Chapter 4
Structures – part 2

STATICS, AGE-1330

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Chapter 4:

Analysis of Simple Structures

Outline

- Method of Sections
- Frames
- Machines*

1. Method of Sections

Methods of Truss Analysis

1) **Method of joints**

2) **Method of Sections**

2. Method of Sections

- The method of **joints** is good if we have to find the internal forces in **all the truss members**. In situations where we need to find the internal forces only **in a few specific members** of a truss, the method of sections is **more appropriate**.

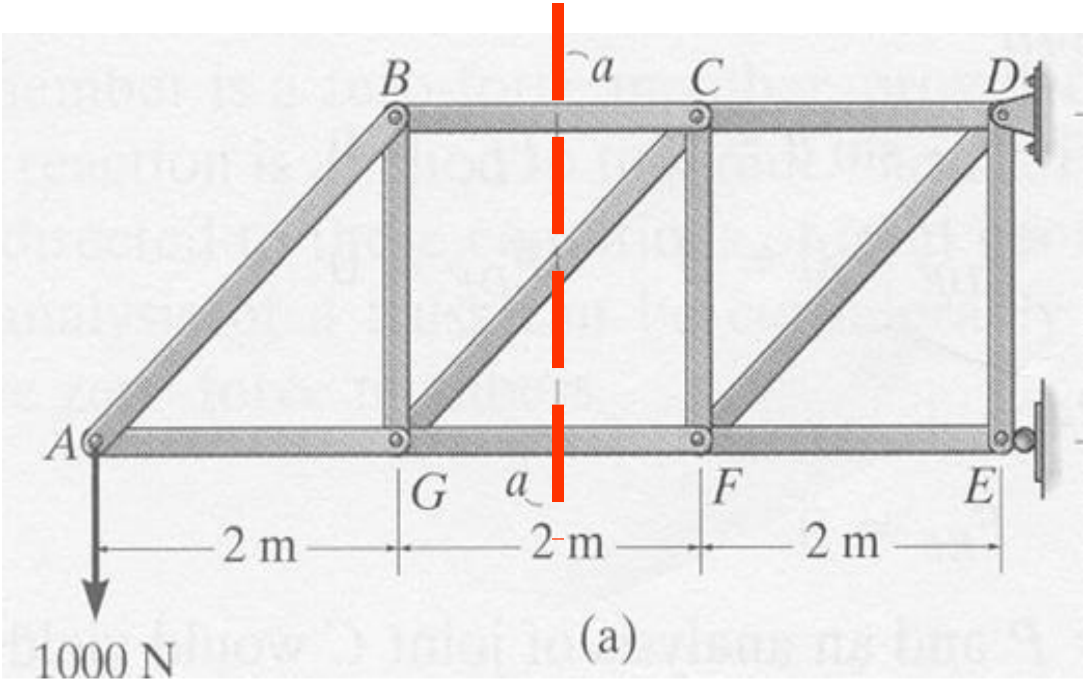
- The Method of Sections is based on the two dimensional equilibrium of rigid bodies (**$\Sigma F_x = 0$**, **$\Sigma F_y = 0$** & **$\Sigma M = 0$**).

- This method has the basic advantage that the force in almost any desired member may be found directly from an analysis of a section which has **cut that member**.

❖ In **choosing a section** of the truss, in general, **not more than three members** whose forces are **unknown should be cut**, since there are **only three available independent equilibrium** relations.

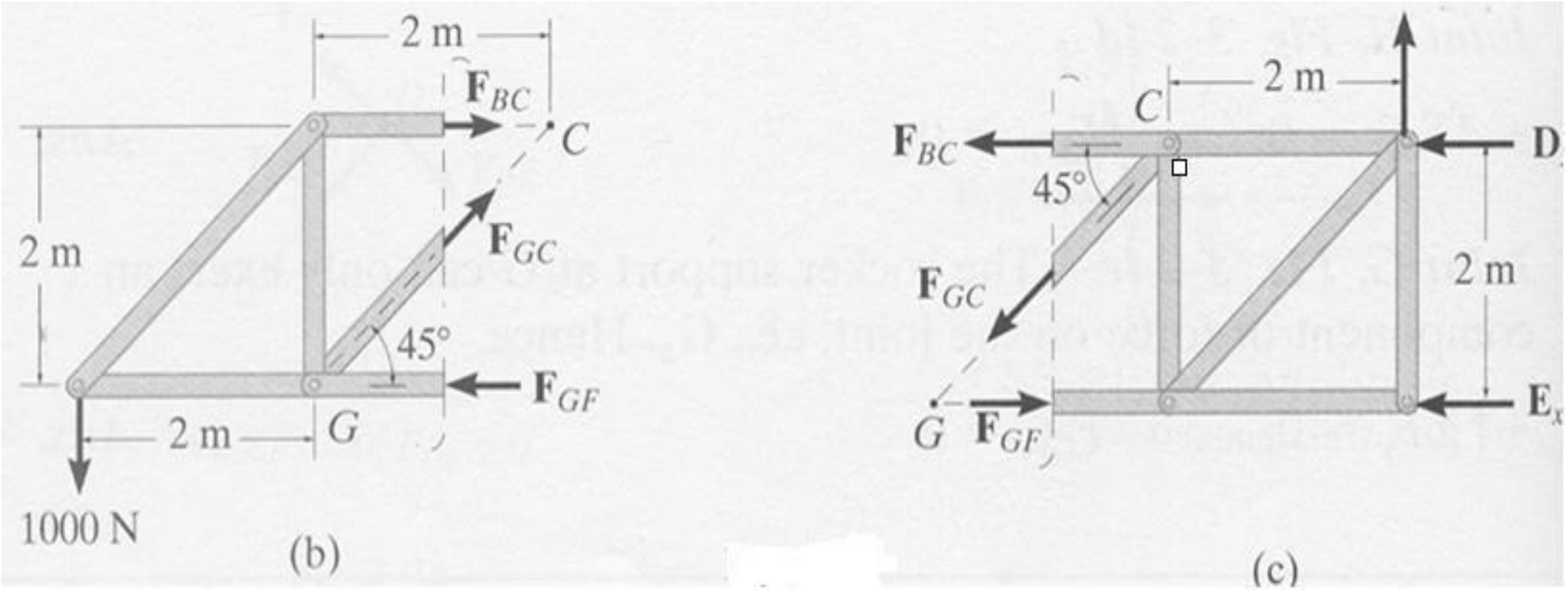
Method of Sections cont.

Let see the sample truss to determine member BC,GF and CG.



Method of Sections cont.

=> Each of section **PART** of the truss is in Equilibrium



Method of Sections - Procedure

A) Free Body Diagram

- Determine external reactions of entire truss
- Decide how to section truss

  - **Three unknown forces at the most**

Method of Sections – Procedure (cont.)

Free Body Diagram

- Draw FBD of one part
  - Choose part with least number of forces

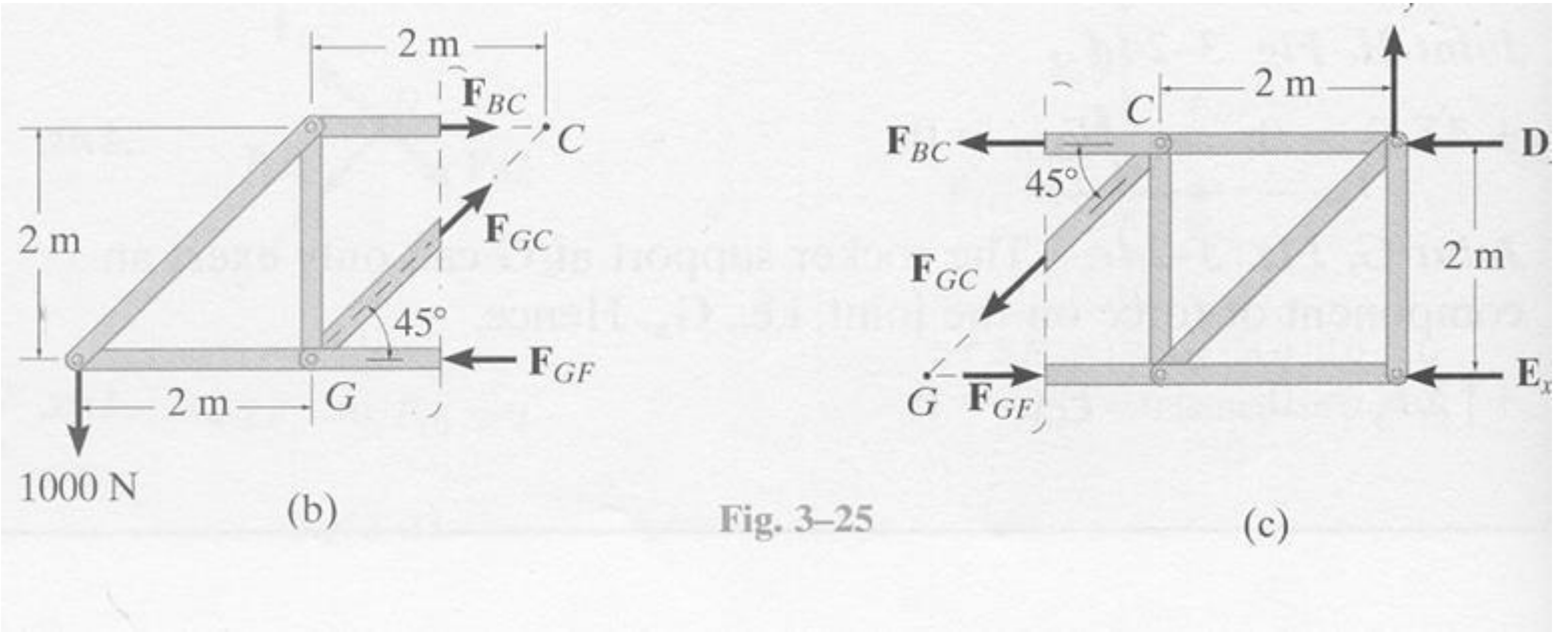


Fig. 3–25

Method of Sections – Procedure (cont.)

Free Body Diagram

- Establish direction of unknown forces

  - Assume **all forces cause tension** in member

  - **Numerical results**:

    (+) Force is in tension

    (-) Force in opposite direction i.e compression

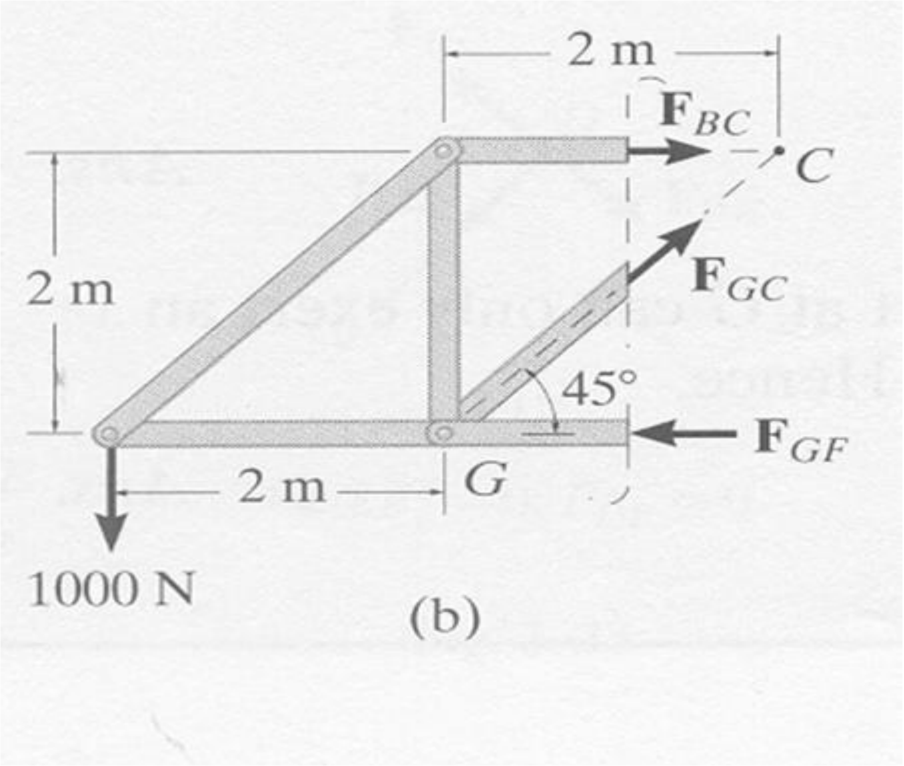
Method of Sections – Procedure (cont'd)

B) Equations of Equilibrium

$$\sum F_x = 0$$

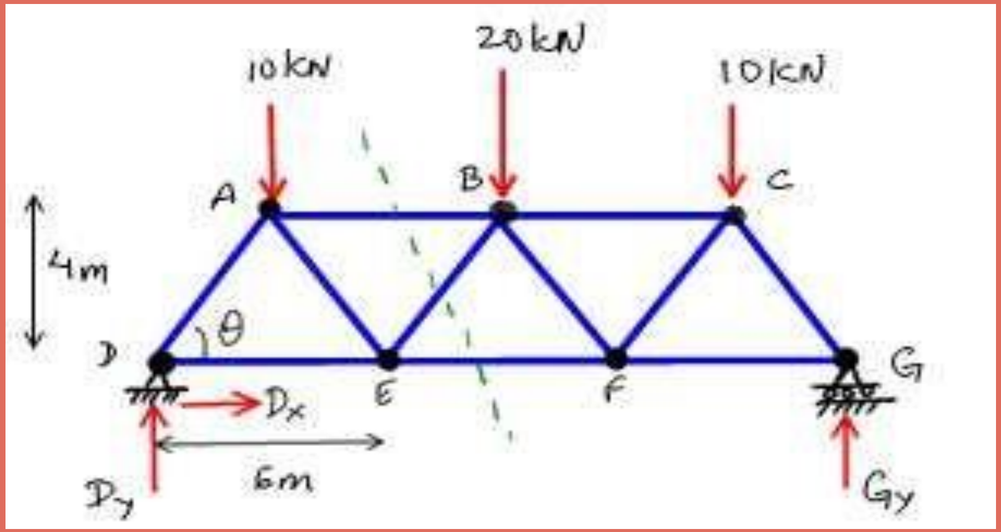
$$\sum F_y = 0$$

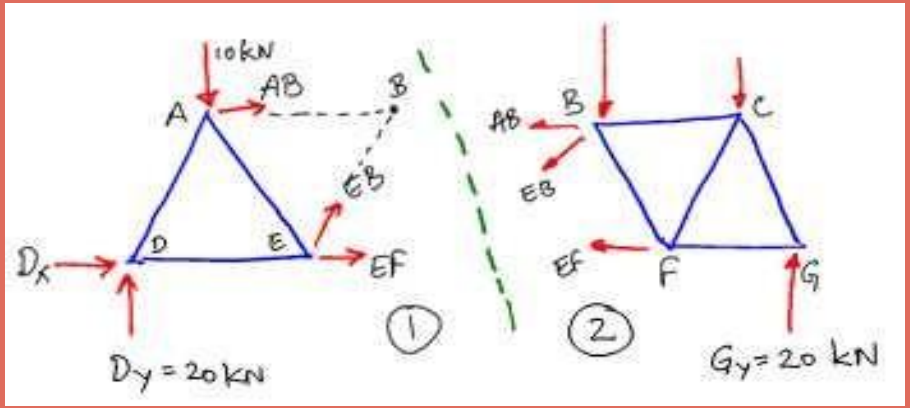
$$\sum M = 0$$



- **Take moments about a point that lies on the intersection of the lines of action of two unknown forces**

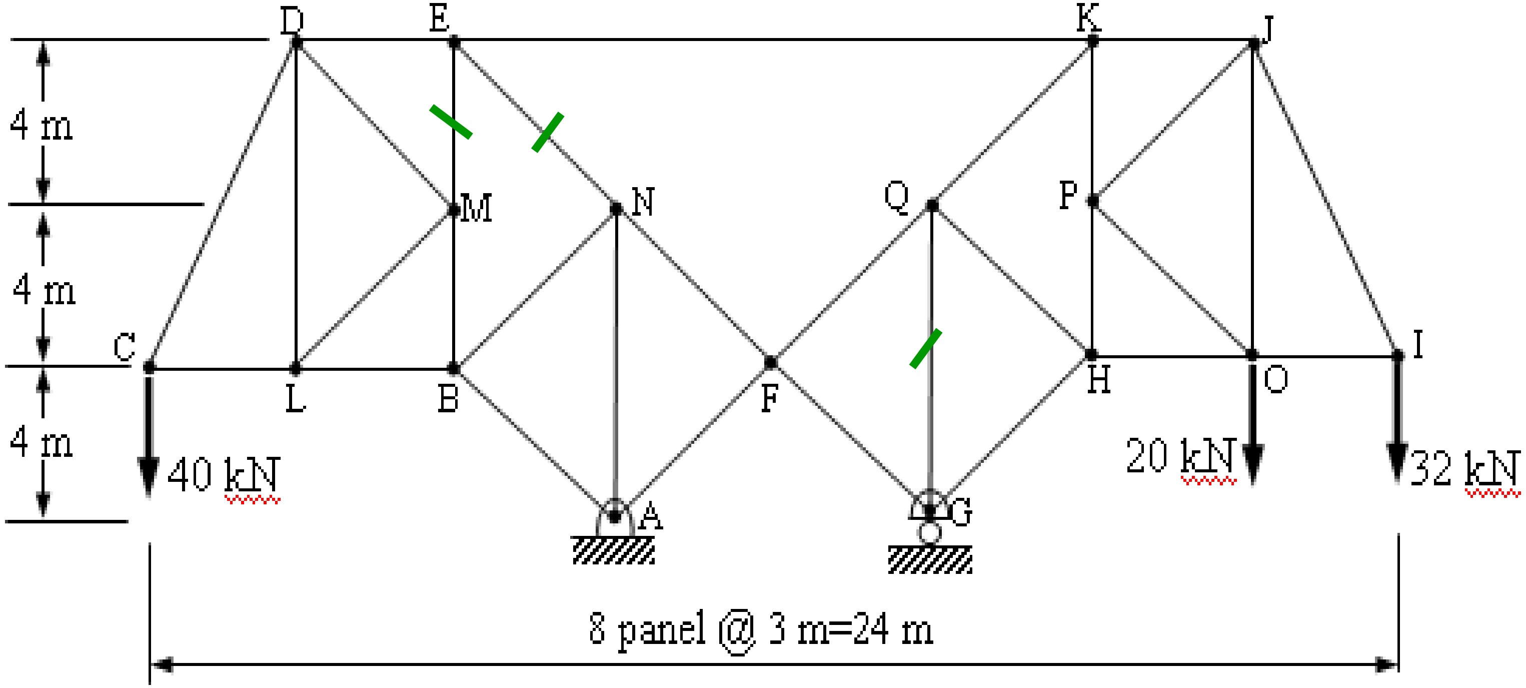
For example, find the force in member EF:



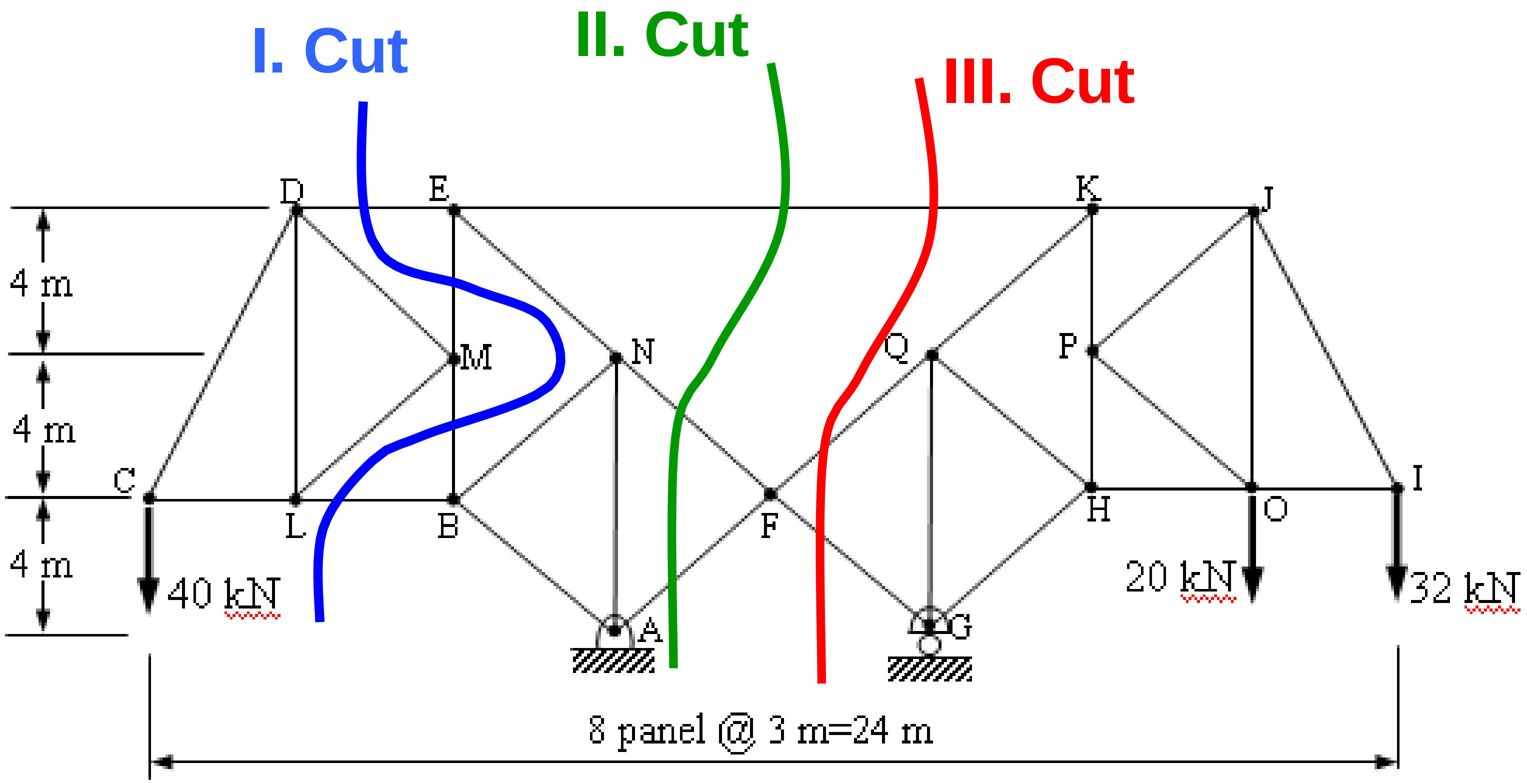


Example

Determine the forces in members **ME**, **NE** and **QG**.

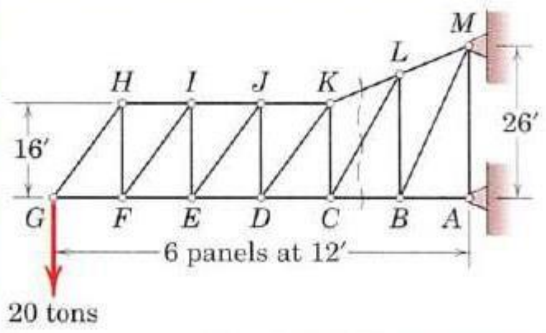


What do you understand from this section(cut)?



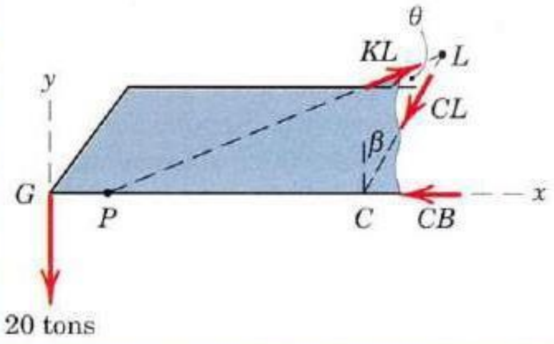
Sample Problem 4/2

Calculate the forces induced in members *KL*, *CL*, and *CB* by the 20-ton load on the cantilever truss.



Solution. Although the vertical components of the reactions at *A* and *M* are statically indeterminate with the two fixed supports, all members other than *AM* are statically determinate. We may pass a section directly through members *KL*, *CL*, and *CB* and analyze the portion of the truss to the left of this section as a statically determinate rigid body. ①

The free-body diagram of the portion of the truss to the left of the section is shown. A moment sum about *L* quickly verifies the assignment of *CB* as compression, and a moment sum about *C* quickly discloses that *KL* is in tension. The direction of *CL* is not quite so obvious until we observe that *KL* and *CB* intersect at a point *P* to the right of *G*. A moment sum about *P* eliminates reference to *KL* and *CB* and shows that *CL* must be compressive to balance the moment of the 20-ton force about *P*. With these considerations in mind the solution becomes straightforward, as we now see how to solve for each of the three unknowns independently of the other two.



② Summing moments about *L* requires finding the moment arm $\overline{BL} = 16 + (26 - 16)/2 = 21$ ft. Thus,

$[\Sigma M_L = 0]$ $\quad 20(5)(12) - CB(21) = 0 \quad CB = 57.1$ tons *C* *Ans.*

Next we take moments about *C*, which requires a calculation of cos θ. From the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos\theta = 12/13$. Therefore,

$[\Sigma M_C = 0]$ $\quad 20(4)(12) - \frac{12}{13}KL(16) = 0 \quad KL = 65.0$ tons *T* *Ans.*

Finally, we may find *CL* by a moment sum about *P*, whose distance from *C* is given by $\overline{PC}/16 = 24/(26 - 16)$ or $\overline{PC} = 38.4$ ft. We also need β, which is given by $\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(12/21) = 29.7°$ and $\cos\beta = 0.868$. We now have

③ $[\Sigma M_P = 0]$ $\quad 20(48 - 38.4) - CL(0.868)(38.4) = 0$

$CL = 5.76$ tons *C* *Ans.*

Helpful Hints

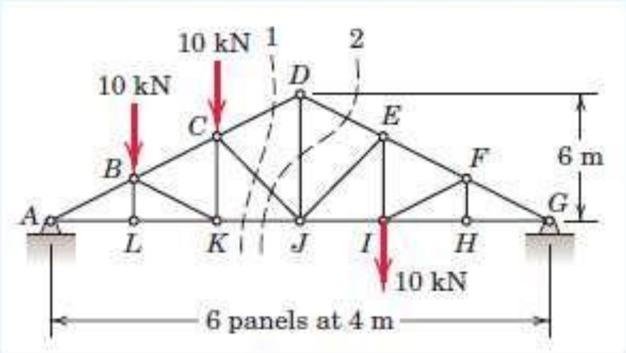
① We note that analysis by the method of joints would necessitate working with eight joints in order to calculate the three forces in question. Thus, the method of sections offers a considerable advantage in this case.

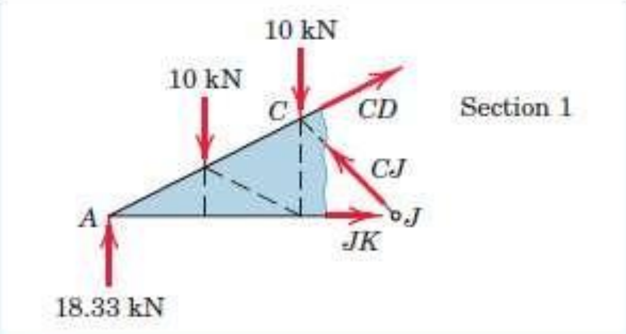
② We could have started with moments about *C* or *P* just as well.

③ We could also have determined *CL* by a force summation in either the *x*- or *y*-direction.

SAMPLE PROBLEM 4/4

Calculate the force in member *DJ* of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.





Solution. It is not possible to pass a section through *DJ* without cutting four members whose forces are unknown. Although three of these cut by section 2 are concurrent at *J* and therefore the moment equation about *J* could be used to obtain *DE*, the force in *DJ* cannot be obtained from the remaining two equilibrium principles. It is necessary to consider first the adjacent section 1 before analyzing section 2.

The free-body diagram for section 1 is drawn and includes the reaction of 18.33 kN at *A*, which is previously calculated from the equilibrium of the truss as a whole. In assigning the proper directions for the forces acting on the three cut members, we see that a balance of moments about *A* eliminates the effects of *CD* and *JK* and clearly requires that *CJ* be up and to the left. A balance of moments about *C* eliminates the effect of the three forces concurrent at *C* and indicates that *JK* must be to the right to supply sufficient counterclockwise moment. Again it should be fairly obvious that the lower chord is under tension because of the bending tendency of the truss. Although it should also be apparent that the top chord is under compression, for purposes of illustration the force in *CD* will (1) be arbitrarily assigned as tension.

By the analysis of section 1, *CJ* is obtained from

$[\Sigma M_A = 0] \qquad 0.707CJ(12) - 10(4) - 10(8) = 0 \qquad CJ = 14.14 \text{ kN } C$

In this equation the moment of *CJ* is calculated by considering its horizontal and vertical components acting at point *J*. Equilibrium of moments about *J* requires

$[\Sigma M_J = 0] \qquad 0.894CD(6) + 18.33(12) - 10(4) - 10(8) = 0$

$CD = -18.63 \text{ kN}$

The moment of *CD* about *J* is calculated here by considering its two components (2) as acting through *D*. The minus sign indicates that *CD* was assigned in the wrong direction.

Helpful Hints

(1) There is no harm in assigning one or more of the forces in the wrong direction, as long as the calculations are consistent with the assumption. A negative answer will show the need for reversing the direction of the force.

(2) If desired, the direction of *CD* may be changed on the free-body diagram and the algebraic sign of *CD* reversed in the calculations, or else the work may be left as it stands with a note stating the proper direction.

wrong direction.

Hence, $CD = 18.63 \text{ kN } C$

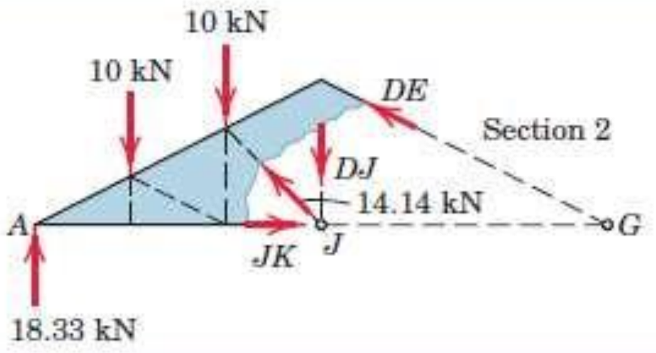
From the free-body diagram of section 2, which now includes the known value of CJ, a balance of moments about G is seen to eliminate DE and JK. Thus, 3

$$[\Sigma M_G = 0] \qquad 12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$$

$$DJ = 16.67 \text{ kN } T \qquad \textit{Ans.}$$

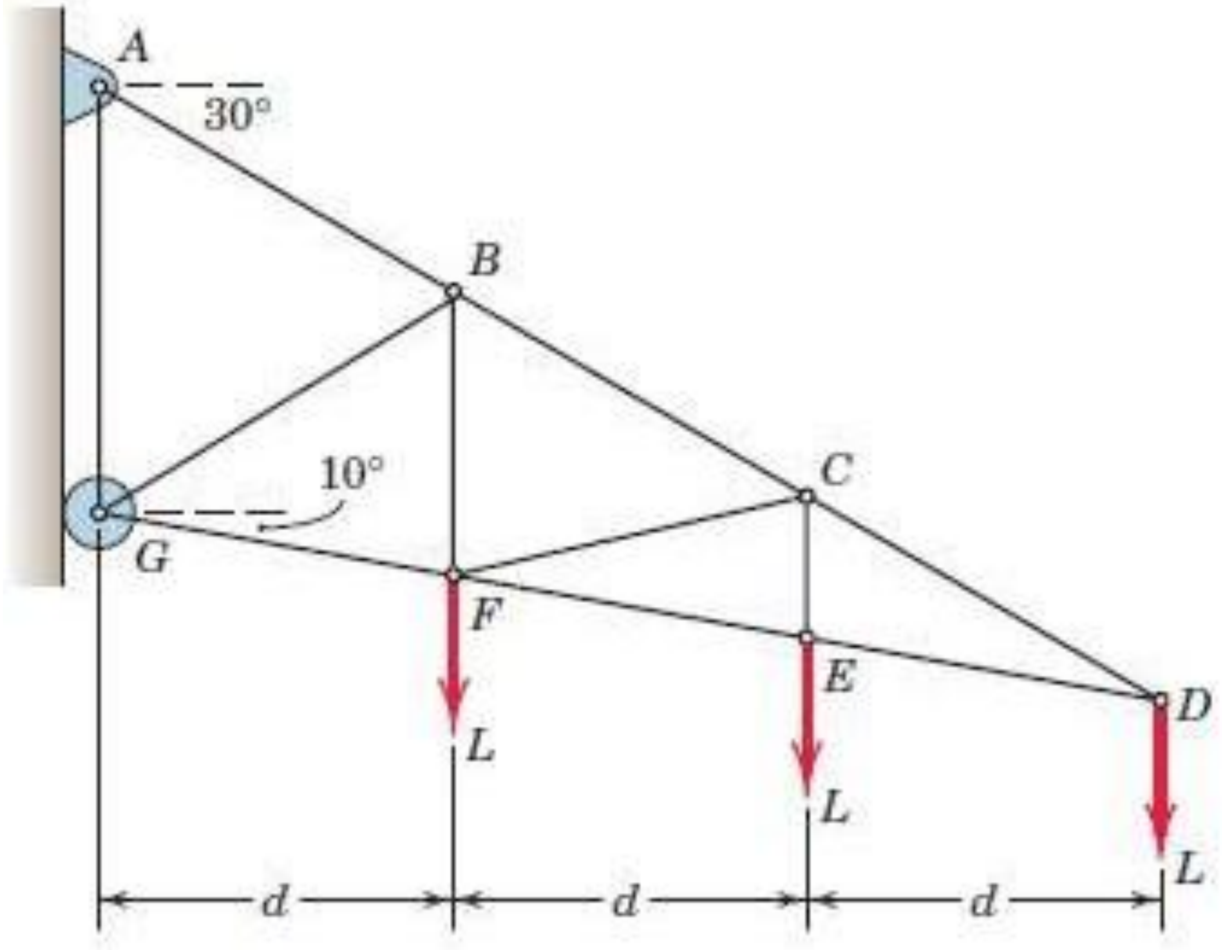
Again the moment of CJ is determined from its components considered to be acting at J. The answer for DJ is positive, so that the assumed tensile direction is correct.

An alternative approach to the entire problem is to utilize section 1 to determine CD and then use the method of joints applied at D to determine DJ.

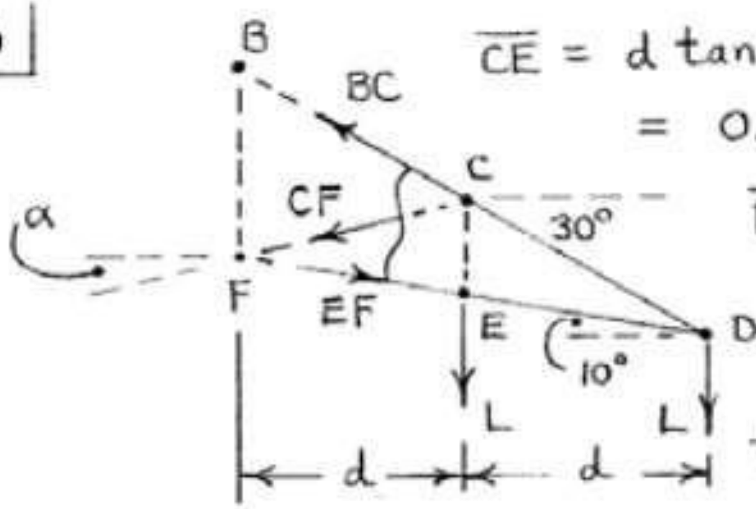


3 Observe that a section through members CD, DJ, and DE could be taken which would cut only three unknown members. However, since the forces in these three members are all concurrent at D, a moment equation about D would yield no information about them. The remaining two force equations would not be sufficient to solve for the three unknowns.

Problem 2: Determine the forces in members BC, CF, and EF of the loaded truss.



4/30



$\overline{CE} = d\tan 30° - d\tan 10°$

$= 0.401d$

$\overline{BF} = 2\overline{CE}$

$= 0.802d$

$\alpha = \tan^{-1}\left(\frac{0.401d - d\tan 10°}{d}\right)$

$= 12.66°$

$\circlearrowleft + \Sigma M_C = 0: \; -Ld + EF\cos 10°(0.401d) = 0$

$EF = 2.53L \;\; C$

$\circlearrowleft + \Sigma M_F = 0: \; -Ld - L(2d) + BC\cos 30°(0.802d) = 0$

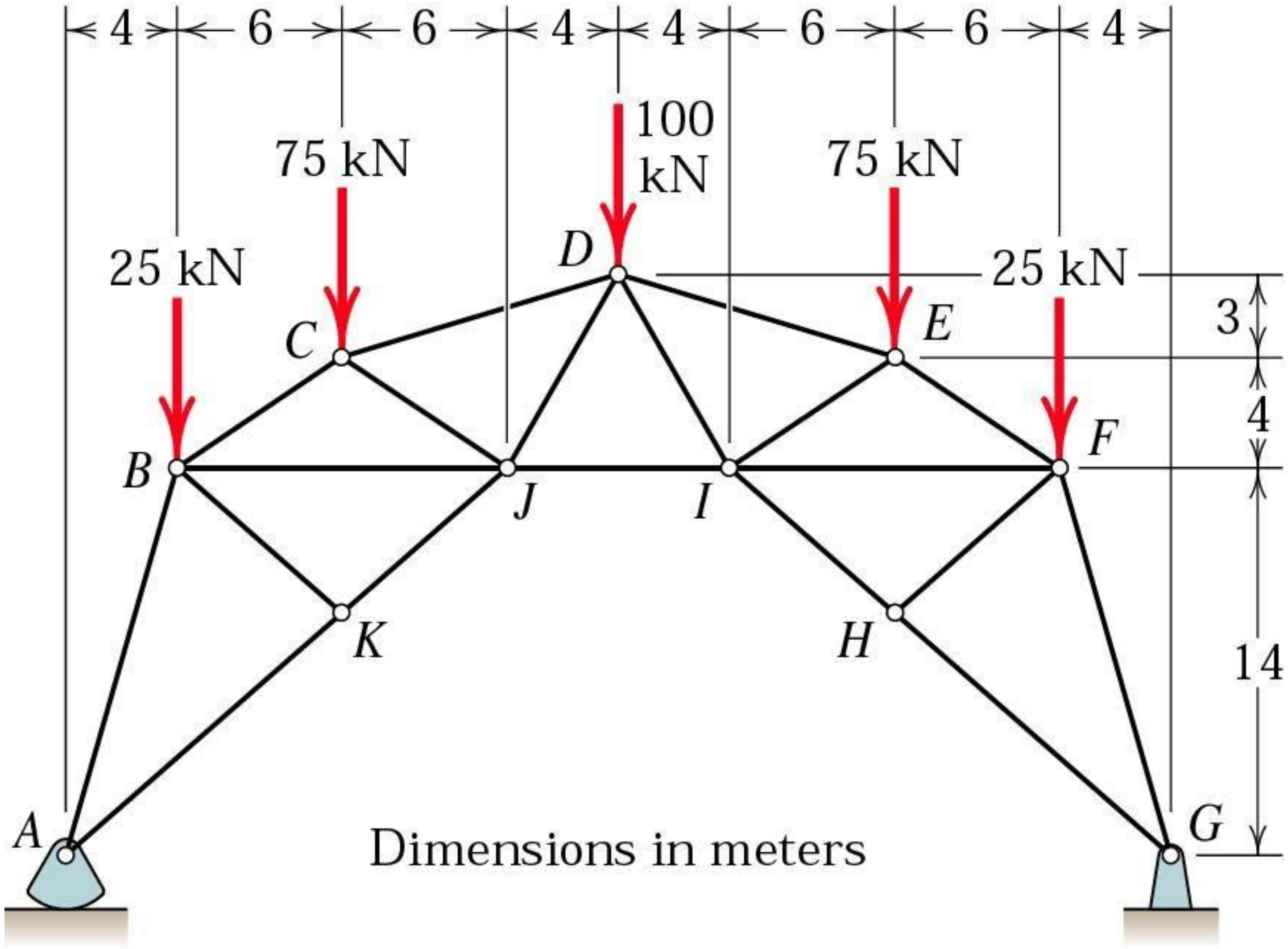
$BC = 4.32L \;\; T$

$\circlearrowleft + \Sigma M_D = 0: \; Ld + CF\cos 12.66°(d\tan 30°)$

$+ CF\sin 12.66°(d) = 0 \qquad CF = -1.278L$

or $CF = 1.278L \;\; C$

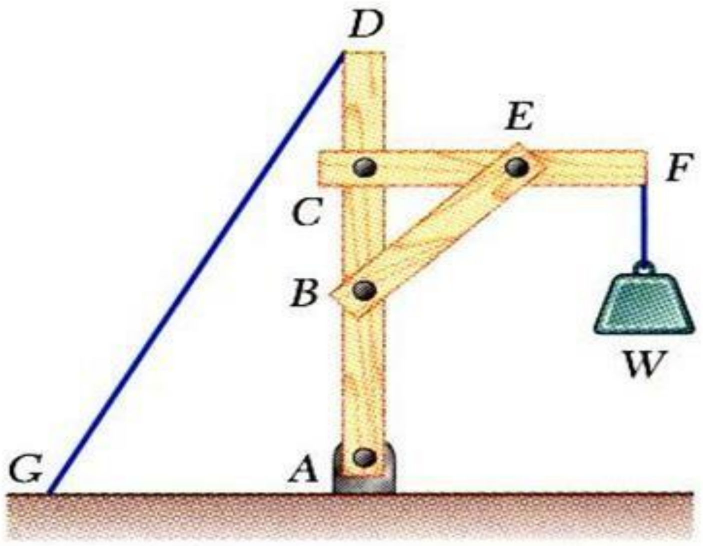
4. Determine the forces in members DE, EI, FI and HI.



2. Frames

Frames

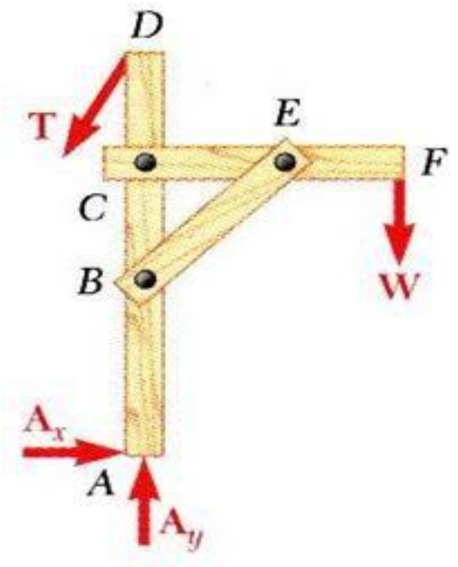
- Frames are structures with at least one multi-force member, i.e. at least one member that has 3 or more forces acting on it at different points.
- Frames are structures that contain pin connected multi-force members (members with more forces).Frames are used to support the system of the loads while remaining stationary.



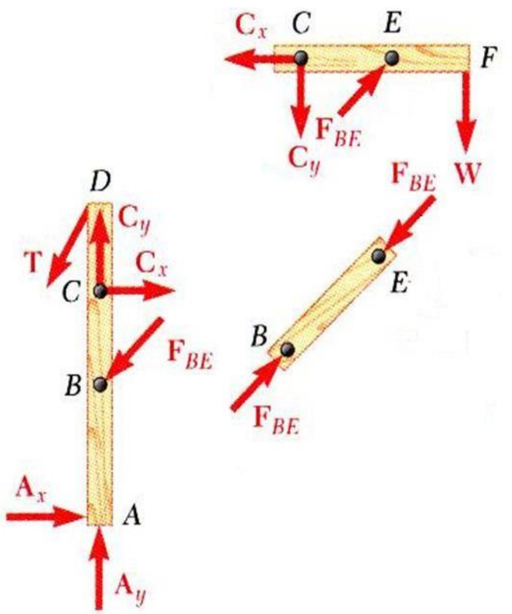
Frame: an assembly of rigid members (of which at least one is a multi–force member) intended to be a stationary structure for supporting a load.

Frame analysis involves determining:

(i) External Reactions



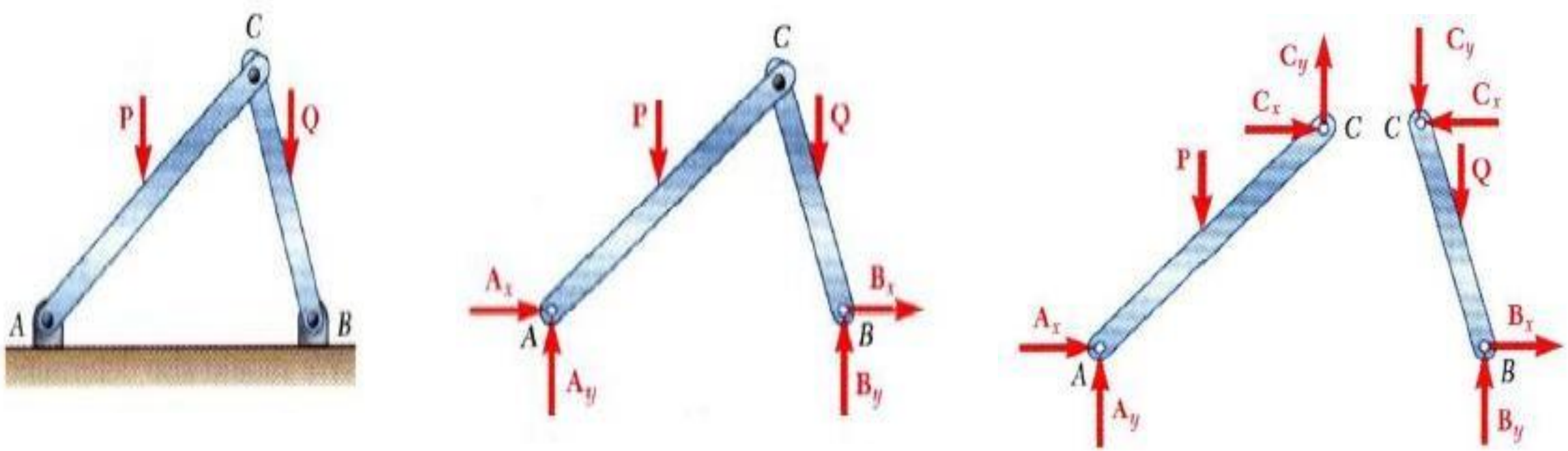
(ii) Internal forces at the joints



Note: Follow Newton's 3rd Law

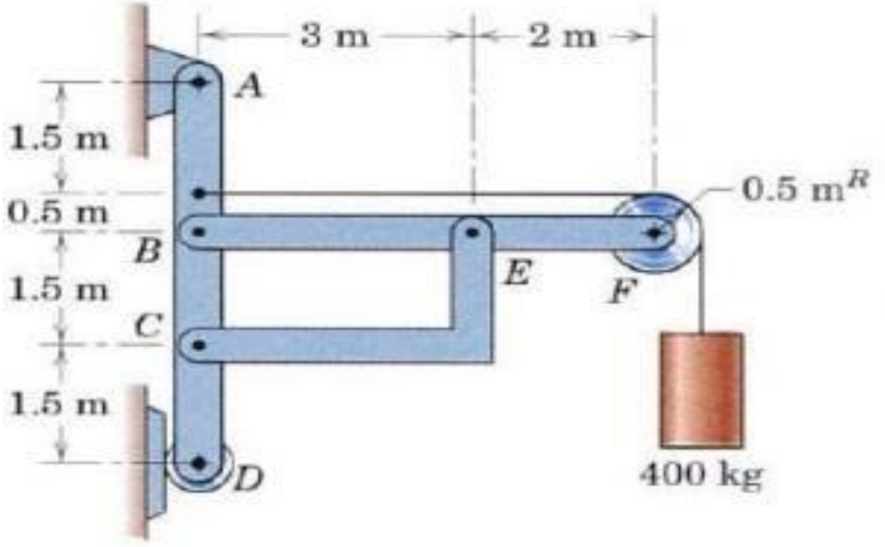
Frames that are **not internally** Rigid

- When a frame is not internally rigid, it has to be provided with additional external supports to make it rigid.
- The support reactions for such frames cannot be simply determined by external equilibrium.
- One has to draw the FBD of all the component parts to find out whether the frame is determinate or indeterminate.



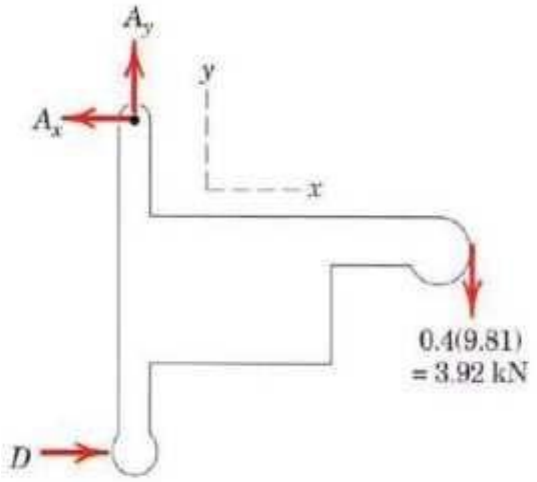
Example

- The frame supports the 400 kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.



Solution

- From the free-body diagram of the entire frame we determine the **external reactions**.Thus



$\circlearrowleft \sum M_A = 0; \quad 5.5\text{m} \times 0.4 \times 9.81\text{kN} - 5\text{m} \times D = 0; \longrightarrow D = 4.32 \text{ kN}$

$\longrightarrow \sum F_x = 0; \; D - A_x = 0; \; 4.32 \text{ kN} - A_x = 0; \longrightarrow A_x = 4.32 \text{ kN}$

$\uparrow \sum F_y = 0; \; A_y - 3.92 \text{ kN} = 0; \longrightarrow A_y = 3.92 \text{ kN}$

Solution

- To compute the horizontal and vertical components of all forces acting in each members

Member BF:

$$\Sigma M_B = 0$$

$$3.92 \times 5 - C_x \times (1/2) \times 3 = 0$$

$$C_x = 13.08 \text{ kN} = E_x$$

$$C_y = 0.5 \times C_x = 6.54 \text{ kN} = E_y$$

$$\Sigma F_X = 0$$

$$B_x + 3.92 - 13.08 = 0 \Longrightarrow B_x = 9.15 \text{ kN}$$

$$\Sigma F_Y = 0$$

$$B_y + 3.92 - 13.0 /2 = 0 \Longrightarrow B_y = 2.62 \text{ kN}$$

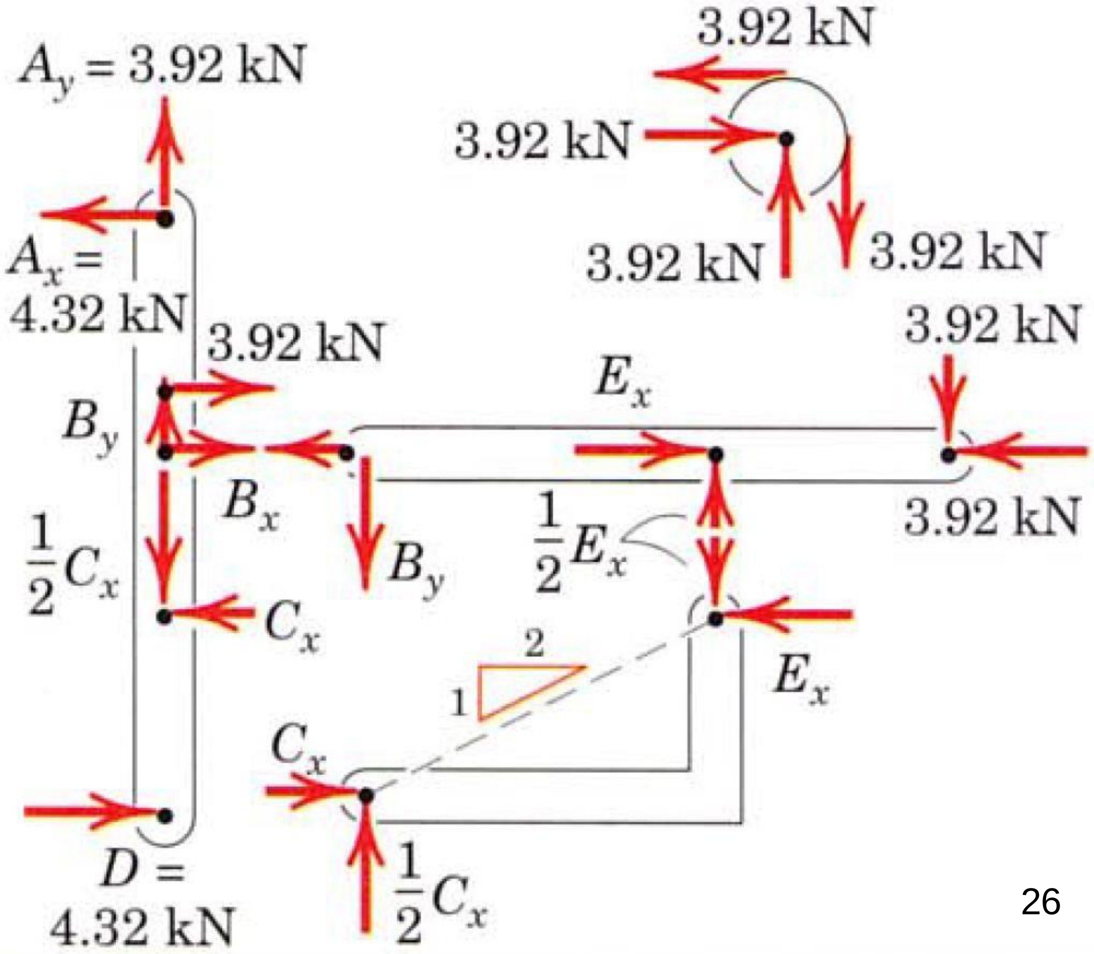
As a check we take member AD:

Apply equations of equilibrium

$$[\Sigma M_C = 0] \quad 4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$$

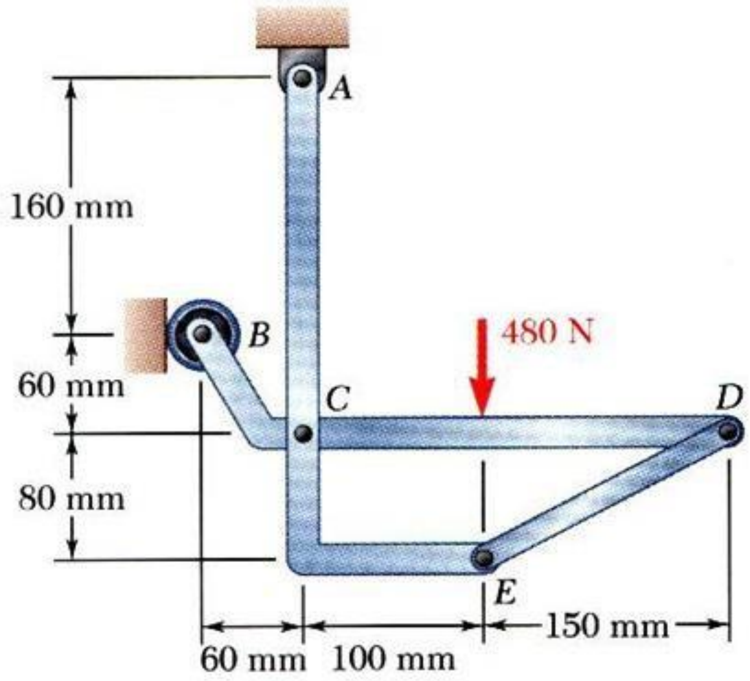
$$[\Sigma F_x = 0] \quad -4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$$

$$[\Sigma F_y = 0] \quad -13.08/2 + 2.62 + 3.92 = 0$$



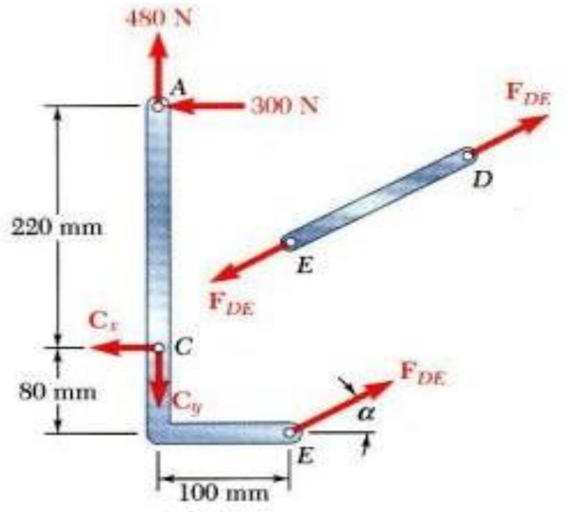
Example-2

- Determine the components of the forces acting on each member of the frame shown

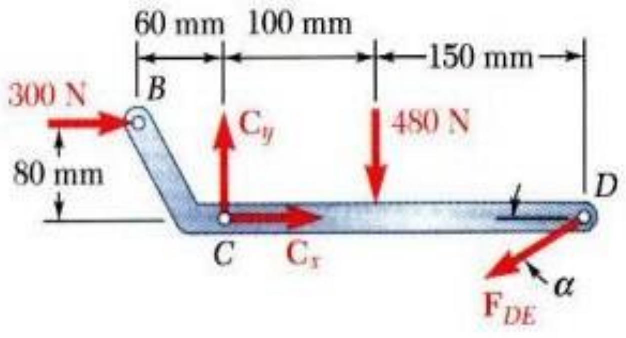


Solution

For checking



Member BCD



$$\sum M_C = 0 = (F_{DE} \sin\alpha)(250\ \text{mm}) + (300\ \text{N})(60\ \text{mm}) + (480\ \text{N})(100\ \text{mm})$$

$$F_{DE} = -561\ \text{N} \qquad \boxed{F_{DE} = 561\ \text{N}\ \ C}$$

- Sum of forces in the x and y directions may be used to find the force components at C.

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\ \text{N}$$

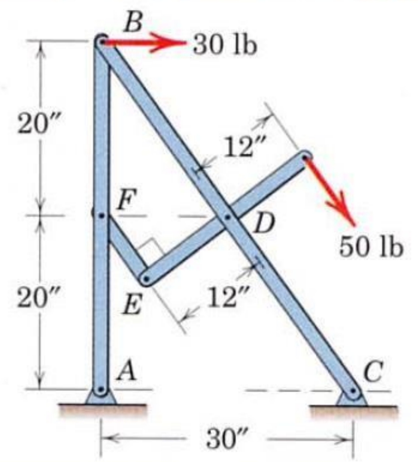
$$0 = C_x - (-561\ \text{N})\cos\alpha + 300\ \text{N} \qquad \boxed{C_x = -795\ \text{N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\ \text{N}$$

$$0 = C_y - (-561\ \text{N})\sin\alpha - 480\ \text{N} \qquad \boxed{C_y = 216\ \text{N}}$$

Sample Problem 4/6

Neglect the weight of the frame and compute the forces acting on all of its members.

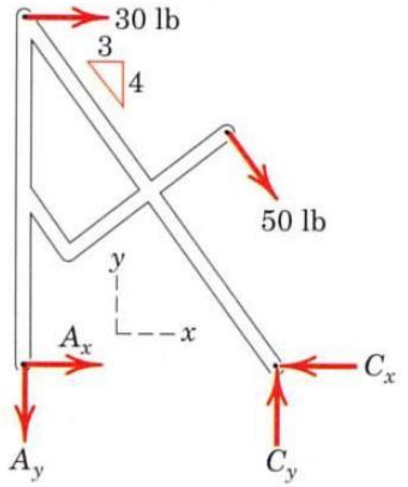


Solution. We note first that the frame is not a rigid unit when removed from its supports since $BDEF$ is a movable quadrilateral and not a rigid triangle. Consequently the external reactions cannot be completely determined until the individual members are analyzed. However, we can determine the vertical components of the reactions at A and C from the free-body diagram of the frame as a whole. Thus,

① ②

$[\Sigma M_C = 0]$ $\quad 50(12) + 30(40) - 30A_y = 0 \quad A_y = 60 \text{ lb}$ *Ans.*

$[\Sigma F_y = 0]$ $\quad C_y - 50(4/5) - 60 = 0 \quad C_y = 100 \text{ lb}$ *Ans.*



Next we dismember the frame and draw the free-body diagram of each part. Since EF is a two-force member, the direction of the force at E on ED and at F on AB is known. We assume that the 30-lb force is applied to the pin as a part ③ of member BC. There should be no difficulty in assigning the correct directions for forces E, F, D, and B_x. The direction of B_y, however, may not be assigned by inspection and therefore is arbitrarily shown as downward on AB and upward on BC.

Member ED. The two unknowns are easily obtained by

$[\Sigma M_D = 0]$ $\quad 50(12) - 12E = 0 \quad E = 50$ lb *Ans.*

$[\Sigma F = 0]$ $\quad D - 50 - 50 = 0 \quad D = 100$ lb *Ans.*

Member EF. Clearly F is equal and opposite to E with the magnitude of 50 lb.

Member AB. Since F is now known, we solve for B_x, A_x, and B_y from

$[\Sigma M_A = 0]$ $\quad 50(3/5)(20) - B_x(40) = 0 \quad B_x = 15$ lb *Ans.*

$[\Sigma F_x = 0]$ $\quad A_x + 15 - 50(3/5) = 0 \quad A_x = 15$ lb *Ans.*

$[\Sigma F_y = 0]$ $\quad 50(4/5) - 60 - B_y = 0 \quad B_y = -20$ lb *Ans.*

The minus sign shows that we assigned B_y in the wrong direction.

Member BC. The results for B_x, B_y, and D are now transferred to BC, and the
④ remaining unknown C_x is found from

$[\Sigma F_x = 0]$ $\quad 30 + 100(3/5) - 15 - C_x = 0 \quad C_x = 75$ lb *Ans.*

We may apply the remaining two equilibrium equations as a check. Thus,

$[\Sigma F_y = 0]$ $\quad 100 + (-20) - 100(4/5) = 0$

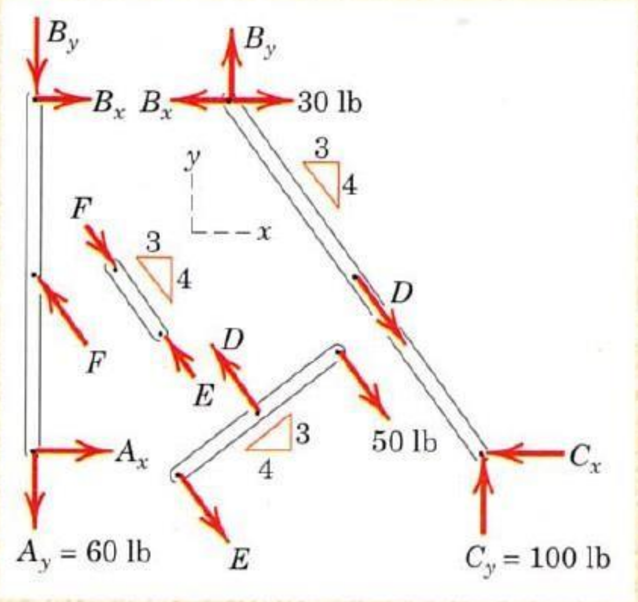
$[\Sigma M_C = 0]$ $\quad (30 - 15)(40) + (-20)(30) = 0$

Helpful Hints

① We see that this frame corresponds to the category illustrated in Fig. 4/14*b*.

② The directions of A_x and C_x are not obvious initially and can be assigned arbitrarily to be corrected later if necessary.

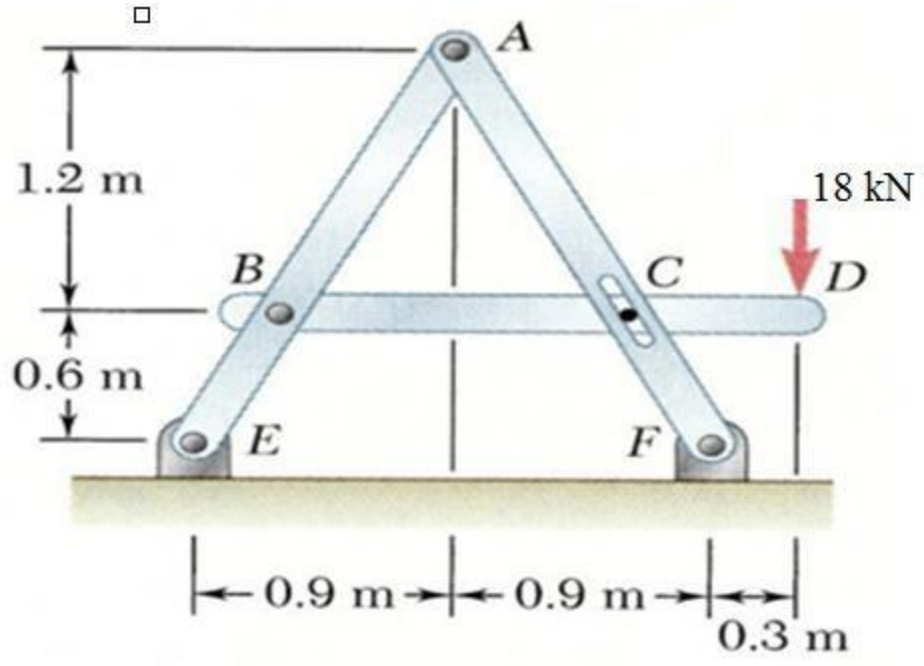
③ Alternatively the 30-lb force could be applied to the pin considered a part of BA, with a resulting change in the reaction B_x.



④ Alternatively we could have returned to the free-body diagram of the frame as a whole and found C_x.

Example-1

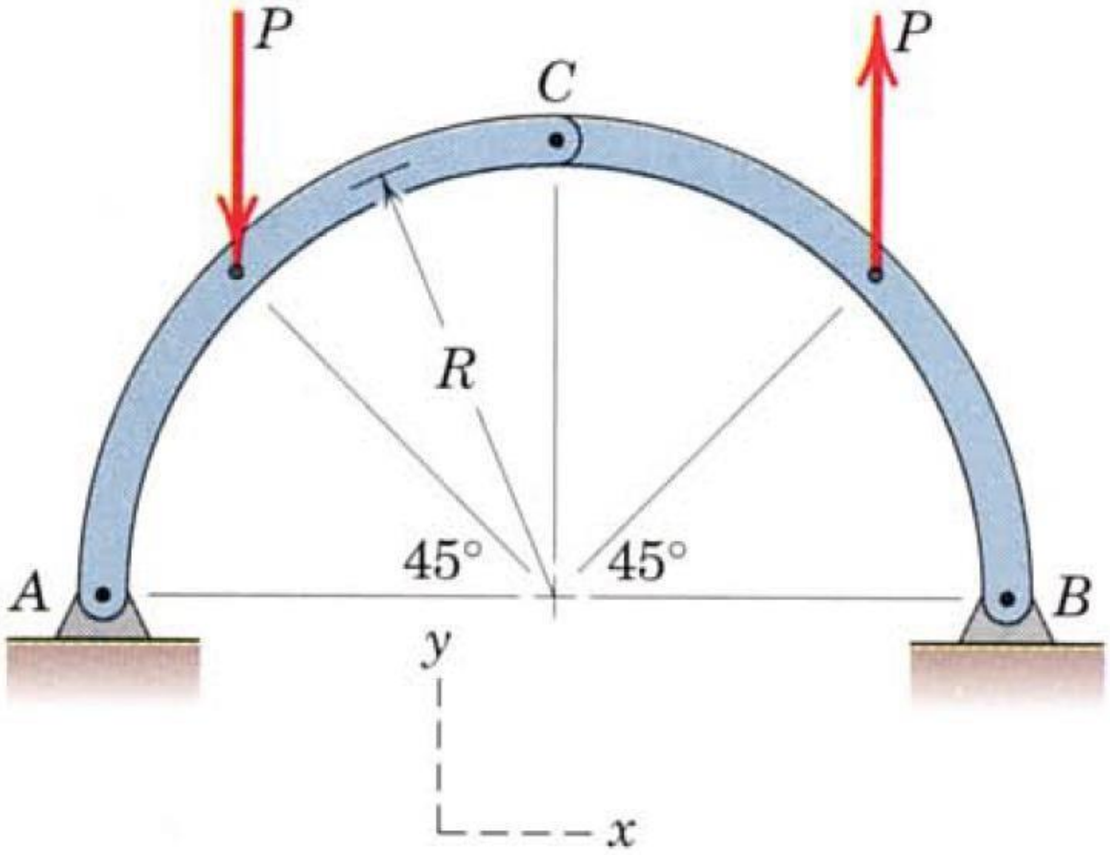
- Determine the components of the forces acting on each member of the frame shown



4/69 Determine the components of all forces acting on each member of the loaded truss. What is the primary difference between this problem and Prob. 4/68?

Ans. $A_x = C_x = B_x = 0$

$A_y = 0.707P$, $B_y = -0.707P$, $C_y = 0.293P$



3. Machines

Machines

- **Machine:** is an assembly of rigid members designed to do mechanical work by transmitting a given set of input loading forces into another set of output forces.

Hole Punch

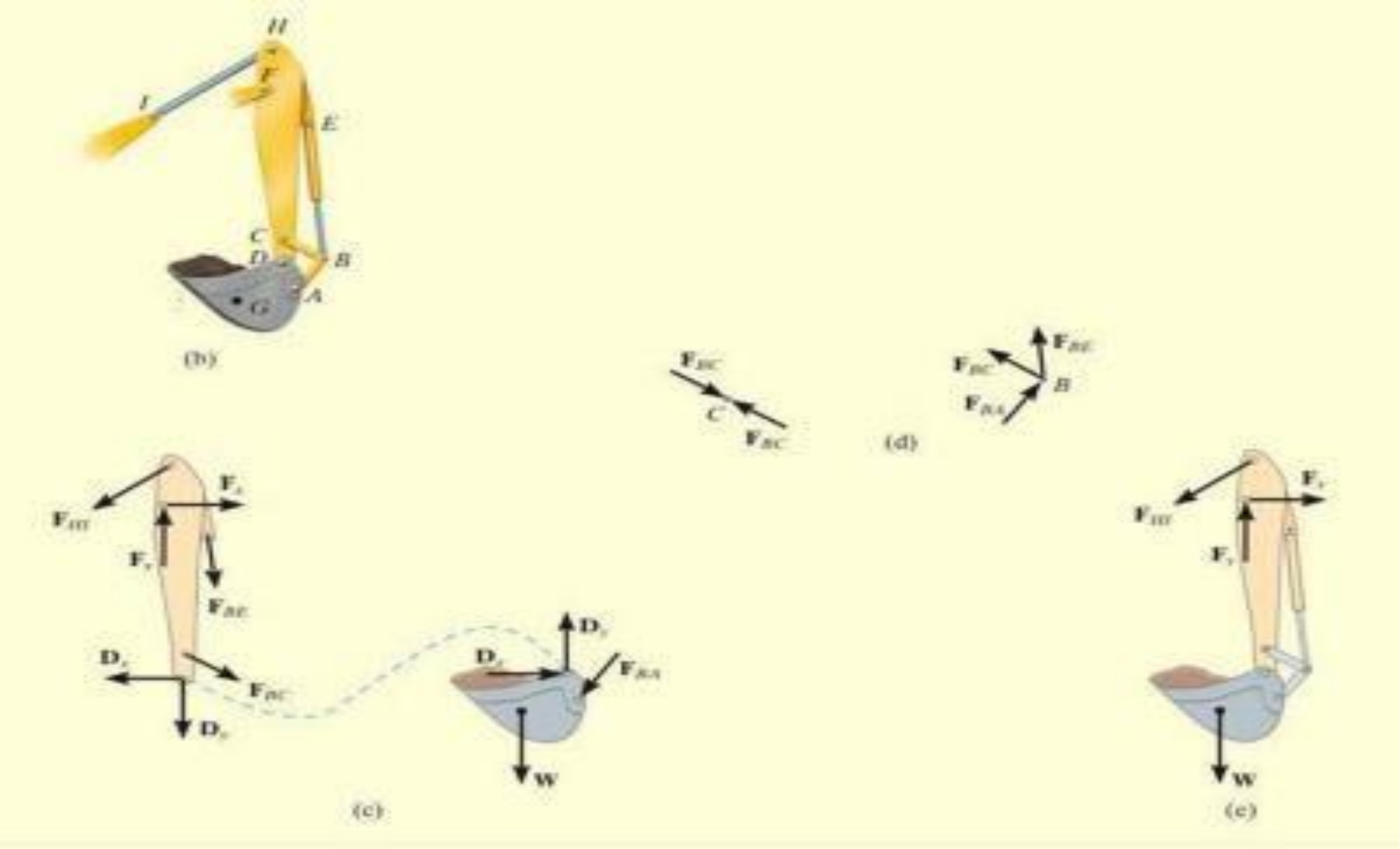
Excavator

Machines

- Machines structures contains moving parts and are designed to transmit or modify forces. Their main purpose is to transform *input forces* into *output forces*.
- Machines are usually non-rigid internally. So we use the components of the machine as a free-body.

Example: pliers, front end loaders, back hole, etc.

Solution



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C
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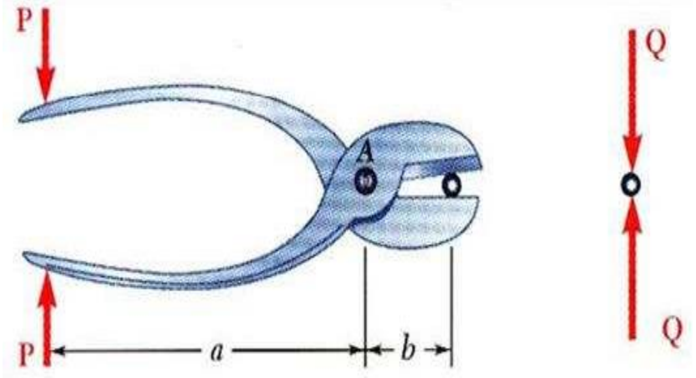
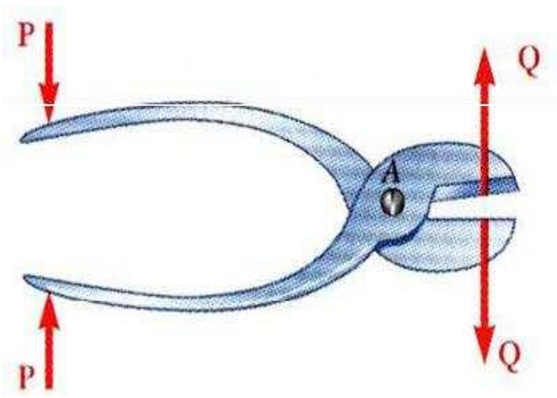
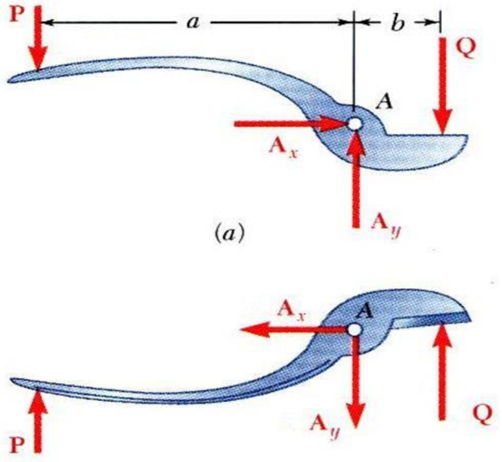
F_BE
F_BC
B
F_BA
(d)
F_HI
F_x
F_y
F_BE
D_x
F_BC
D_y
D_y
D_x
F_BA
W
(c)
F_HI
F_x
F_y
W
(e)

Example - 2

- Given the magnitude of ***P***, determine the magnitude of ***Q***.



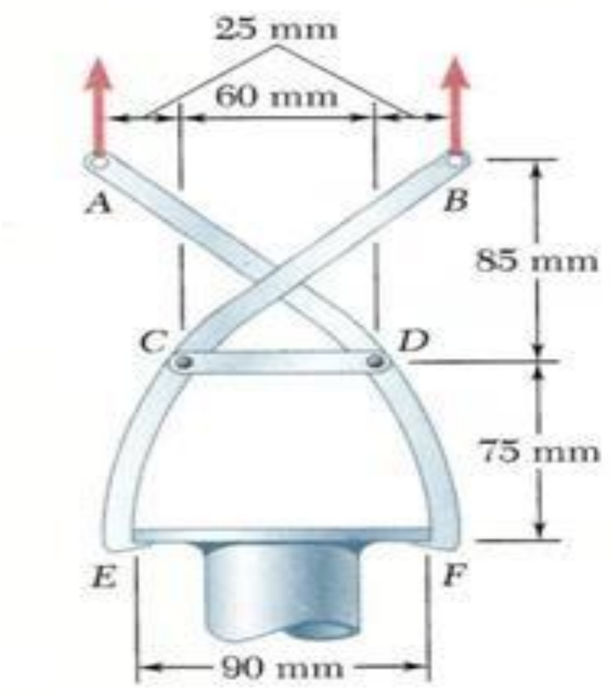


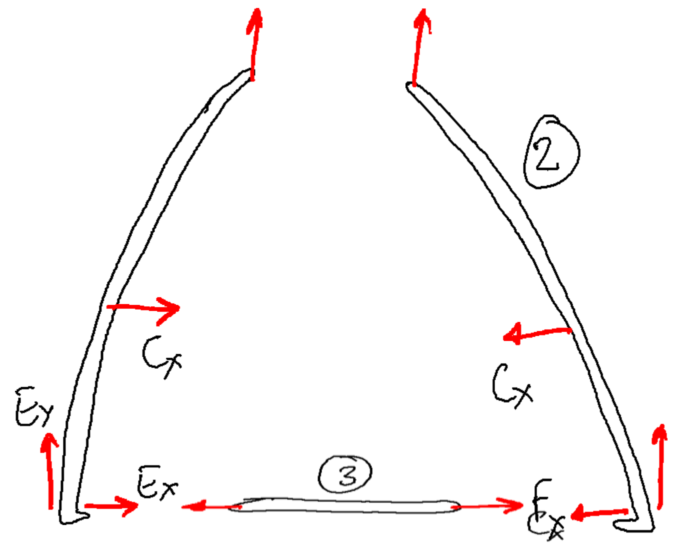


(a)

Example -3

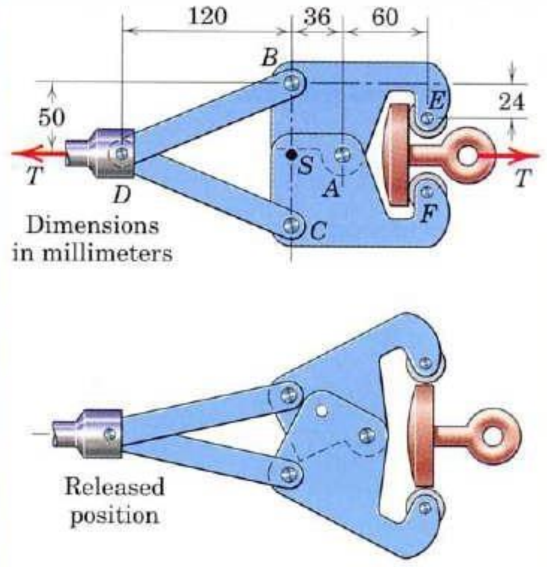
- The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the force exerted at D and F on tong ADF.

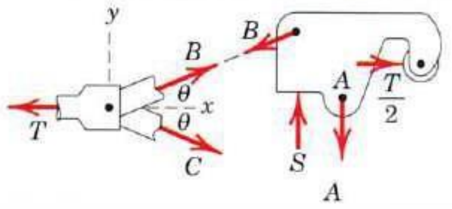




Sample Problem 4/7

The machine shown is designed as an overload protection device which releases the load when it exceeds a predetermined value T. A soft metal shear pin S is inserted in a hole in the lower half and is acted on by the upper half. When the total force on the pin exceeds its strength, it will break. The two halves then rotate about A under the action of the tensions in BD and CD, as shown in the second sketch, and rollers E and F release the eye bolt. Determine the maximum allowable tension T if the pin S will shear when the total force on it is 800 N. Also compute the corresponding force on the hinge pin A.





Solution. Because of symmetry we analyze only one of the two hinged members. The upper part is chosen, and its free-body diagram along with that for the connection at D is drawn. Because of symmetry the forces at S and A have no x-components. The two-force members BD and CD exert forces of equal magnitude $B = C$ on the connection at D. Equilibrium of the connection gives ①

$$[\Sigma F_x = 0] \qquad B\cos\theta + C\cos\theta - T = 0 \qquad 2B\cos\theta = T$$

$$B = T/(2\cos\theta)$$

From the free-body diagram of the upper part we express the equilibrium of moments about point A. Substituting $S = 800$ N and the expression for B gives

② $$[\Sigma M_A = 0] \qquad \frac{T}{2\cos\theta}(\cos\theta)(50) + \frac{T}{2\cos\theta}(\sin\theta)(36) - 36(800) - \frac{T}{2}(26) = 0$$

Substituting $\sin\theta/\cos\theta = \tan\theta = 5/12$ and solving for T give

$$T\left(25 + \frac{5(36)}{2(12)} - 13\right) = 28\ 800$$

$$T = 1477\text{ N} \quad \text{or} \quad T = 1.477\text{ kN} \qquad \textit{Ans.}$$

Finally, equilibrium in the y-direction gives us

$$[\Sigma F_y = 0] \qquad S - B\sin\theta - A = 0$$

$$800 - \frac{1477}{2(12/13)}\frac{5}{13} - A = 0 \qquad A = 492\text{ N} \qquad \textit{Ans.}$$

Helpful Hints

① It is always useful to recognize symmetry. Here it tells us that the forces acting on the two parts behave as mirror images of each other with respect to the x-axis. Thus, we cannot have an action on one member in the plus x-direction and its reaction on the other member in the negative x-direction. Consequently the forces at S and A have no x-components.

② Be careful not to forget the moment of the y-component of B. Note that our units here are newton-millimeters.