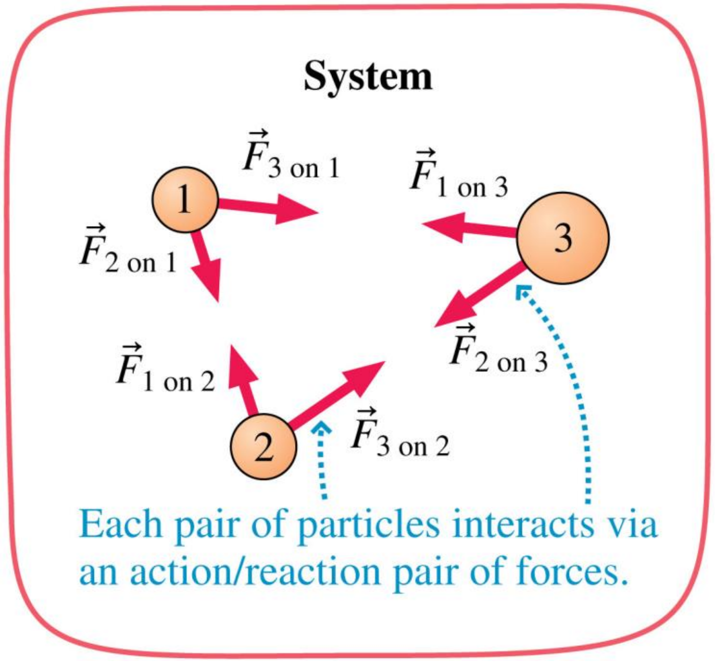
Conservation of Momentum

- The forces acting on two balls during a collision form an action/reaction pair. They have equal magnitude but opposite directions (Newton's third law).
- If the momentum of ball 1 increases, the momentum of ball 2 will decrease by the same amount.



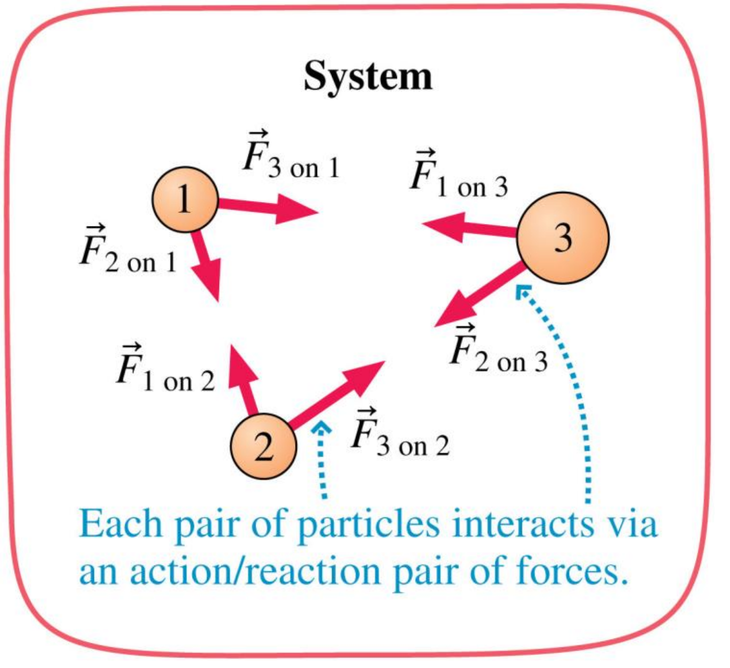
Law of Conservation of Momentum

- **There is no change in the *total* momentum of the system** no matter how complicated the forces are between the particles.
- The total momentum of the system is *conserved*.



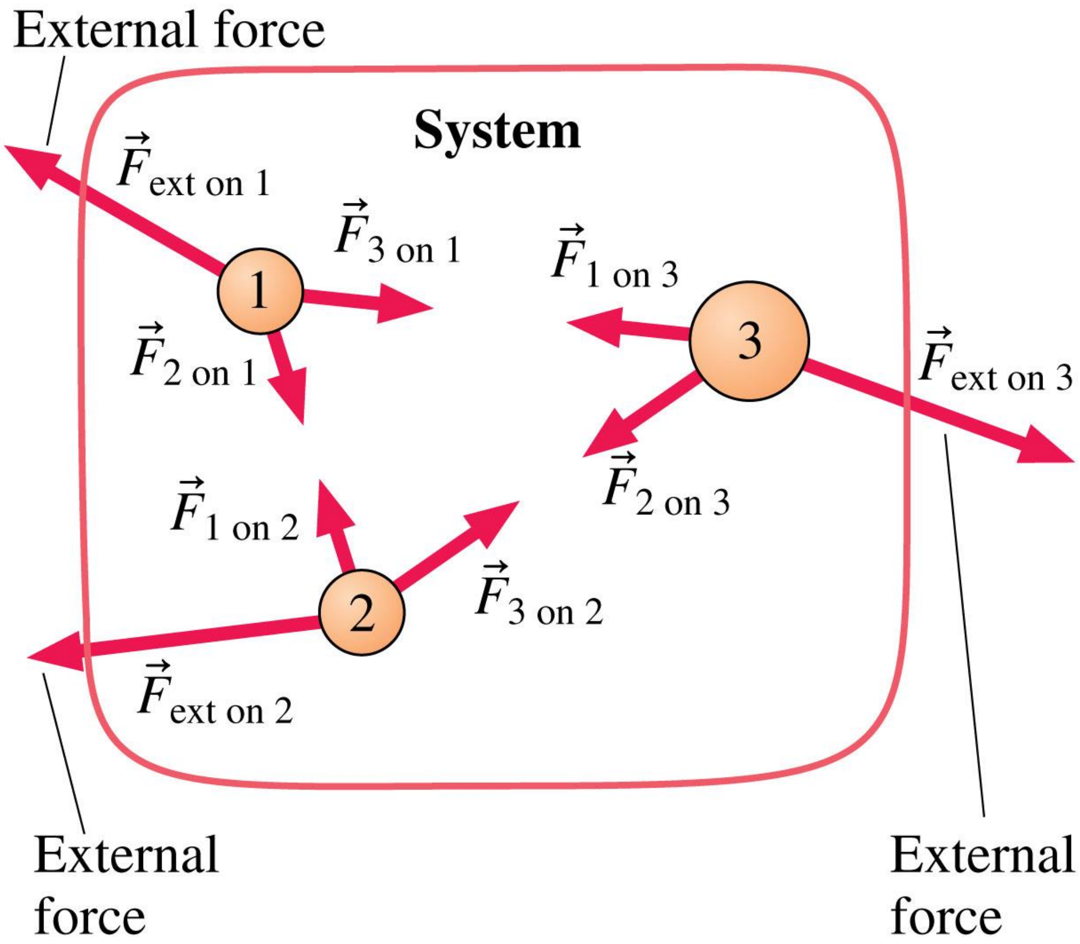
Law of Conservation of Momentum

- **Internal forces** act only between particles within a system.
- **The total momentum of a system subjected to only internal forces is conserved.**



Law of Conservation of Momentum

- **External forces** are forces from agents outside the system.
- External forces *can* change the momentum of the system.



Law of Conservation of Momentum

- The change in the total momentum is

$$\begin{aligned}\Delta\vec{P} &= \Delta\vec{p}_1 + \Delta\vec{p}_2 + \Delta\vec{p}_3 \\ &= (\vec{F}_{\text{ext on 1}}\,\Delta t) + (\vec{F}_{\text{ext on 2}}\,\Delta t) + (\vec{F}_{\text{ext on 3}}\,\Delta t) \\ &= (\vec{F}_{\text{ext on 1}} + \vec{F}_{\text{ext on 2}} + \vec{F}_{\text{ext on 3}})\,\Delta t \\ &= \vec{F}_{\text{net}}\,\Delta t\end{aligned}$$

- $\vec{F}_{\text{net}}$ is the net force due to *external forces.*
- **If $\vec{F}_{\text{net}} = \vec{0}$, the *total* momentum $\vec{P}$ of the system does not change.**
- An **isolated system** is a system with no net external force acting on it, leaving the momentum unchanged.

Law of Conservation of Momentum

Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

- The law of conservation of momentum for an isolated system is written

$$\vec{P}_f = \vec{P}_i$$

Law of conservation of momentum for an isolated system

- **The total momentum after an interaction is equal to the total momentum before the interaction.**

Law of Conservation of Momentum

- Since momentum is a vector, we can rewrite the law of conservation of momentum for an isolated system:

Final momentum / Initial momentum

$$x\text{-component:}\quad \underbrace{(p_{1x})_\text{f} + (p_{2x})_\text{f} + (p_{3x})_\text{f} + \cdots}_{\text{Final momentum}} = \underbrace{(p_{1x})_\text{i} + (p_{2x})_\text{i} + (p_{3x})_\text{i} + \cdots}_{\text{Initial momentum}}$$

Particle 1 Particle 2 Particle 3

$$y\text{-component:}\quad (p_{1y})_\text{f} + (p_{2y})_\text{f} + (p_{3y})_\text{f} + \cdots = (p_{1y})_\text{i} + (p_{2y})_\text{i} + (p_{3y})_\text{i} + \cdots$$

Law of Conservation of Momentum

Conservation of momentum problems

We can use the law of conservation of momentum to relate the momenta and velocities of objects *after* an interaction to their values *before* the interaction.

PREPARE Clearly define the *system*.

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is then conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapter 10, conservation of energy.

Law of Conservation of Momentum

Conservation of momentum problems

Following Tactics Box 9.1, draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

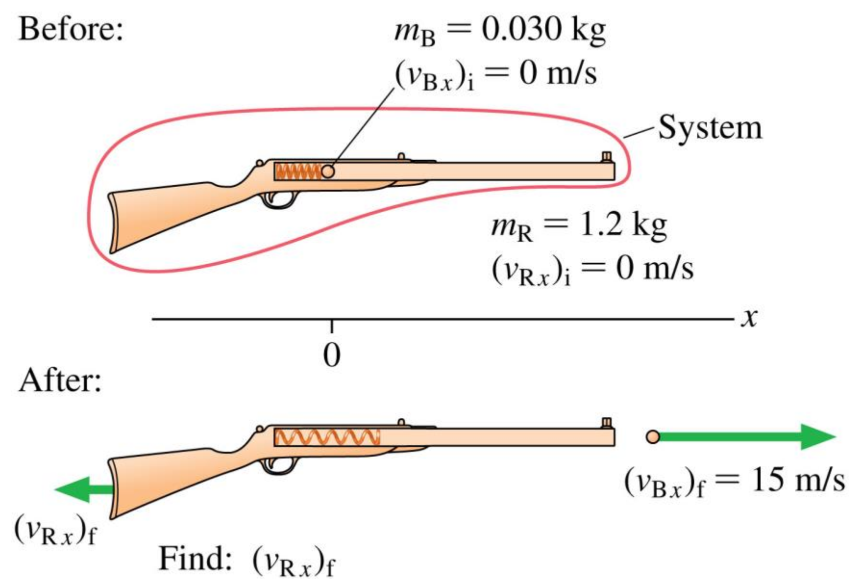
SOLVE The mathematical representation is based on the law of conservation of momentum, Equations 9.15. Because we generally want to solve for the velocities of objects, we usually use Equations 9.15 in the equivalent form

$$m_1(v_{1x})_f + m_2(v_{2x})_f + \cdots = m_1(v_{1x})_i + m_2(v_{2x})_i + \cdots$$
$$m_1(v_{1y})_f + m_2(v_{2y})_f + \cdots = m_1(v_{1y})_i + m_2(v_{2y})_i + \cdots$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Example 9.7 Recoil speed of a rifle

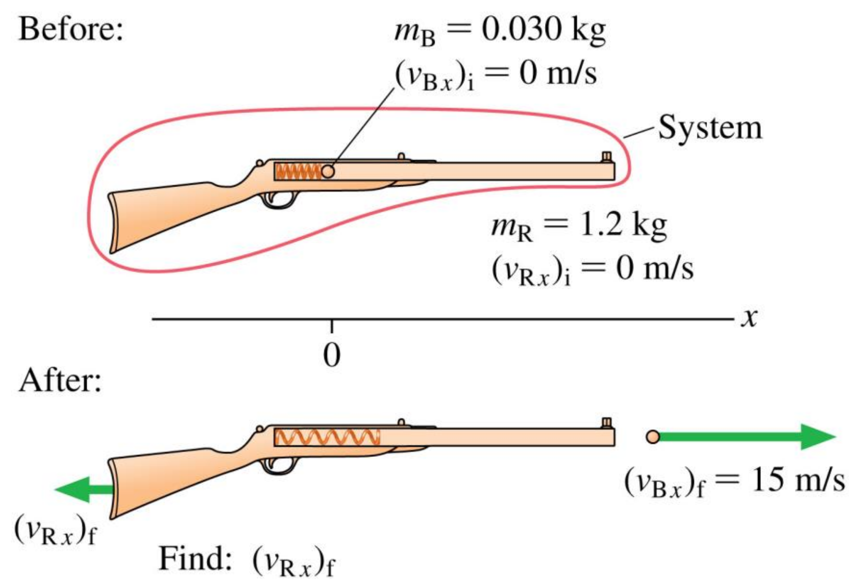
A 30 g ball is fired from a 1.2 kg spring-loaded toy rifle with a speed of 15 m/s. What is the recoil speed of the rifle?



PREPARE As the ball moves down the barrel, there are complicated forces exerted on the ball and on the rifle. However, if we take the system to be the ball + rifle, these are *internal* forces that do not change the total momentum.

Example 9.7 Recoil speed of a rifle (cont.)

The *external* forces of the rifle's and ball's weights are balanced by the external force exerted by the person holding the rifle, so $\vec{F}_{\text{net}} = \vec{0}$. This is an isolated system and the law of conservation of momentum applies.



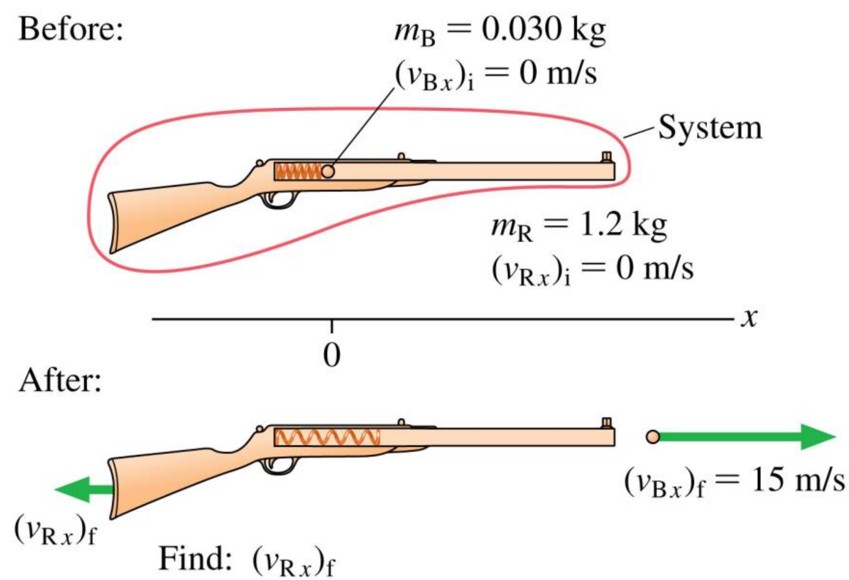
The figure shows a visual overview before and after the ball is fired. We'll assume the ball is fired in the $+x$-direction.

Example 9.7 Recoil speed of a rifle (cont.)

SOLVE The x-component of the total momentum is

$P_x = p_{Bx} + p_{Rx}$.

Everything is at rest before the trigger is pulled, so the initial momentum is zero.



After the trigger is pulled, the internal force of the spring pushes the ball down the barrel and pushes the rifle backward. Conservation of momentum gives

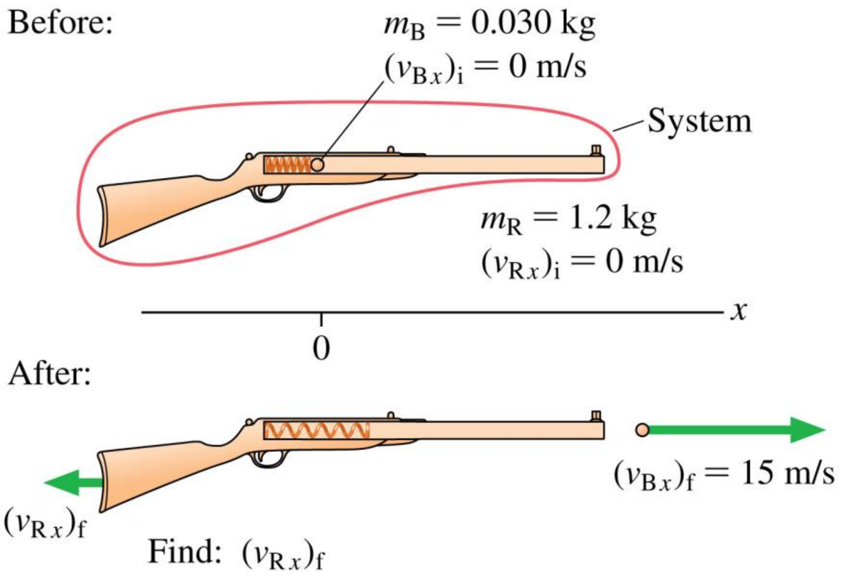
$$(P_x)_f = m_B(v_{Bx})_f + m_R(v_{Rx})_f = (P_x)_i = 0$$

Example 9.7 Recoil speed of a rifle (cont.)

Solving for the rifle's velocity, we find

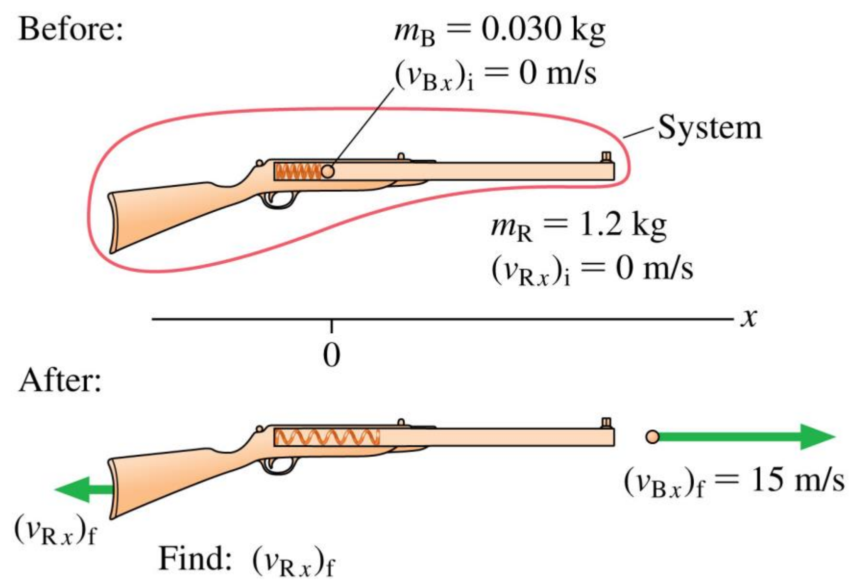
$$(v_{Rx})_f = -\frac{m_B}{m_R}(v_{Bx})_f = -\frac{0.030\text{ kg}}{1.2\text{ kg}} \times 15\text{ m/s} = -0.38\text{ m/s}$$

The minus sign indicates that the rifle's recoil is to the left. The recoil *speed* is 0.38 m/s.



Example 9.7 Recoil speed of a rifle (cont.)

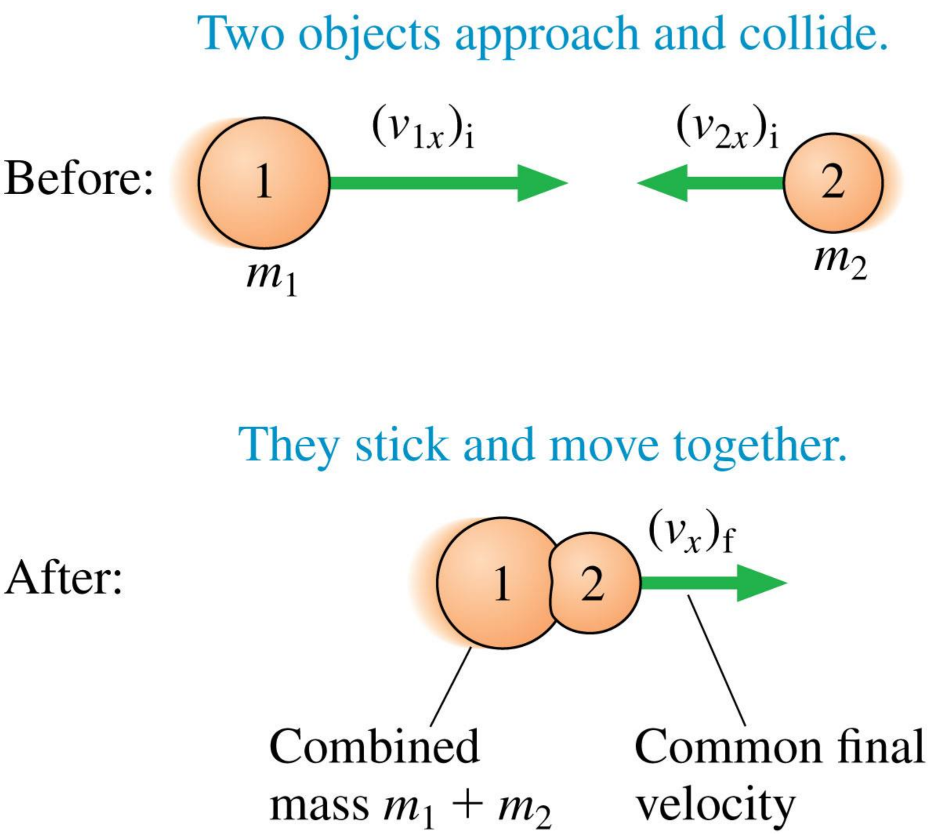
ASSESS Real rifles fire their bullets at much higher velocities, and their recoil is correspondingly higher. Shooters need to brace themselves against the "kick" of the rifle back against their shoulder.



Section 9.5 Inelastic Collisions

Inelastic Collisions

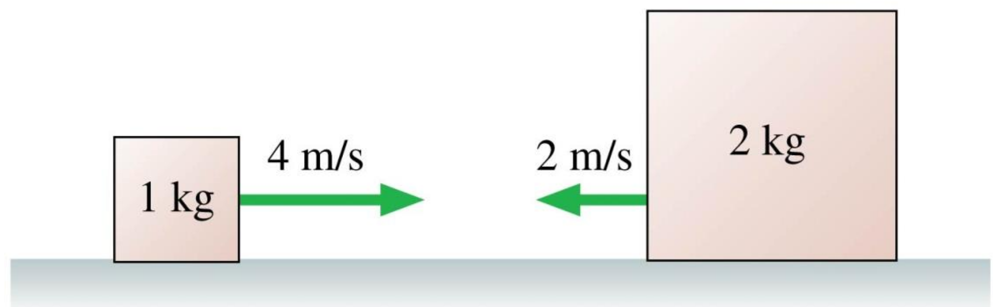
- A **perfectly inelastic collision** is a collision in which the two objects stick together and move with a common final velocity.
- Examples of perfectly inelastic collisions include clay hitting the floor and a bullet embedding itself in wood.



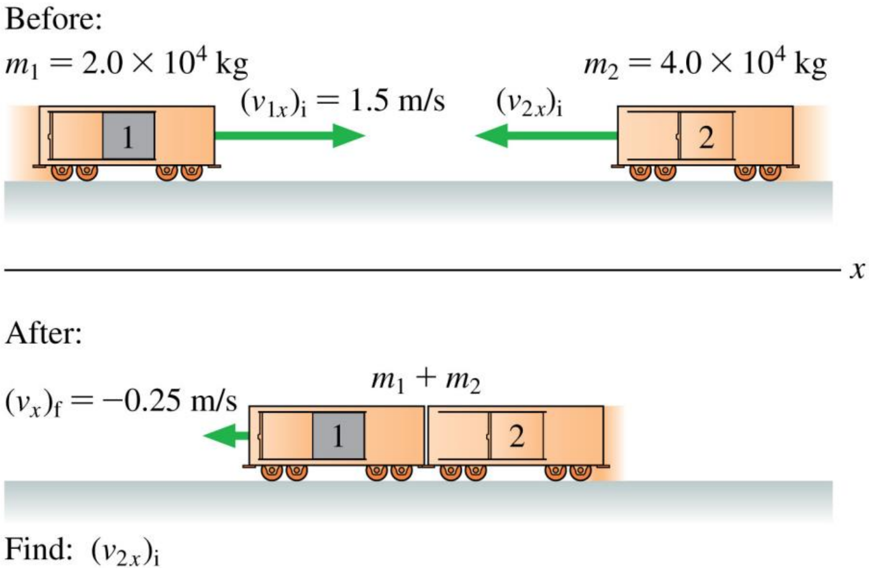
QuickCheck 9.9

- The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the
two boxes is

A. 2 m/s to the left
B. 1 m/s to the left
C. 0 m/s, at rest
D. 1 m/s to the right
E. 2 m/s to the right



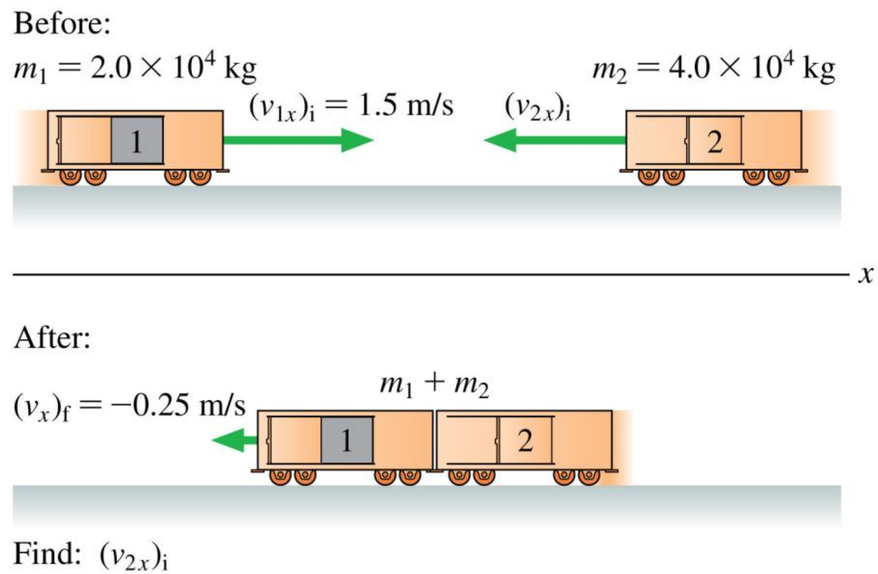
Example 9.8 A perfectly inelastic collision of railroad cars



In assembling a train from several railroad cars, two of the cars, with masses 2.0×10^4 kg and 4.0×10^4 kg, are rolled toward each other. When they meet, they couple and stick together. The lighter car has an initial speed of 1.5 m/s; the collision causes it to reverse direction at 0.25 m/s. What was the initial speed of the heavier car?

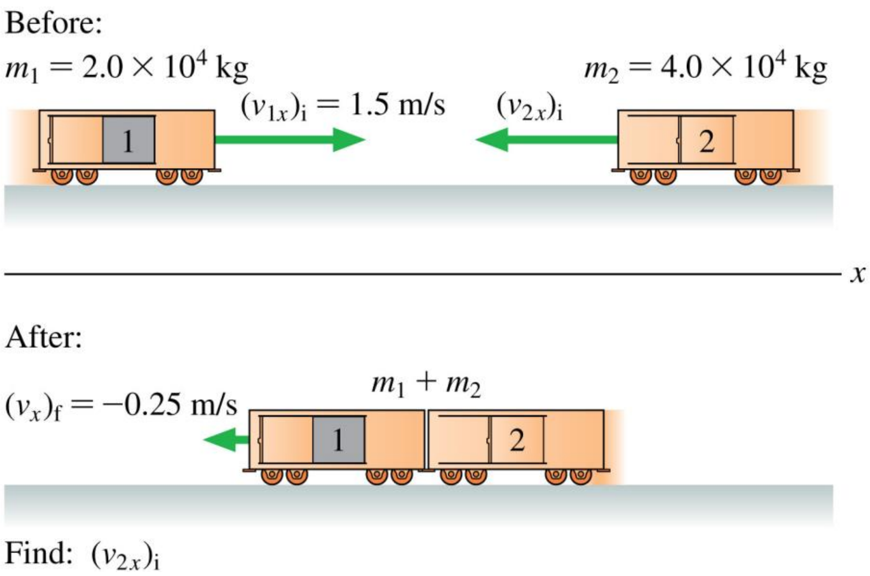
Example 9.8 A perfectly inelastic collision of railroad cars (cont.)

PREPARE We model the cars as particles and define the two cars as the system. This is an isolated system, so its total momentum is conserved in the collision. The cars stick together, so this is a perfectly inelastic collision.



The figure shows a visual overview. We've chosen to let the 2.0×10^4 kg car (car 1) start out moving to the right, so $(v_{1x})_i$ is a positive 1.5 m/s. The cars move to the left after the collision, so their common final velocity is $(v_x)_f = -0.25$ m/s. You can see that velocity $(v_{2x})_i$ must be negative in order to "turn around" both cars.

Example 9.8 A perfectly inelastic collision of railroad cars (cont.)



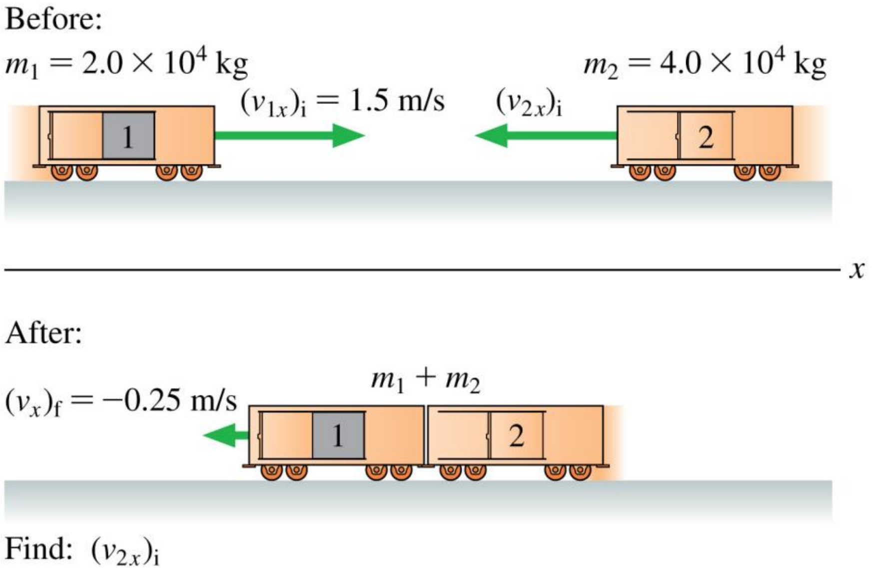
SOLVE The law of conservation of momentum, $(P_x)_f = (P_x)_i$, is

$$(m_1 + m_2)(v_x)_f = m_1(v_{1x})_i + m_2(v_{2x})_i$$

where we made use of the fact that the combined mass $m_1 + m_2$ moves together after the collision.

Example 9.8 A perfectly inelastic collision of railroad cars (cont.)

We can easily solve for the initial velocity of the 4.0×10^4 kg car:

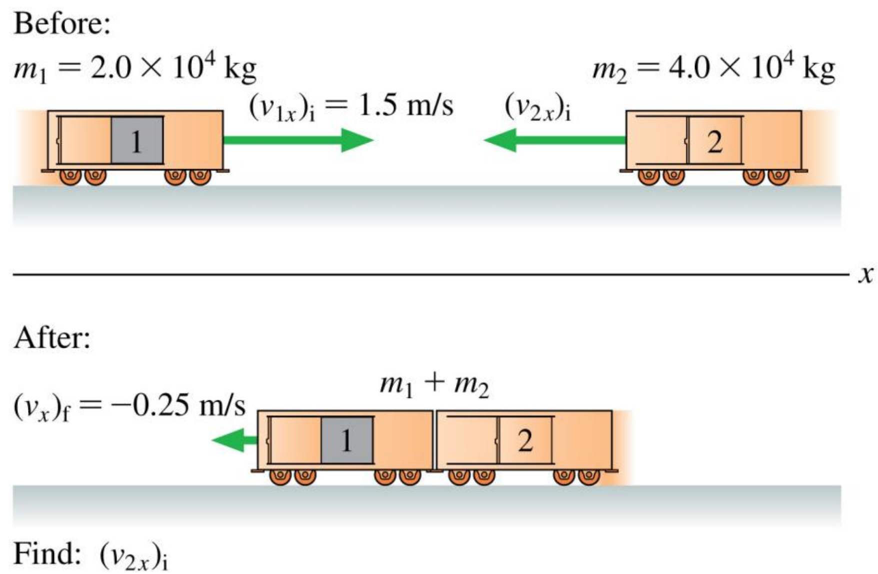


$$(v_{2x})_i = \frac{(m_1 + m_2)(v_x)_f - m_1(v_{1x})_i}{m_2}$$

$$= \frac{(6.0 \times 10^4 \text{ kg})(-0.25 \text{ m/s}) - (2.0 \times 10^4 \text{ kg})(1.5 \text{ m/s})}{4.0 \times 10^4 \text{ kg}}$$

$$= -1.1 \text{ m/s}$$

Example 9.8 A perfectly inelastic collision of railroad cars (cont.)



The negative sign, which we anticipated, indicates that the heavier car started out moving to the left. The initial speed of the car, which we were asked to find, is 1.1 m/s.

ASSESS The key step in solving inelastic collision problems is that both objects move after the collision with the same velocity. You should thus choose a single symbol (here, $(v_x)_f$) for this common velocity.

Reading Question 9.1

Impulse is

A. A force that is applied at a random time.
B. A force that is applied very suddenly.
C. The area under the force curve in a force-versus-time graph.
D. The time interval that a force lasts.

Reading Question 9.3

The total momentum of a system is conserved

A. Always.
B. If no external forces act on the system.
C. If no internal forces act on the system.
D. Never; it's just an approximation.