06 – Friction

STATICS, AGE-1330

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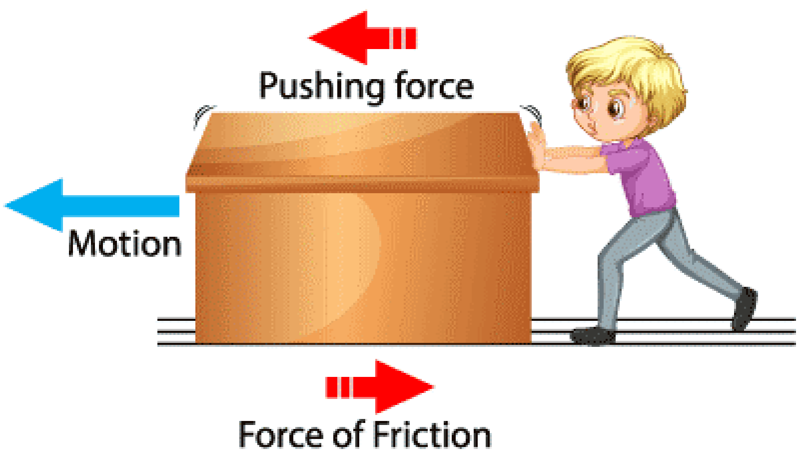
What Is Friction?

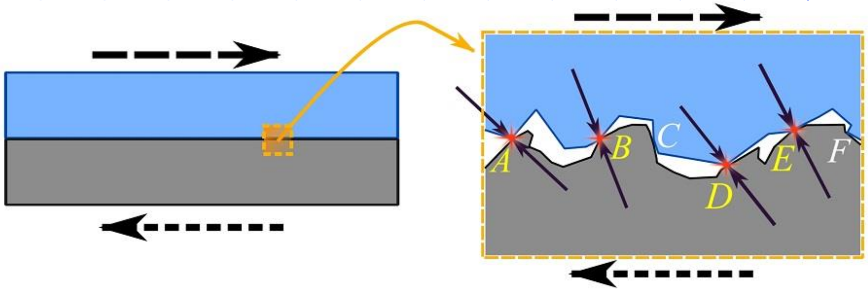
- Force between contacting surfaces
- Opposes motion or tendency to move
- Always present in machines and daily life
- Can cause energy loss as heat
- Needed for walking, driving, braking



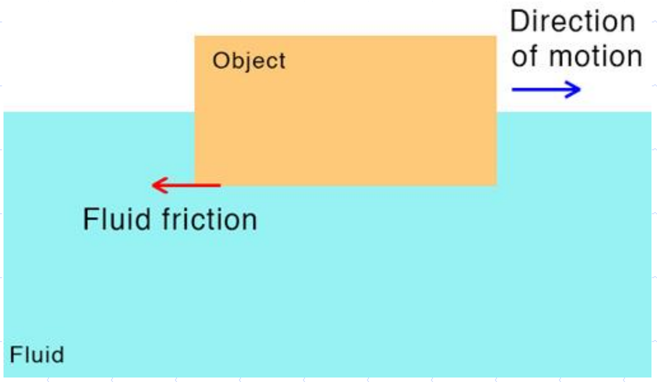
Why Study Friction?

- Many machine parts rely on friction (brakes, clutches, screws)
- Must be considered when accuracy and safety matter
- Lubrication reduces friction and wear
- Situations when friction can be neglected: *ideal*
- Situations when friction must be considered: *real*

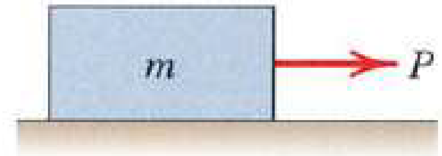
Types of Friction

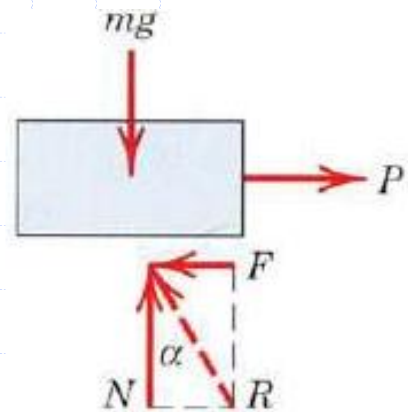


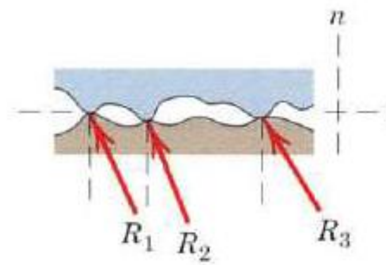
- Dry Friction
  - Contact between unlubricated solids
  - Caused by microscopic surface roughness
  - Friction increases with applied load
- Fluid Friction
  - Surfaces separated by a fluid film
  - Depends on viscosity and relative speed
- Internal Friction
  - Resistance inside materials during deformation
  - Not the focus of this chapter



Dry Friction Basics

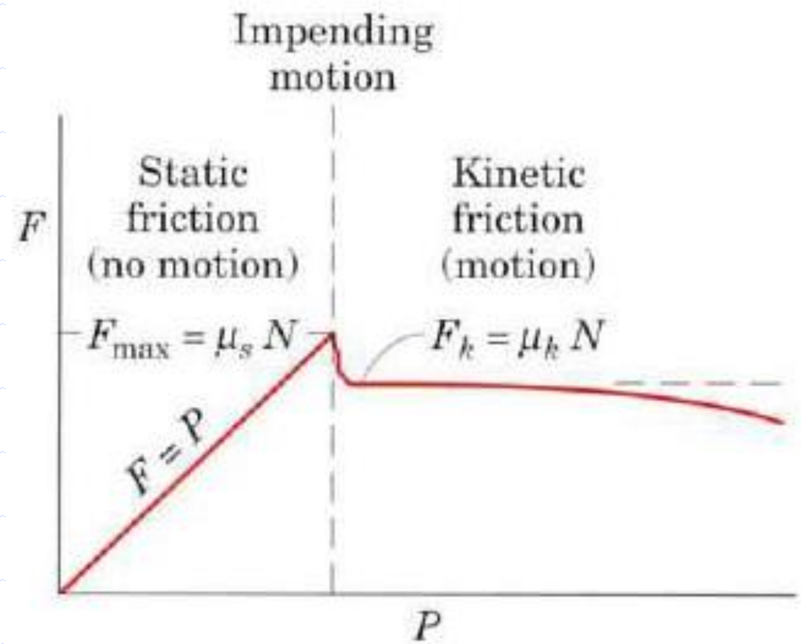






Friction Behavior

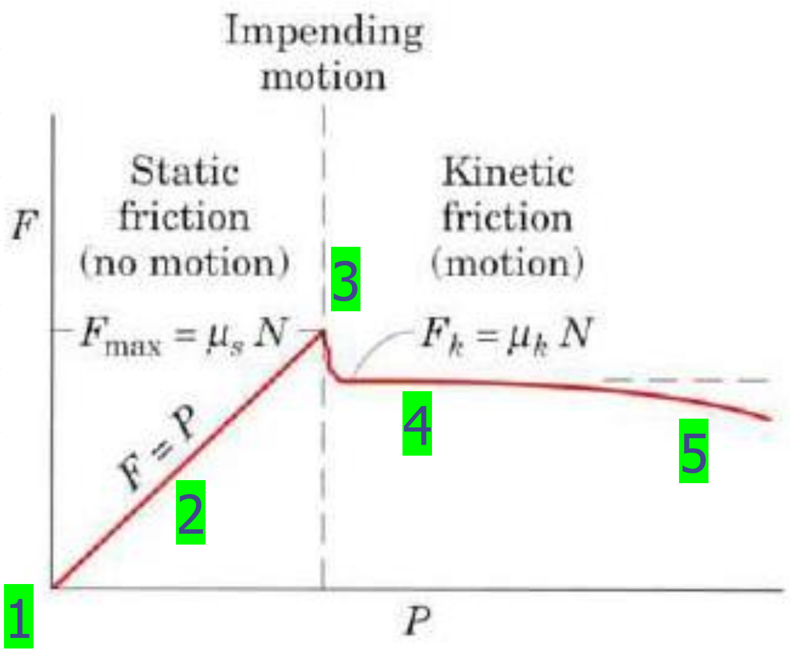
- Applied force increases → friction increases
- Motion begins at the **limiting** friction
- Before sliding: F = applied force
- At impending motion: $F = F_{max}$



Dry Friction Basics

Friction Behavior (cont.)

1. P = 0 → F = **0**
2. P = **F** ($F < \mu_s N$)
3. Impending motion:
   $\boldsymbol{F = F_{max} = \mu_s N}$
   μ_s: coefficient of static friction
4. Motion starts:
   $\boldsymbol{F = F_k = \mu_k N}$
   μ_k: coefficient of kinetic friction
   $\mu_k < \mu_s$
5. Velocity increases →
   F **decreases** (slightly)



Dry Friction Basics

CONTACTING SURFACE	TYPICAL VALUES OF COEFFICIENT OF FRICTION	
	STATIC, μ_s	KINETIC, μ_k
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.04	0.04
Steel on babbitt (dry)	0.4	0.3
Steel on babbitt (greasy)	0.1	0.07
Brass on steel (dry)	0.5	0.4
Brake lining on cast iron	0.4	0.3
Rubber tires on smooth pavement (dry)	0.9	0.8
Wire rope on iron pulley (dry)	0.2	0.15
Hemp rope on metal	0.3	0.2
Metal on ice		0.02

Dry Friction Basics

Friction Angle

- At impending motion:

$$F = F_{max} = \mu_s N$$

$$\tan \phi_s = \frac{F}{N} = \mu_s$$

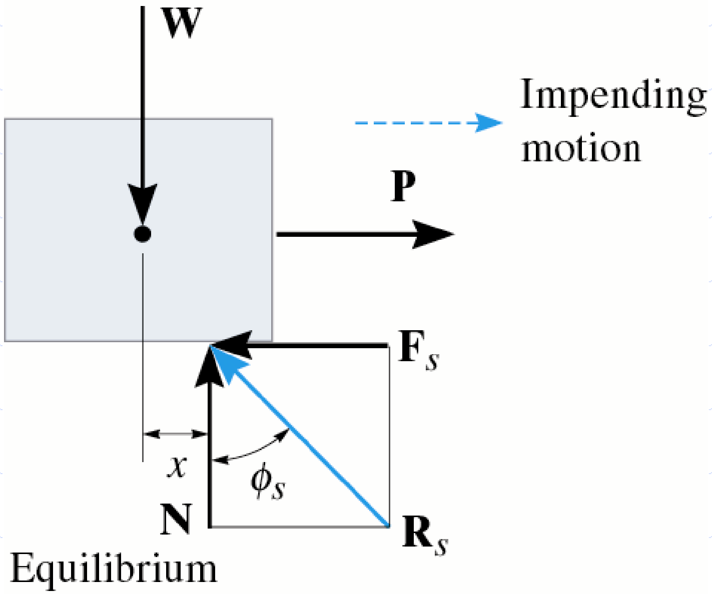
ϕ_s: angle of static friction

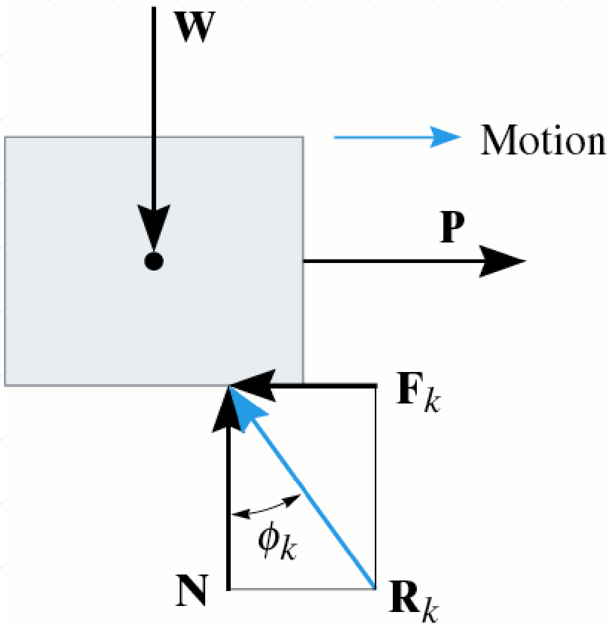
- When motion occurs:

$$F = F_k = \mu_k N$$

$$\tan \phi_k = \frac{F}{N} = \mu_k$$

ϕ_k: angle of kinetic friction

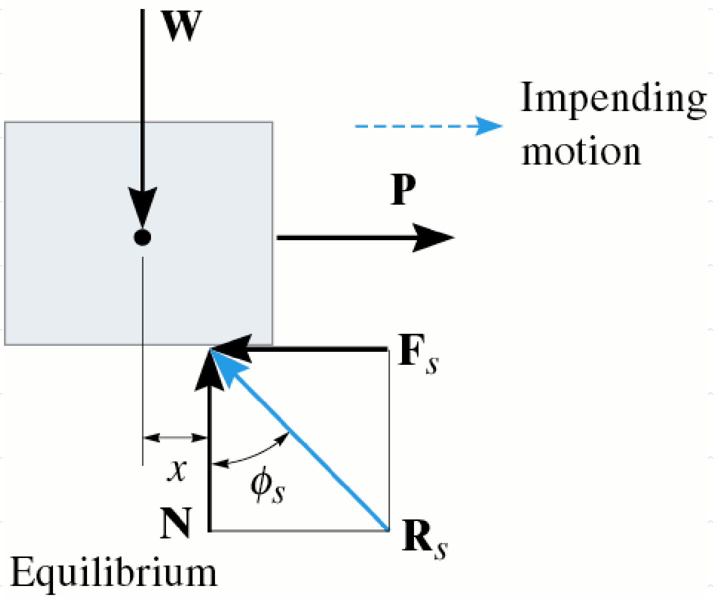




Types of Friction Problems

Category 1 – Limiting Static Friction (F_{max}):

- Object is **about to** move (i.e. impending motion)
- Equilibrium conditions apply
- Use $F = F_{max} = \mu_s N$



Types of Friction Problems

Category 2 – Condition unknown:

- Motion or no motion? We don't know (at first)
- So what to do?
- First: assume equilibrium
- Second: find F and compare: $\boldsymbol{F}$ vs. $\boldsymbol{F_{max} = \mu_s N}$

a) If $\boldsymbol{F} < (\boldsymbol{F_{max} = \mu_s N}) \rightarrow$ body is in static equilibrium ✔
$\rightarrow$ determine $\boldsymbol{F}$ using eqns. of equilibrium (only)

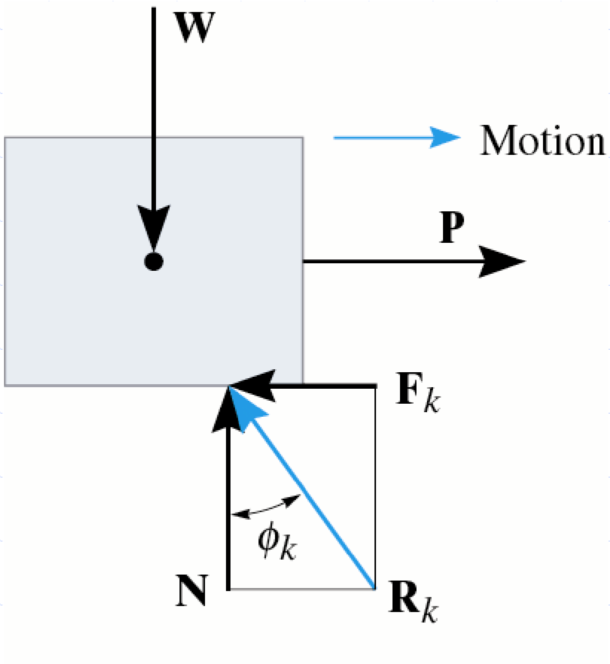
b) If $\boldsymbol{F} = (\boldsymbol{F_{max} = \mu_s N}) \rightarrow$ body is in static equilibrium ✔
$\rightarrow$ motion is impending $\rightarrow$ use equilibrium + $\boldsymbol{F_{max}}$

c) If $\boldsymbol{F} > (\boldsymbol{F_{max} = \mu_s N}) \rightarrow$ impossible condition! $\rightarrow$ body is *not* in static equilibrium $\rightarrow$ use: $\boldsymbol{F = F_k = \mu_k N}$

Types of Friction Problems

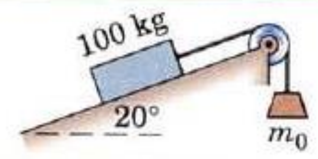
Category 3 – Kinetic Motion:

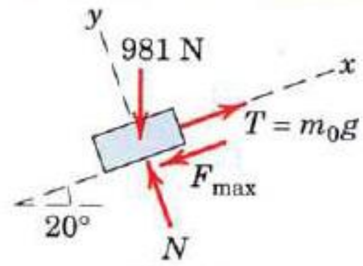
- Surfaces already sliding
- Use: $F = F_k = \mu_k N$



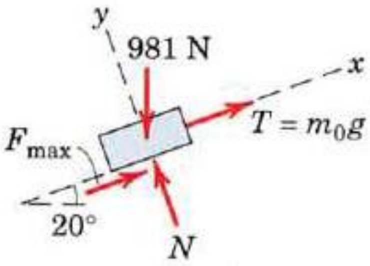
Sample Problem 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.





Case I



Case II

Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

$[\Sigma F_y = 0]$ $\quad N - 981 \cos 20° = 0 \quad N = 922$ N

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.30(922) = 277$ N

$[\Sigma F_x = 0]$ $\quad m_0(9.81) - 277 - 981 \sin 20° = 0 \quad m_0 = 62.4$ kg *Ans.*

The minimum value of m_0 is determined when motion is impending down the
① plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x-direction requires

$[\Sigma F_x = 0]$ $\quad m_0(9.81) + 277 - 981 \sin 20° = 0 \quad m_0 = 6.01$ kg *Ans.*

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

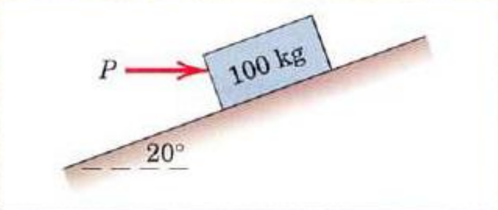
In both cases equilibrium requires that the resultant of F_{max} and N be concurrent with the 981-N weight and the tension T.

Helpful Hint

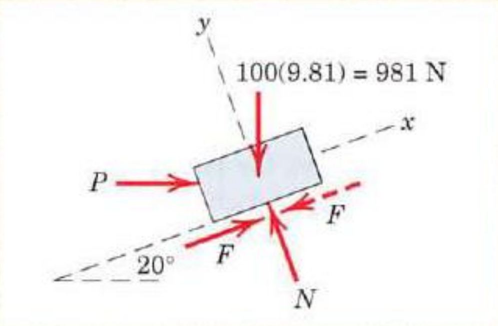
① We see from the results of Sample Problem 6/1 that the block would slide down the incline without the restraint of attachment to m_0 since $\tan 20° > 0.30$. Thus, a value of m_0 will be required to maintain equilibrium.

Sample Problem 6/3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500$ N and, second, $P = 100$ N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Solution. There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P. It is therefore necessary that we make an assumption, so we will take the friction force to be up the plane, as shown by the solid arrow. From the free-body diagram a balance of forces in both x- and y-directions gives



$[\Sigma F_x = 0]$ $$P \cos 20^\circ + F - 981 \sin 20^\circ = 0$$

$[\Sigma F_y = 0]$ $$N - P \sin 20^\circ - 981 \cos 20^\circ = 0$$

Case I. $P = 500$ N

Substitution into the first of the two equations gives

$$F = -134.3 \text{ N}$$

The negative sign tells us that *if* the block is in equilibrium, the friction force acting on it is in the direction opposite to that assumed and therefore is down the plane, as represented by the dashed arrow. We cannot reach a conclusion on the magnitude of F, however, until we verify that the surfaces are capable of supporting 134.3 N of friction force. This may be done by substituting $P = 500$ N into the second equation, which gives

$$N = 1093 \text{ N}$$

The maximum static friction force which the surfaces can support is then

$[F_{max} = \mu_s N]$ $$F_{max} = 0.20(1093) = 219 \text{ N}$$

Since this force is greater than that required for equilibrium, we conclude that the assumption of equilibrium was correct. The answer is, then,

$$F = 134.3 \text{ N down the plane}$$ *Ans.*

Case II. $P = 100$ N

Substitution into the two equilibrium equations gives

$$F = 242 \text{ N} \qquad N = 956 \text{ N}$$

But the maximum possible static friction force is

$$[F_{max} = \mu_s N] \qquad F_{max} = 0.20(956) = 191.2 \text{ N}$$

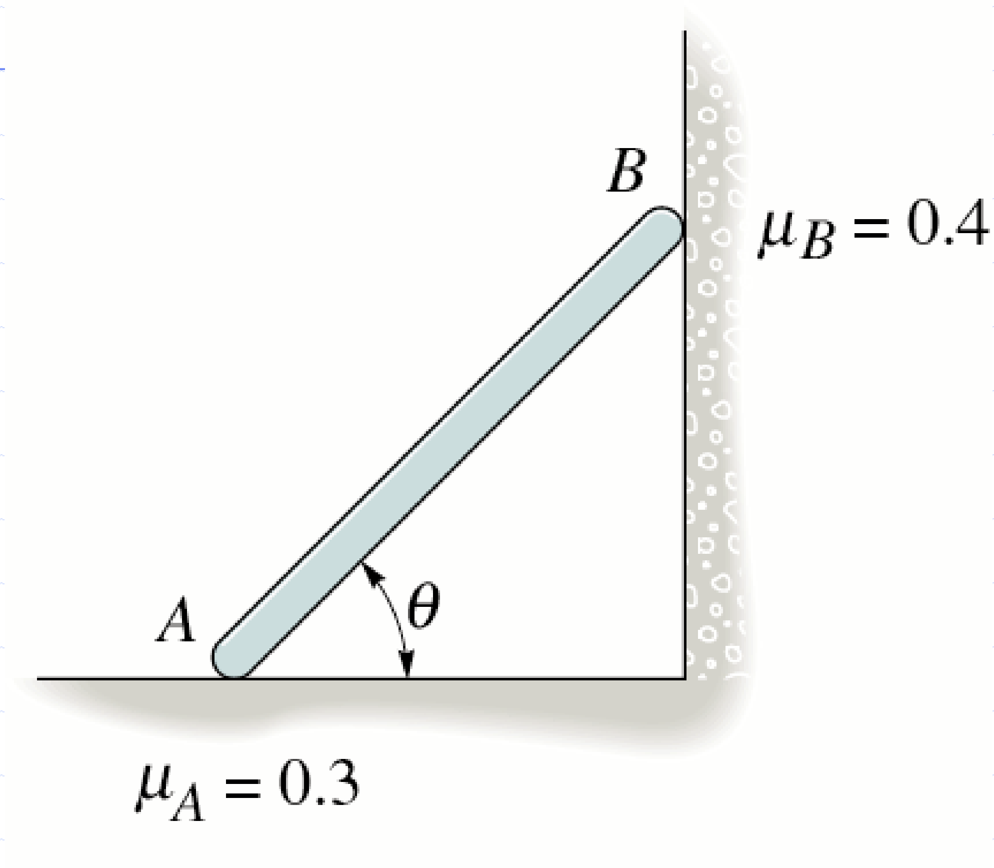
It follows that 242 N of friction cannot be supported. Therefore, equilibrium ① cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane. Hence, the answer is

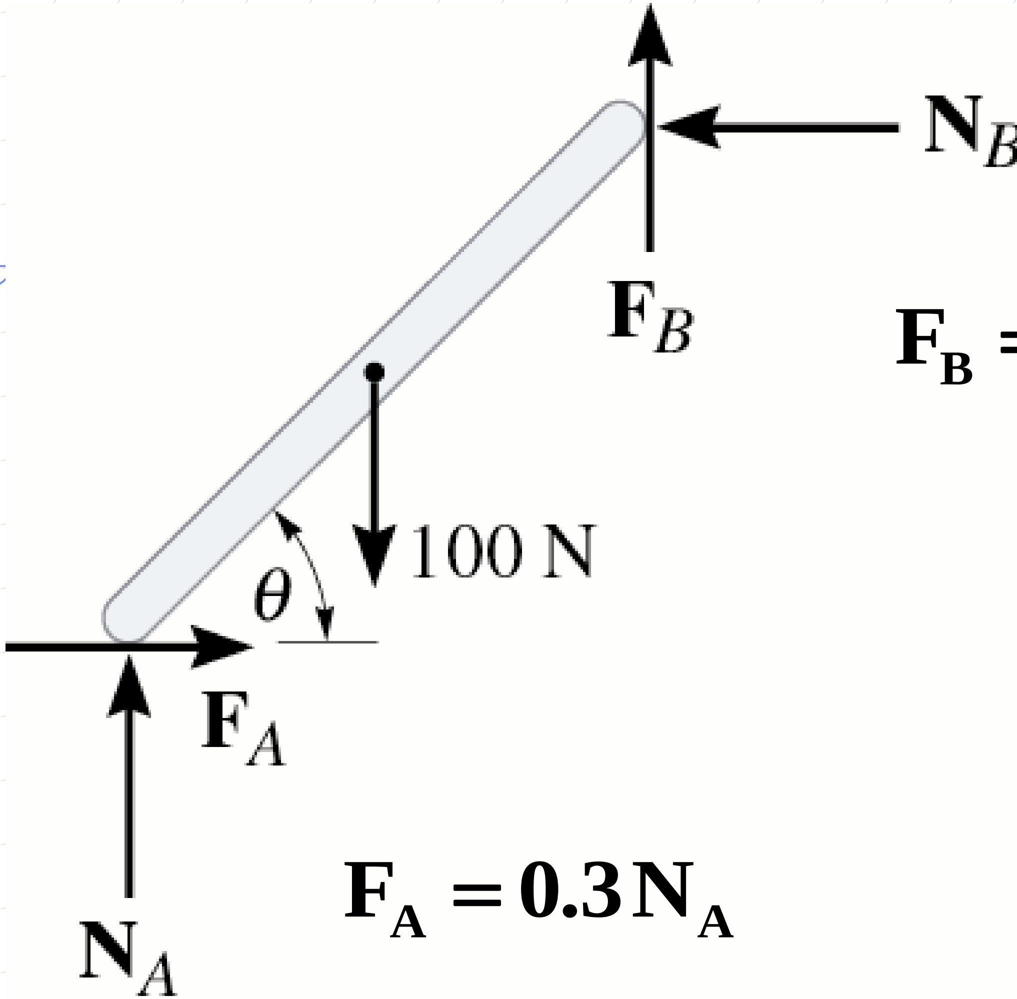
$$[F_k = \mu_k N] \qquad F = 0.17(956) = 162.5 \text{ N up the plane} \qquad \textit{Ans.}$$

Helpful Hint

① We should note that even though ΣF_x is no longer equal to zero, equilibrium does exist in the y-direction, so that $\Sigma F_y = 0$. Therefore, the normal force N is 956 N whether or not the block is in equilibrium.

Example: Impending Motion at All Points (W = 100 N)

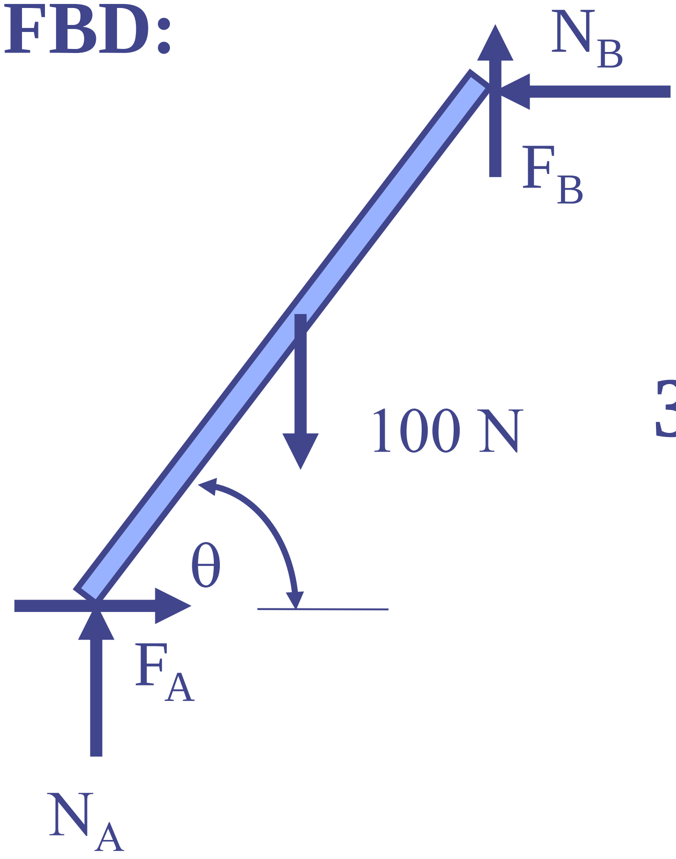




$$F_B = 0.4 N_B$$

$$F_A = 0.3 N_A$$

FBD:



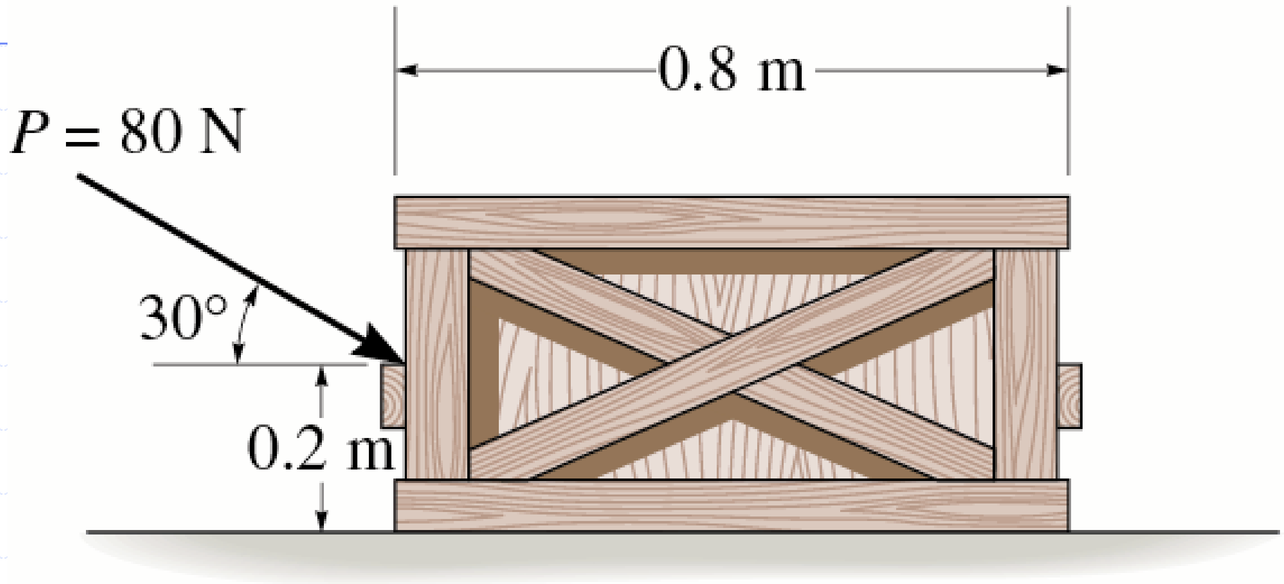
5 Unknowns

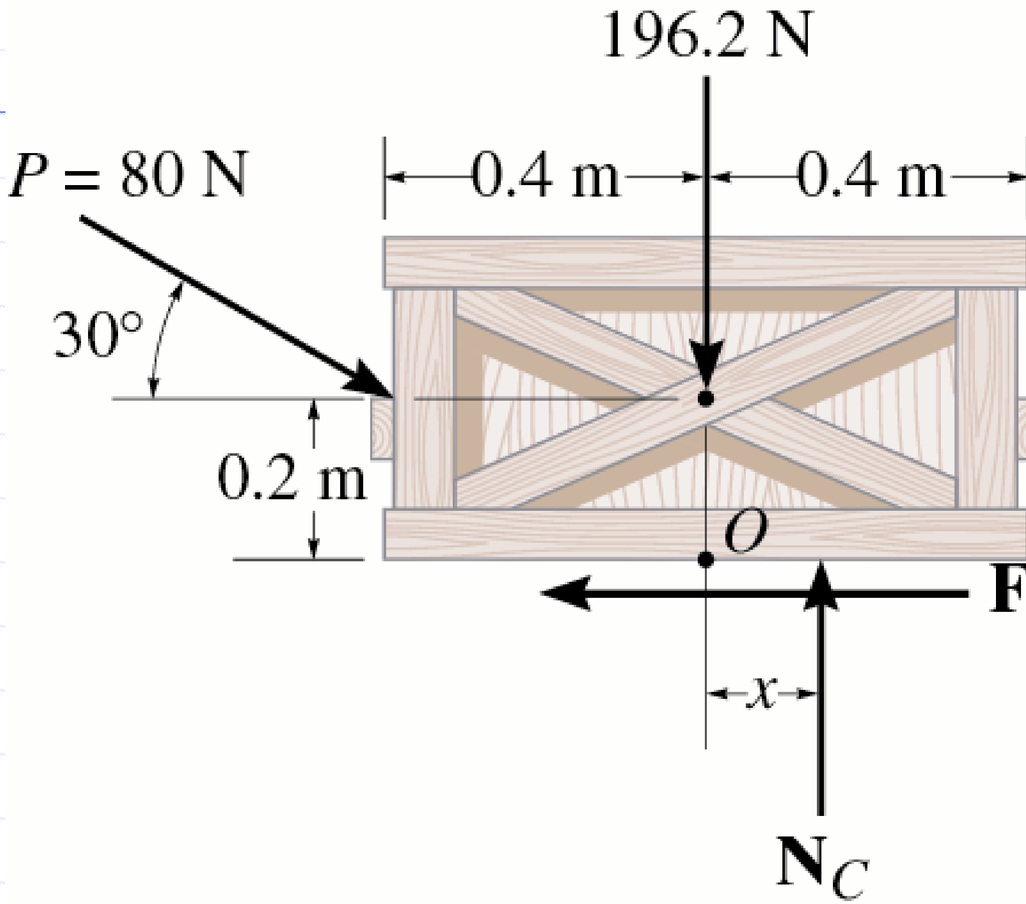
3 Equilibrium Equation

Plus:

$$F_A = 0.3\,N_A$$

$$F_B = 0.4\,N_B$$


0.8 m
P = 80 N
30°
0.2 m


196.2 N
P = 80 N
0.4 m
0.4 m
30°
0.2 m
O
F
x
N_C

3 unknowns - N_C, F, and x
3 equilibrium equations

$$\sum F_x = 0$$

$$80\cos 30^o - F = 0$$

$$\sum F_y = 0$$

$$-80\sin 30^o - 20(9.81) + N_C = 0$$

$$\sum M_O = 0$$

$$80\sin 30^o(0.4) - 80\cos 30^o(0.2) + N_C(x) = 0$$

$$F = 69.3\ N$$

$$N_C = 236\ N$$

$$x = -9.08\ mm$$

Neither sliding or tipping occurs. Crate remains in equilibrium.

$$\sum F_x = 0$$

$$80\cos 30^o - F = 0$$

$$\sum F_y = 0$$

$$-80\sin 30^o - 20(9.81) + N_C = 0$$

$$\sum M_O = 0$$

$$80\sin 30^o(0.4) - 80\cos 30^o(0.2) + N_C(x) = 0$$

$$F = 69.3\ \text{N}$$

$$N_C = 236\ \text{N}$$

$$x = -9.08\ \text{mm}$$