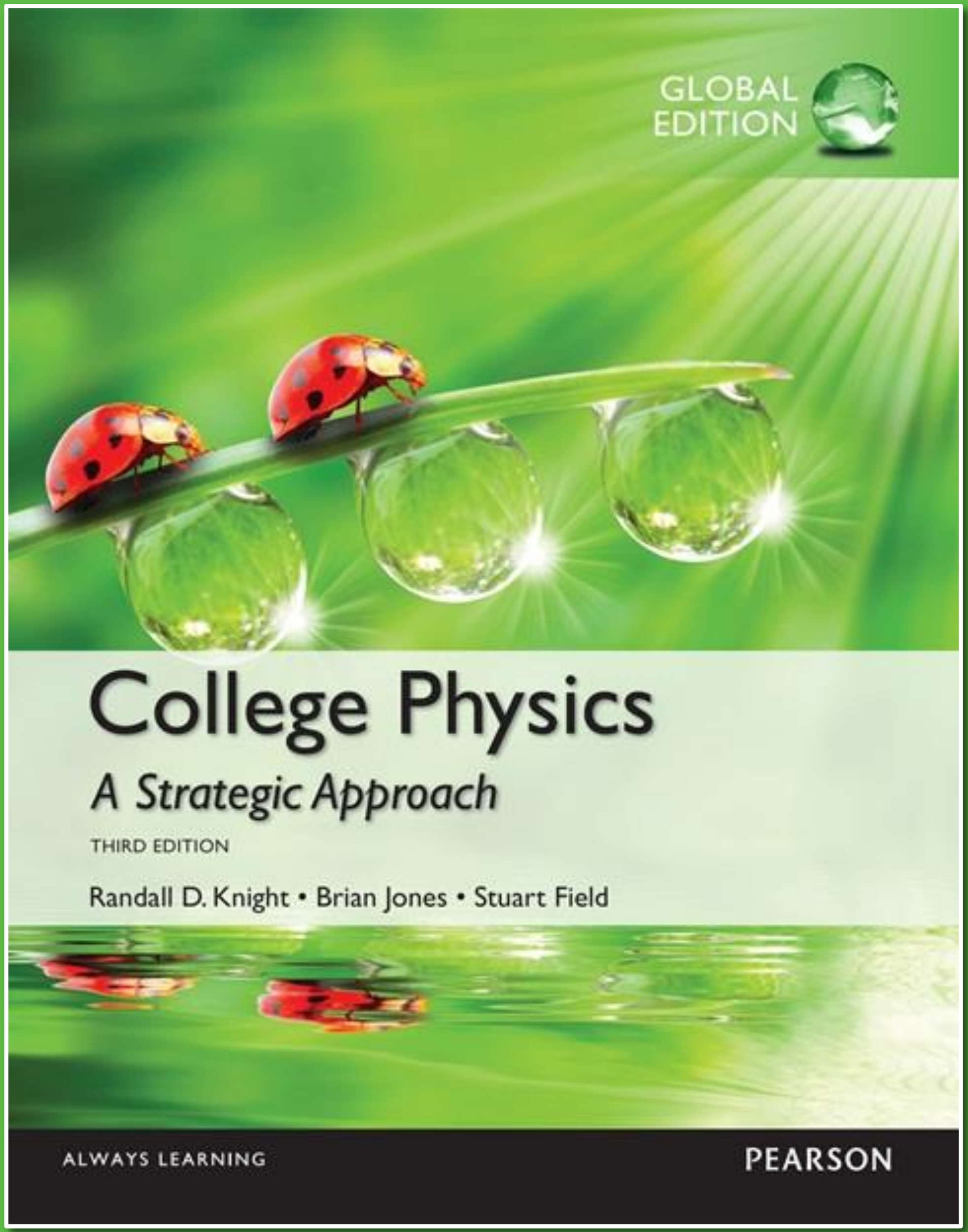


Lecture Presentation

Chapter 8

Equilibrium ~~and Elasticity~~

Chapter 8 Equilibrium

Section 8.1 Torque and Static Equilibrium
Section 8.2 Stability and Balance

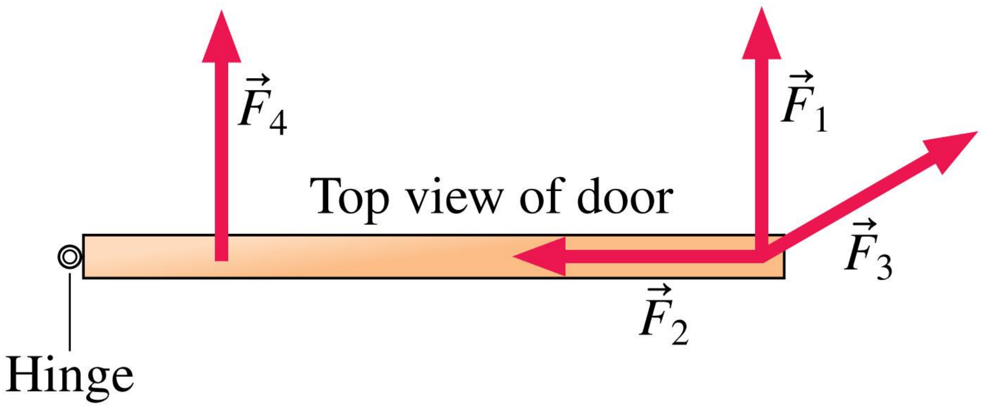
Introduction to the torque

- To start something moving, apply a force. To start something rotating, apply a **torque**, as the sailor is doing to the wheel.

- You’ll see that torque depends on how hard you push and also on *where* you push. A push far from the axle gives a large torque.

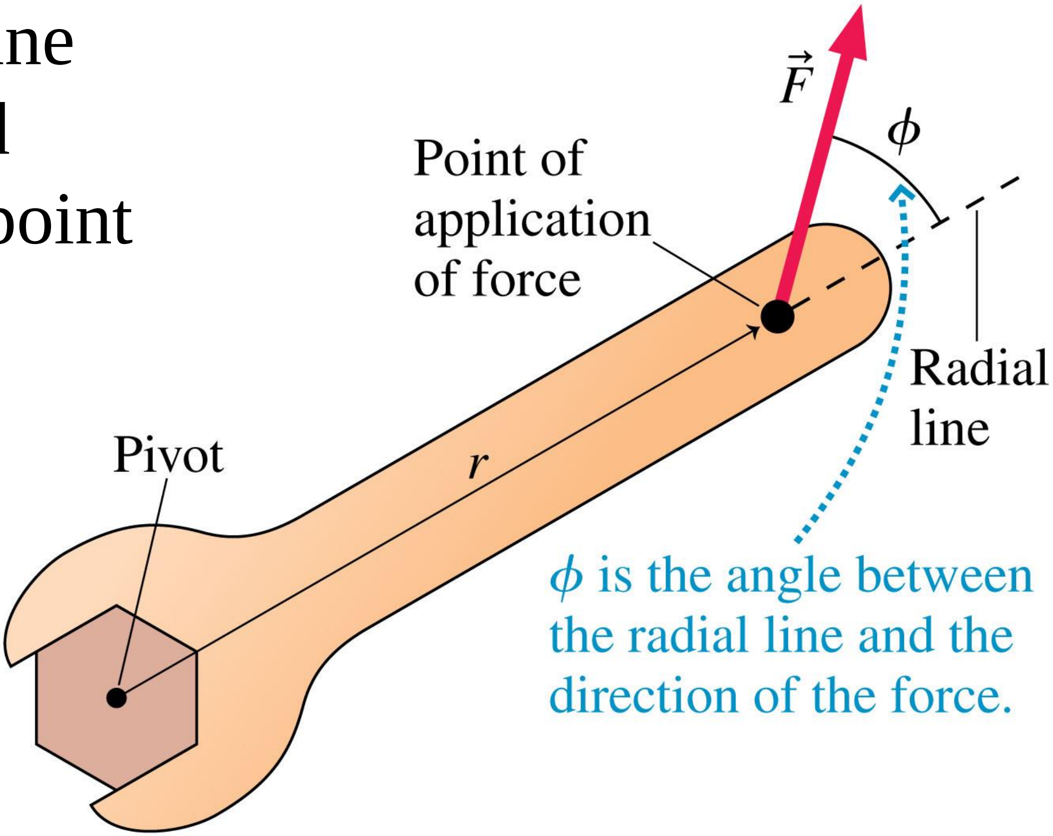
Torque

- Forces with equal strength will have different effects on a swinging door.
- The ability of a force to cause rotation depends on
  - The magnitude *F* of the force.
  - The distance *r* from the pivot—the axis about which the object can rotate—to the point at which force is applied.
  - The angle at which force is applied.



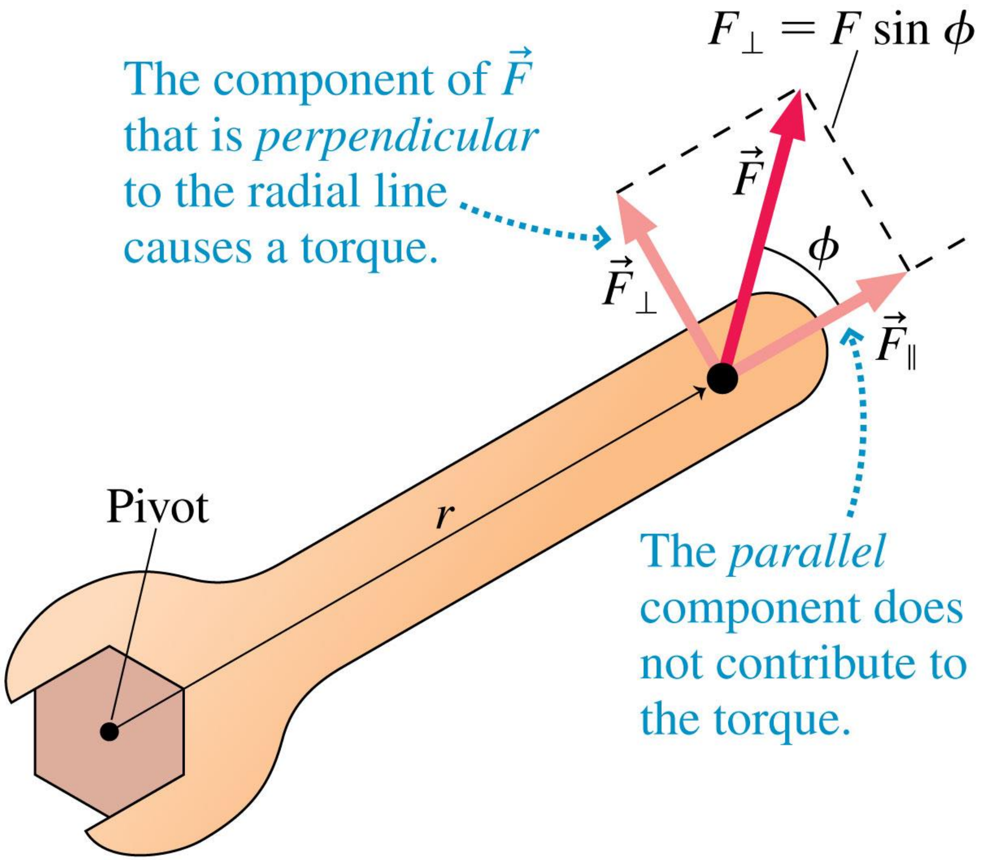
Torque

- The **radial line** is the line starting at the pivot and extending through the point where force is applied.
- The angle ϕ is measured from the radial line to the direction of the force.



Torque

- The **radial line** is the line starting at the pivot and extending through the point where force is applied.
- The angle ϕ is measured from the radial line to the direction of the force.
- Torque is dependent on the perpendicular component of the force being applied.



Torque

- **Torque (τ) is the rotational equivalent of force.**

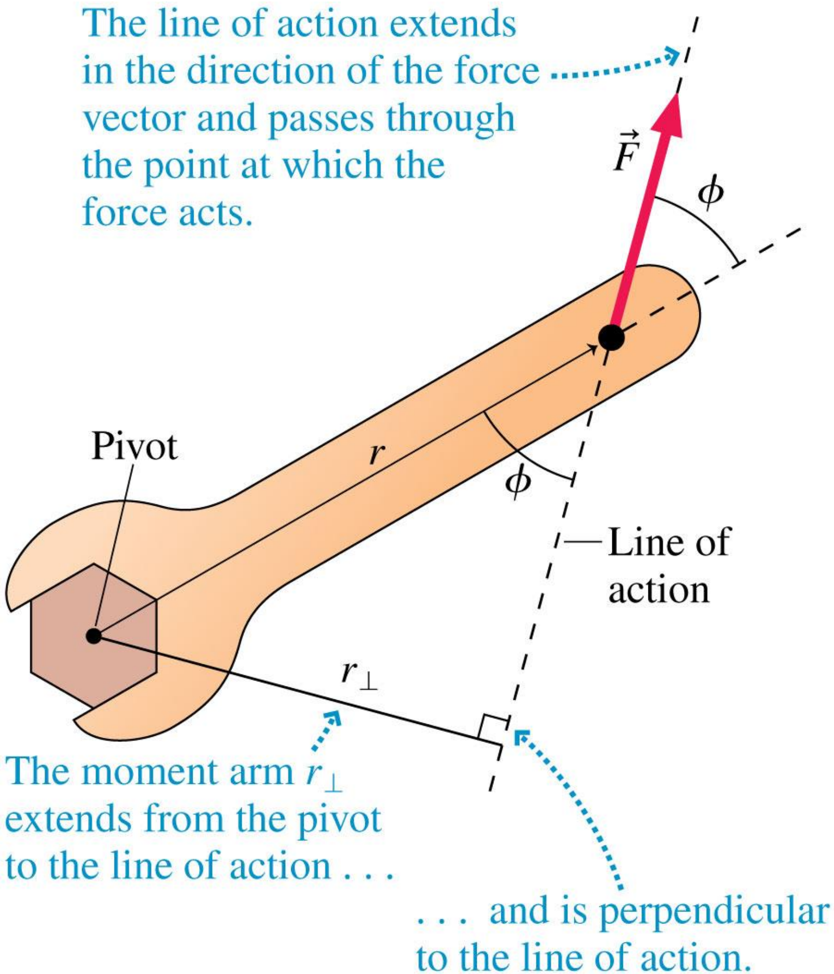
$$\tau = rF_{\perp}$$

Torque due to a force with perpendicular component $F_{\perp}$ acting at a distance r from the pivot

- Torque units are newton-meters, abbreviated $\text{N} \cdot \text{m}$.

Torque

- An alternate way to calculate torque is in terms of the moment arm.
- The **moment arm** (or lever arm) is the perpendicular distance from the *line of action* to the pivot.
- The *line of action* is the line that is in the direction of the force and passes through the point at which the force acts.



Torque

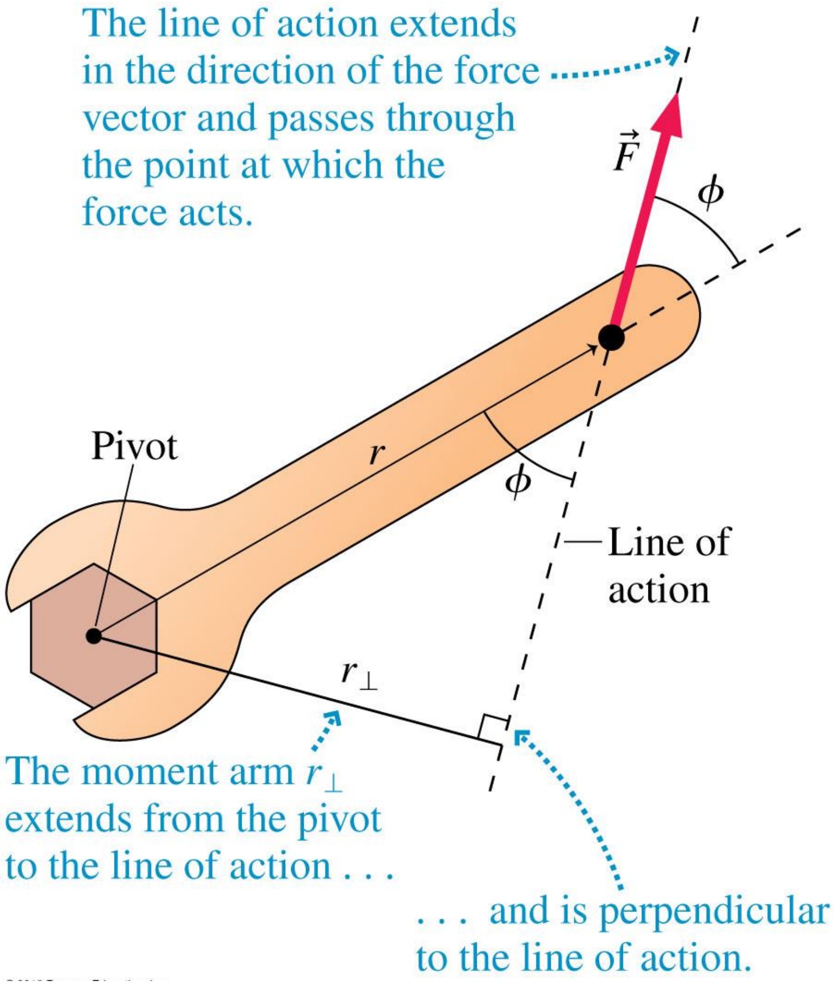
- The equivalent expression for torque is

$$\tau = r_{\perp} F$$

Torque due to a force F with moment arm $r_{\perp}$

- For both methods for calculating torque, the resulting expression is the same:

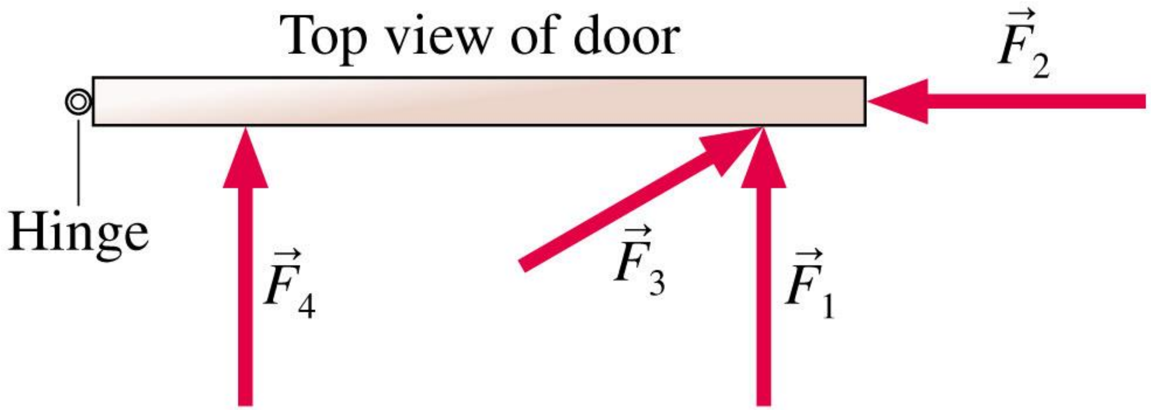
$$\tau = rF\sin\phi$$



QuickCheck

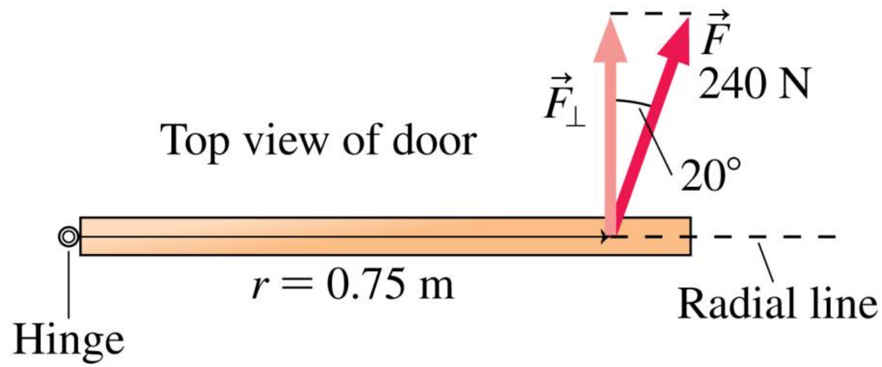
- The four forces shown have the same strength. Which force would be most effective in opening the door?

A. Force F_1
B. Force F_2
C. Force F_3
D. Force F_4
E. Either F_1 or F_3



Example: Torque in opening a door

Ryan is trying to open a stuck door. He pushes it at a point 0.75 m from the hinges with a 240 N force directed 20° away from being perpendicular to the door. There's a natural pivot point, the hinges. What torque does Ryan exert? How could he exert more torque?

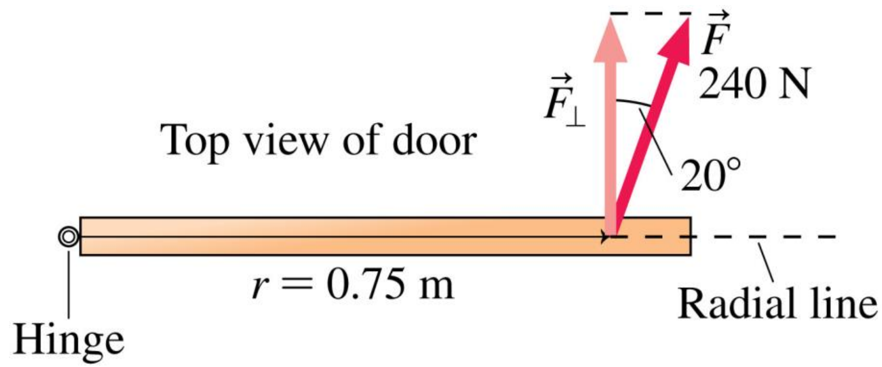


PREPARE In the figure, the radial line is shown drawn from the pivot—the hinge—through the point at which the force $\vec{F}$ is applied. We see that the component of $\vec{F}$ that is perpendicular to the radial line is $F_\perp$ = F cos 20° = 226 N. The distance from the hinge to the point at which the force is applied is r = 0.75 m.

Example: Torque in opening a door (cont.)

SOLVE We can find the torque on the door from Equation 7.10:

$$\tau = rF_{\perp} = (0.75\ \text{m})(226\ \text{N})$$
$$= 170\ \text{N} \cdot \text{m}$$

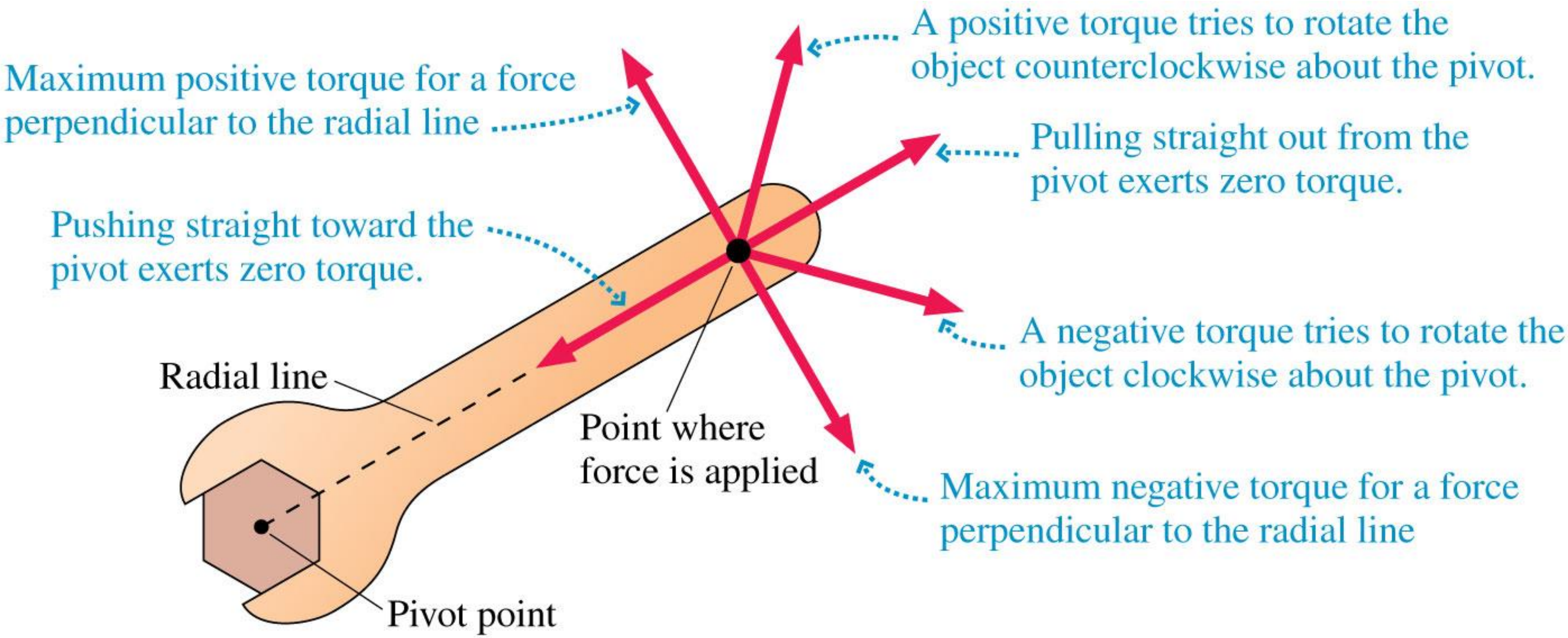


The torque depends on how hard Ryan pushes, where he pushes, and at what angle. If he wants to exert more torque, he could push at a point a bit farther out from the hinge, or he could push exactly perpendicular to the door. Or he could simply push harder!

ASSESS As you'll see by doing more problems, 170 N · m is a significant torque, but this makes sense if you are trying to free a stuck door.

Torque

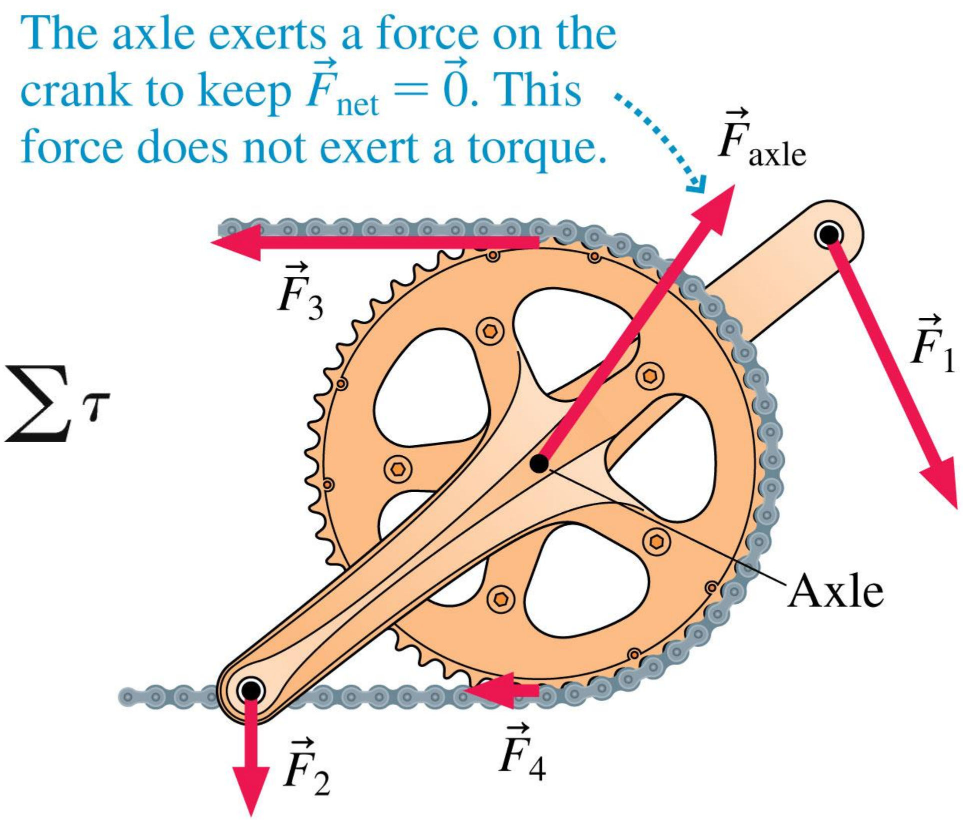
- **A torque that tends to rotate the object in a counter-clockwise direction is positive, while a torque that tends to rotate the object in a clockwise direction is negative.**



Net Torque

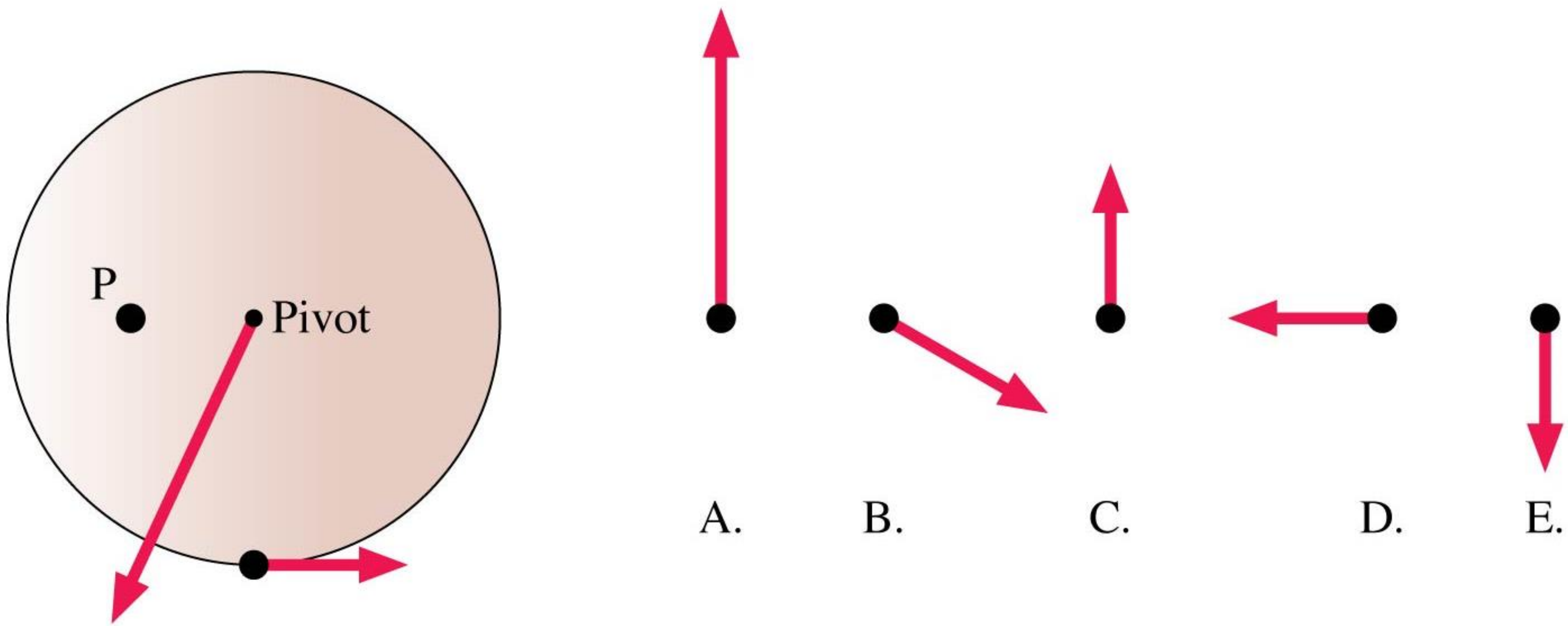
- The *net* torque is the sum of the torques due to the *applied* forces:

$$\tau_{\text{net}} = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \cdots = \sum \tau$$



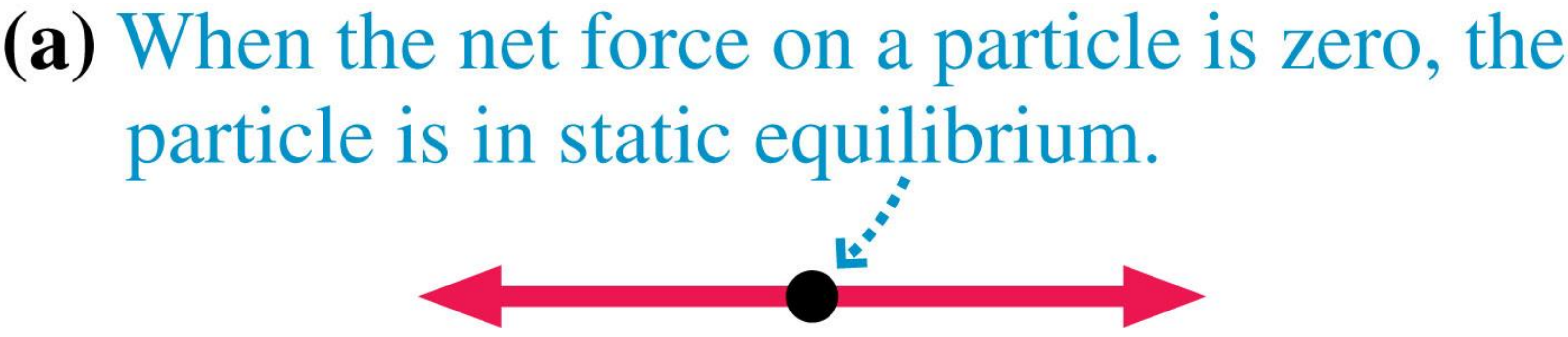
QuickCheck

- Which third force on the wheel, applied at point P, will make the net torque zero?



Section 8.1 Torque and Static Equilibrium

Torque and Static Equilibrium

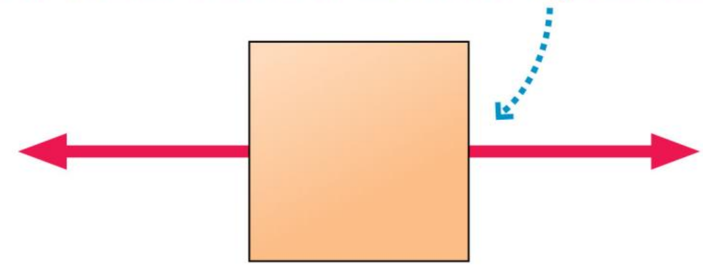


- An object at rest is in *static equilibrium.*
- As long as the object can be modeled as a *particle*, static equilibrium is achieved when the net force on the particle is zero.

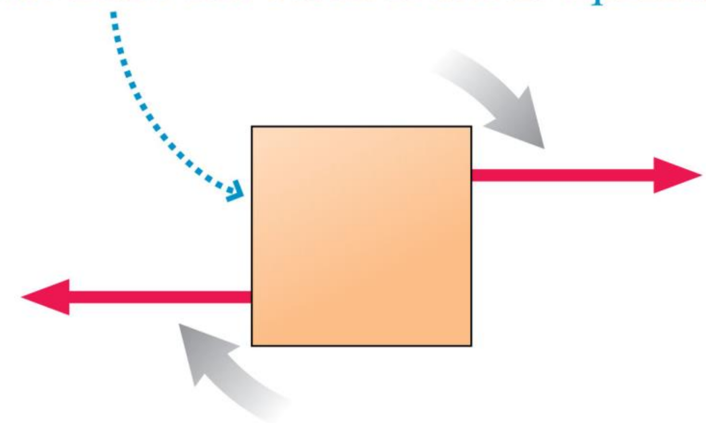
Torque and Static Equilibrium

- For extended objects that can rotate, we must consider the net *torque*, too.
- When the net force and the net torque are zero, the block is in static equilibrium.
- When the net force is zero, but the net torque is *not* zero, the object is *not* in static equilibrium.

(b) Both the net force and the net torque are zero, so the block is in static equilibrium.



(c) The net force is still zero, but the net torque is *not* zero. The block is not in equilibrium.



Torque and Static Equilibrium

- There are two conditions for static equilibrium on an extended object:
  - The net force on the object must be zero.
  - The net torque on the object must be zero.

$$\left.\begin{aligned}\sum F_x &= 0\\ \sum F_y &= 0\end{aligned}\right\} \quad \text{No net force}$$

$$\sum \tau = 0 \quad \} \quad \text{No net torque}$$

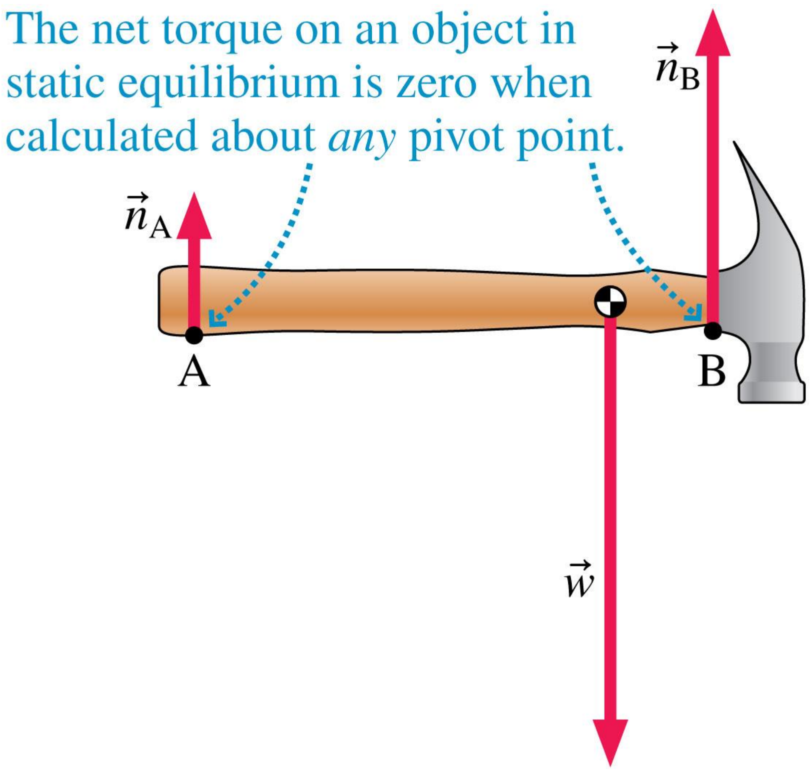
Conditions for static equilibrium of an extended object

QuickCheck 8.1

- Which object is in static equilibrium?

Choosing the Pivot Point

- **For an object in static equilibrium, the net torque about *every* point must be zero.**
- You can choose *any* point you wish as a pivot point for calculating torque.



Choosing the Pivot Point

- Any pivot point will work, but some pivot points can simplify calculations.
- There is a “natural” axis of rotation for many situations. A natural axis is an axis about which rotation *would* occur if the object were not in static equilibrium.

Choosing the Pivot Point

Static equilibrium problems

If an object is in static equilibrium, we can use the fact that there is no net force and no net torque as a basis for solving problems.

PREPARE Model the object as a simple shape. Draw a visual overview that shows all forces and distances. List known information.

- Pick an axis or pivot about which the torques will be calculated.
- Determine the torque about this pivot point due to each force acting on the object. The torques due to any forces acting *at* the pivot are zero.
- Determine the sign of each torque about this pivot point.

SOLVE The mathematical steps are based on the conditions:

$$\vec{F}_{\text{net}} = \vec{0} \qquad \text{and} \qquad \tau_{\text{net}} = 0$$

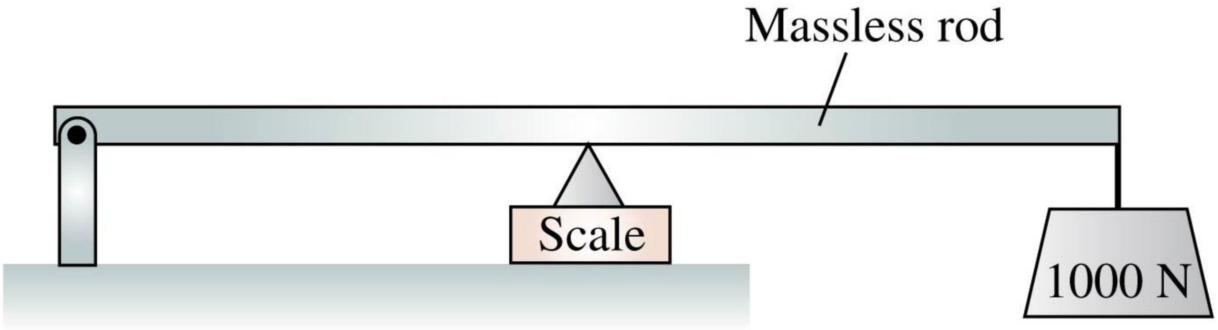
- Write equations for $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau = 0$.
- Solve the resulting equations.

ASSESS Check that your result is reasonable and answers the question.

QuickCheck 8.2

- What does the scale read?

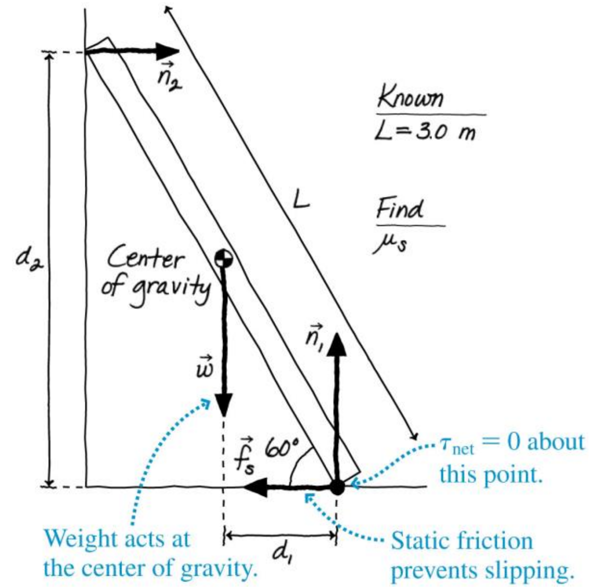
A. 500 N
B. 1000 N
C. 2000 N
D. 4000 N



Answering this requires <u>reasoning</u>, not calculating.

Example 8.4 Will the ladder slip?

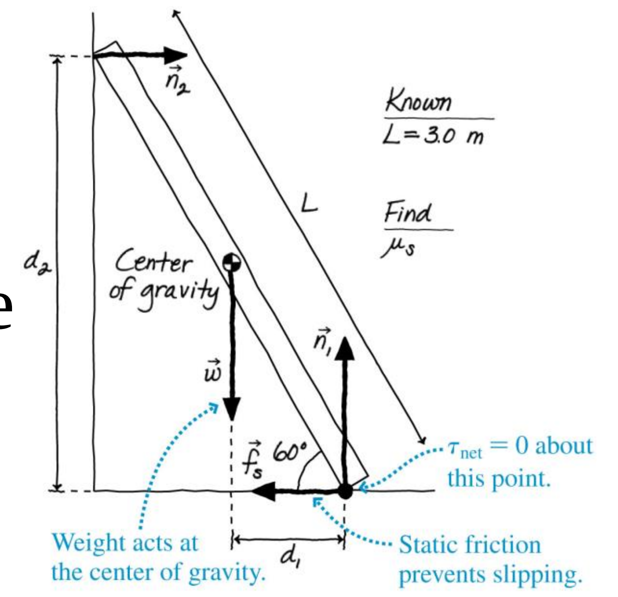
A 3.0-m-long ladder leans against a wall at an angle of 60° with respect to the floor. What is the minimum value of μ_s , the coefficient of static friction with the ground, that will prevent the ladder from slipping? Assume that friction between the ladder and the wall is negligible.



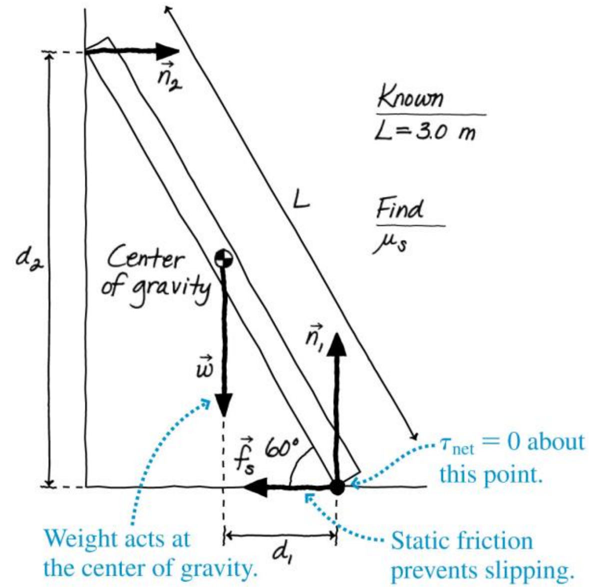
Example 8.4 Will the ladder slip? (cont.)

PREPARE The ladder is a rigid rod of length *L*. To not slip, both the net force and net torque on the ladder must be zero. The figure shows the ladder and the on it.

We are asked to find the necessary coefficient of static friction.



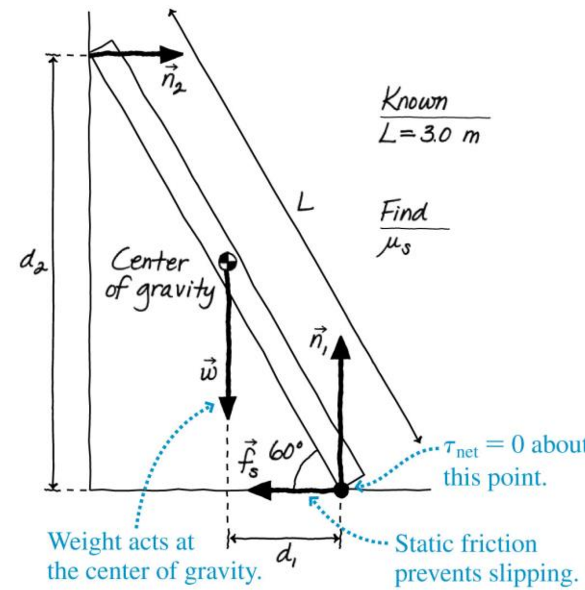
Example 8.4 Will the ladder slip? (cont.)



First, we'll solve for the magnitudes of the static friction force and the normal force. Then we can use these values to determine the necessary value of the coefficient of friction. These forces both act at the bottom corner of the ladder, so even though we are interested in these forces, this is a good choice for the pivot point because two of the forces that act provide no torque, which simplifies the solution.

Example 8.4 Will the ladder slip? (cont.)

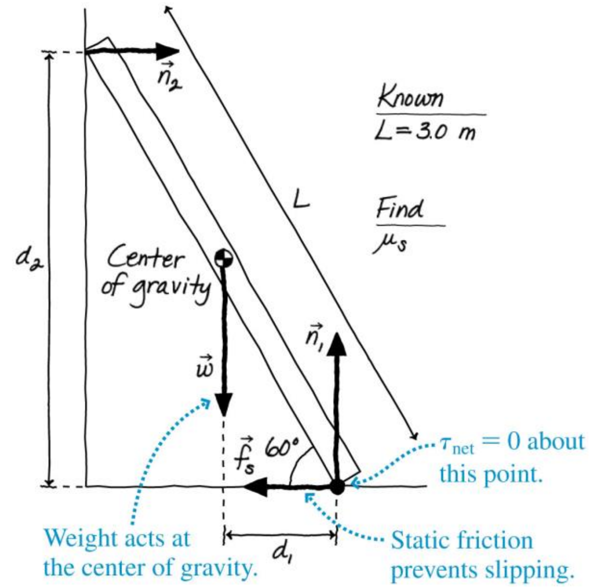
With this choice of pivot, the weight of the ladder, acting at the center of gravity, exerts torque $d_1 w$ and the force of the wall exerts torque $-d_2 n_2$. The signs are based on the observation that $\vec{w}$ would cause the ladder to rotate counterclockwise, while $\vec{n}_2$ would cause it to rotate clockwise.



Example 8.4 Will the ladder slip? (cont.)

SOLVE The *x*- and *y*-components of $\vec{F}_{\text{net}} = \vec{0}$ are

$$\sum F_x = n_2 - f_s = 0$$
$$\sum F_y = n_1 - w = n_1 - Mg = 0$$



The torque about the bottom corner is

$$\tau_{\text{net}} = d_1 w - d_2 n_2 = \frac{1}{2}(L\cos 60°)Mg - (L\sin 60°)n_2 = 0$$

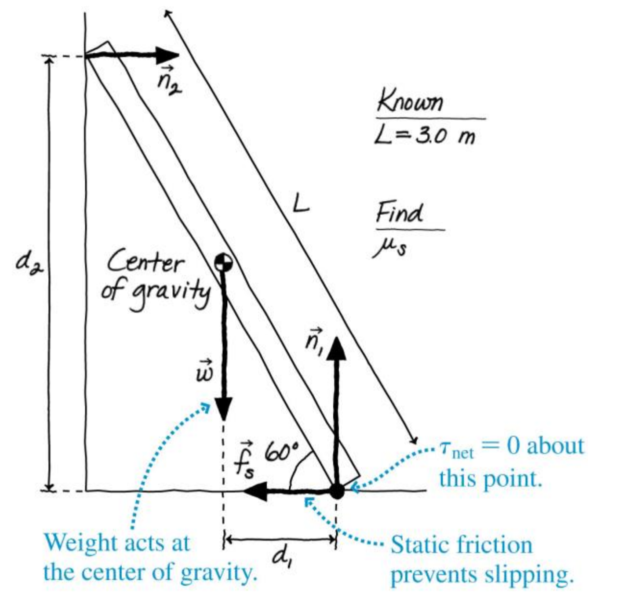
Example 8.4 Will the ladder slip? (cont.)

Altogether, we have three equations with the three unknowns n_1, n_2, and f_s. If we solve the third equation for n_2,

$$n_2 = \frac{\frac{1}{2}(L\cos 60^\circ)Mg}{L\sin 60^\circ} = \frac{Mg}{2\tan 60^\circ}$$

we can then substitute this into the first equation to find

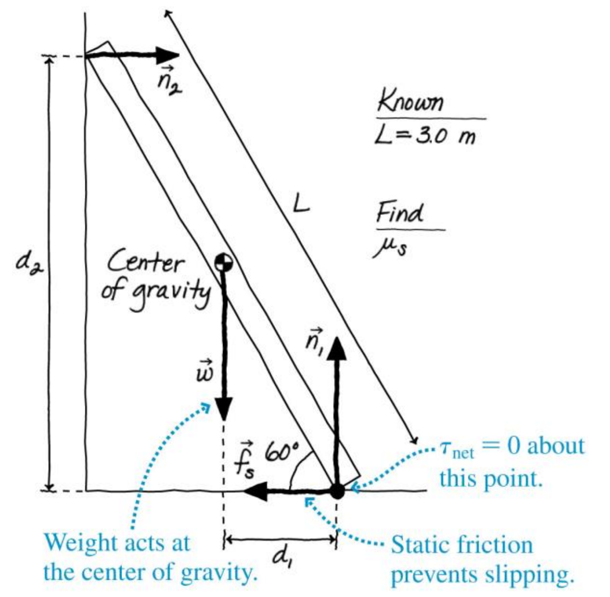
$$f_s = \frac{Mg}{2\tan 60^\circ}$$



Example 8.4 Will the ladder slip? (cont.)

Our model of static friction is $f_s \leq f_{s\,max} = \mu_s n_1$. We can find n_1 from the second equation: $n_1 = Mg$. From this, the model of friction tells us that

$$f_s \leq \mu_s Mg$$



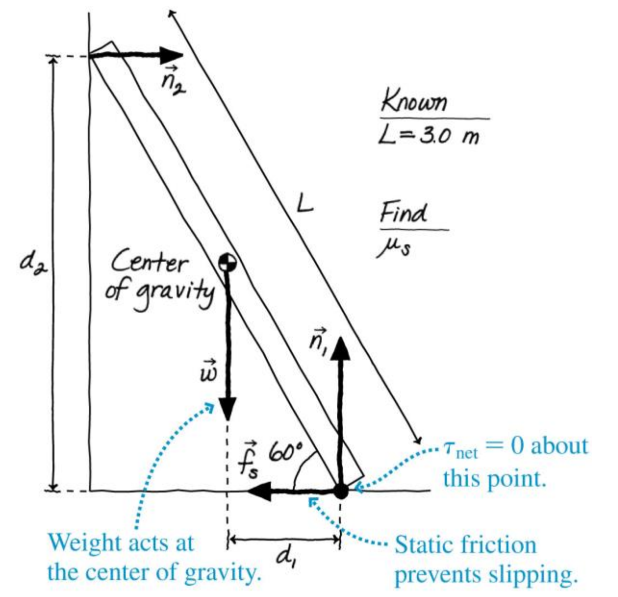
Comparing these two expressions for f_s, we see that μ_s must obey

$$\mu_s \geq \frac{1}{2\tan 60°} = 0.29$$

Thus the minimum value of the coefficient of static friction is 0.29.

Example 8.4 Will the ladder slip? (cont.)

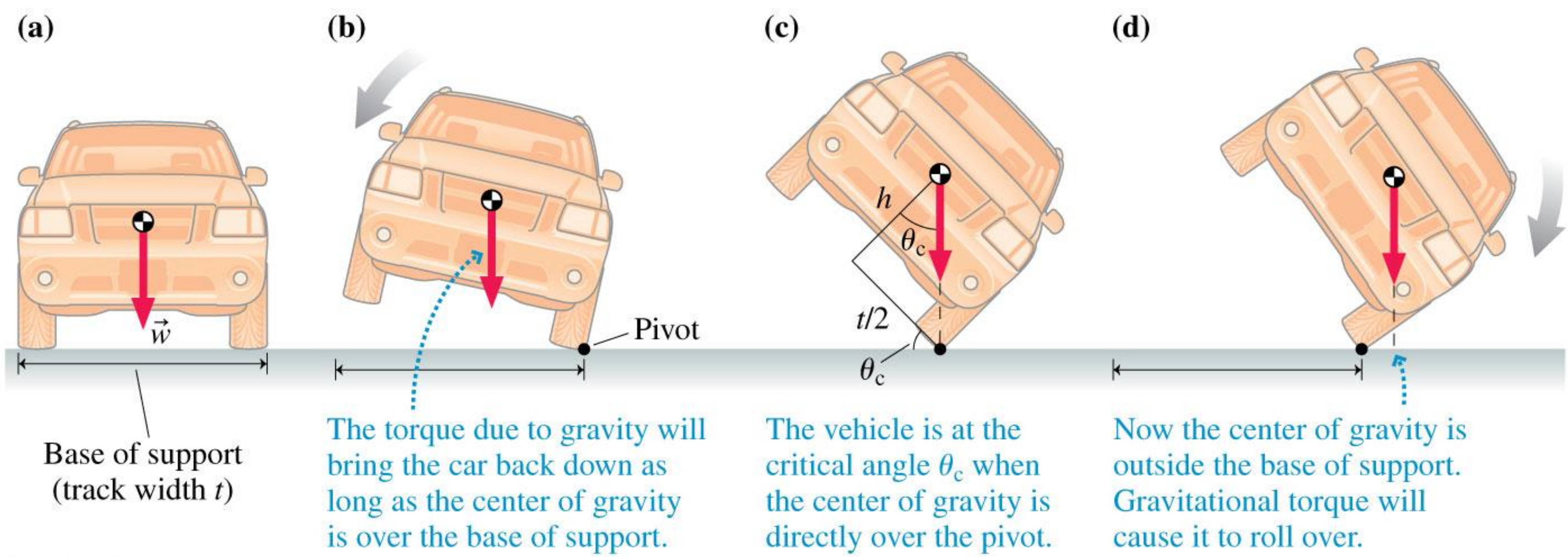
ASSESS You know from experience that you can lean a ladder or other object against a wall if the ground is "rough," but it slips if the surface is too smooth. 0.29 is a "medium" value for the coefficient of static friction, which is reasonable.



Section 8.2 Stability and Balance

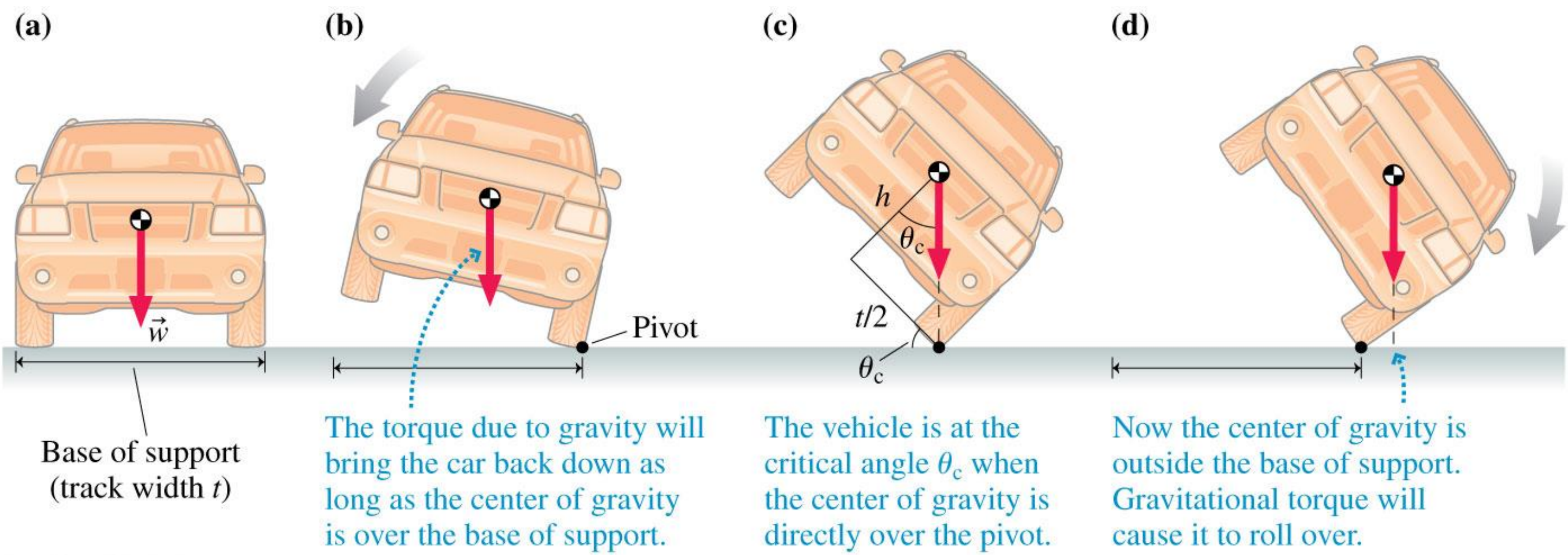
Stability and Balance

- An extended object has a *base of support* on which it rests when in static equilibrium.
- **A wider base of support and/or a lower center of gravity improves stability.**



Stability and Balance

- As long as the object's center of gravity remains over the base of support, torque due to gravity will rotate the object back toward its stable equilibrium position. The object is **stable**.
- If the object's center of gravity moves outside the base of support, the object is **unstable.**



Reading Question 8.1

An object is in equilibrium if

A. $\vec{F}_{\text{net}} = \vec{0}$

B. $\tau_{\text{net}} = 0$

C. Either A or B

D. Both A and B

Reading Question 8.3

An object will be stable if

A. Its center of gravity is below its highest point.
B. Its center of gravity lies over its base of support.
C. Its center of gravity lies outside its base of support.
D. The height of its center of gravity is less than 1/2 its total height.

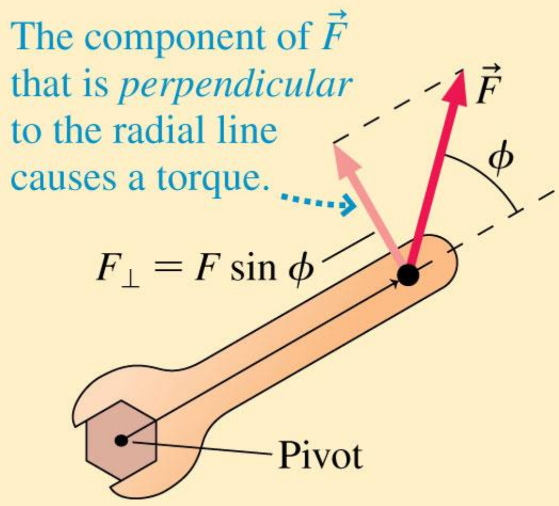
Summary: Important Concepts

Torque

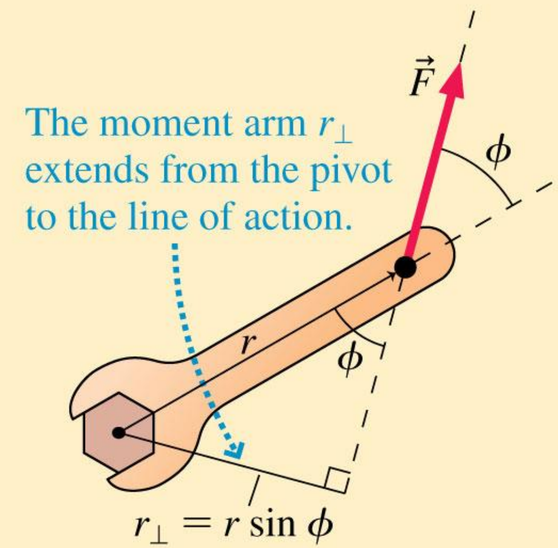
A force causes an object to undergo a linear acceleration, a torque causes an object to undergo an angular acceleration.

There are two interpretations of torque:

Interpretation 1: $\tau = rF_{\perp}$



Interpretation 2: $\tau = r_{\perp} F$



Both interpretations give the same expression for the magnitude of the torque: $\tau = rF \sin \phi$

Summary

GENERAL PRINCIPLES

Static Equilibrium

An object in **static equilibrium** must have no net force on it and no net torque. Mathematically, we express this as

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau = 0$$

Since the net torque is zero about *any* point, the pivot point for calculating the torque can be chosen at any convenient location.

IMPORTANT CONCEPTS

Stability

An object is **stable** if its center of gravity is over its base of support; otherwise, it is **unstable.**