Chapter 4
Structures – part 1

STATICS, AGE-1330

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Chapter 4:

Analysis of Simple Structures

Outline

- What is a Truss?
- What is a Simple truss?
- Assumptions for Truss Design
- Methods of Truss Analysis
- Zero Force Members
- Additional Exercises

1. Introduction

Structure and frames

Introduction

In Chapter 3 we studied the equilibrium of a single rigid body or a system of connected members treated as a single rigid body.

We first drew a free-body diagram of the body showing all forces external to the isolated body and then we applied the force and moment equations of equilibrium.

In Chapter 4 we focus on the determination of the forces internal to a structure—that is, forces of action and reaction between the connected members.

An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.

To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate free-body diagrams of individual members or combinations of members.

This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.

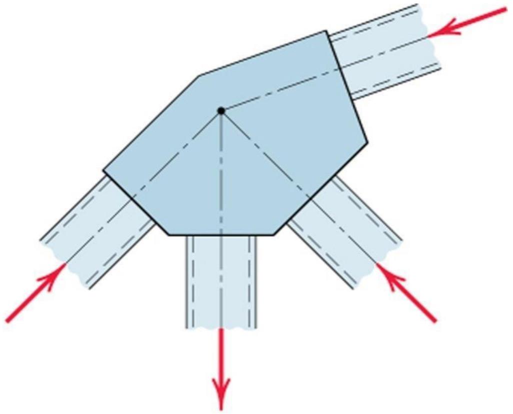
Introduction

- In this chapter, we will find **the internal forces** in the following types of structure.

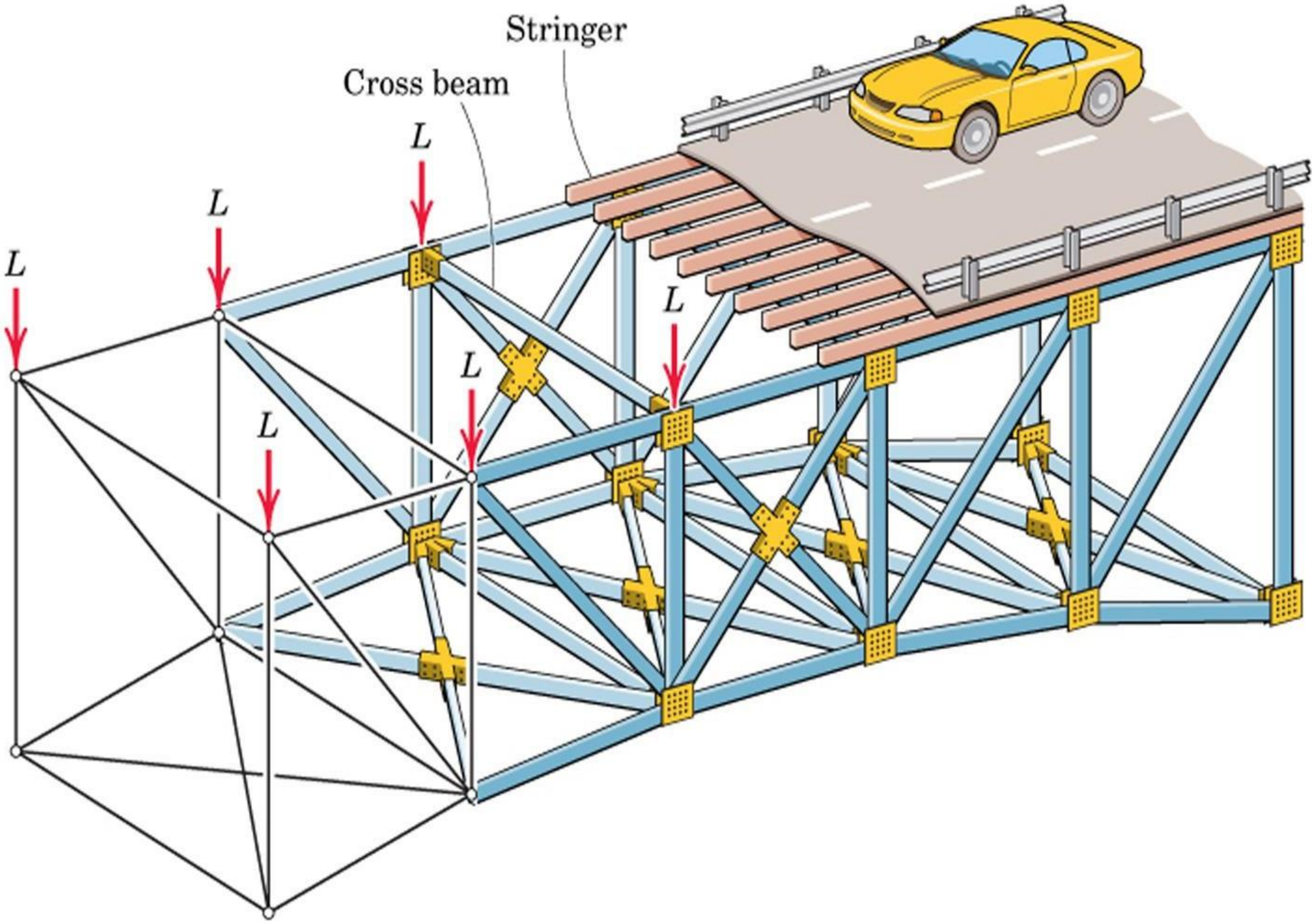
  - Trusses
  - Frames
  - Machines

- Some of the most common structures we see around us are buildings & bridges. A framework **composed of members joined at their ends** to form **a rigid** structure is called a "**truss**". Such as bridges, roof supports, derricks, grid line supports are examples of trusses.

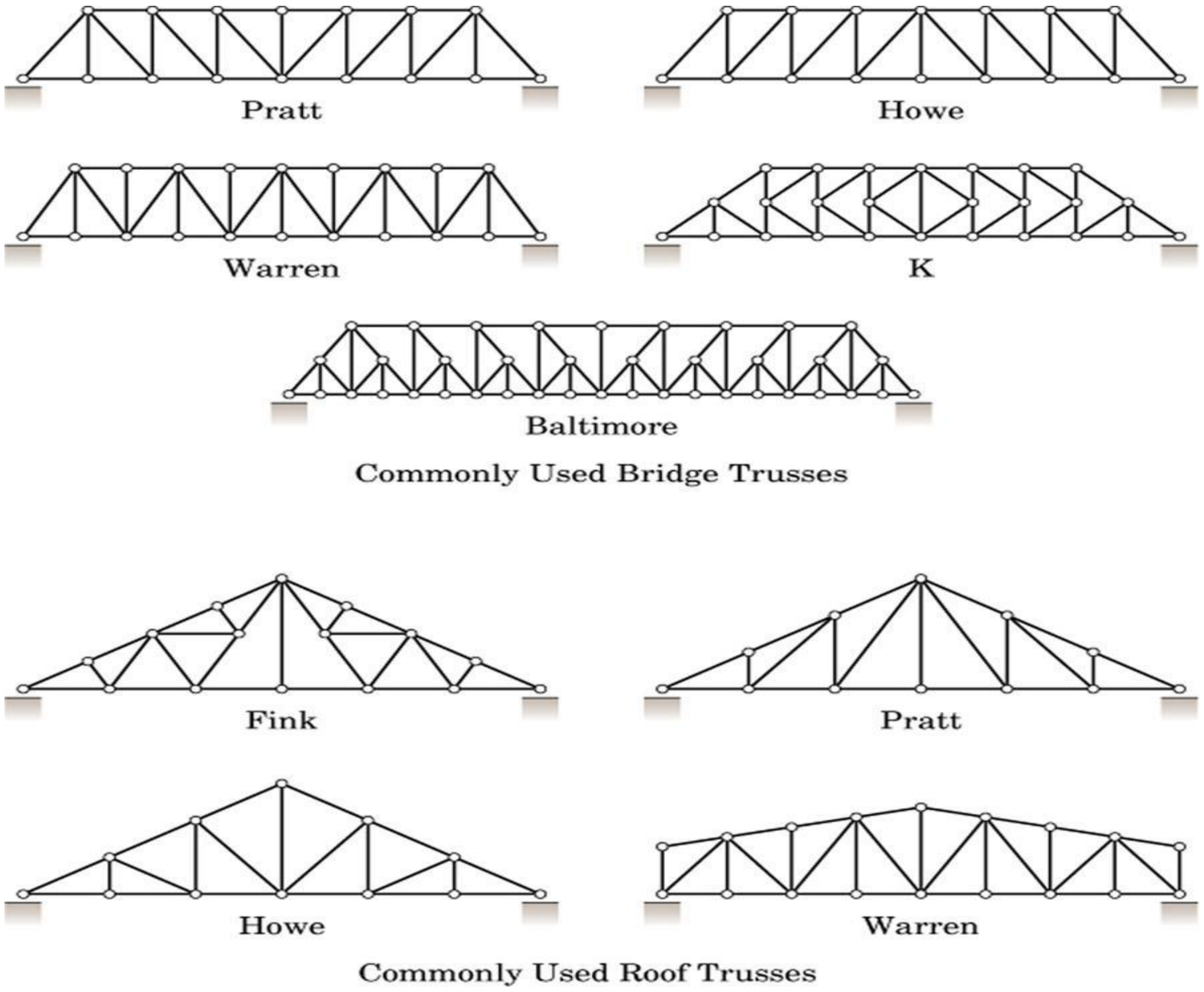
- Structural members commonly used are I-beams, channels, angles, bars and special shapes which are **fastened together** at their ends by welding, riveted connections, or large bolts or pins using large plates named as "gusset plates".



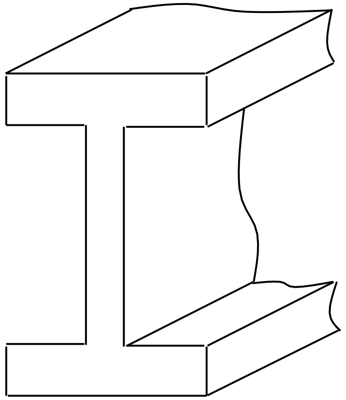
Example of Truss Structure



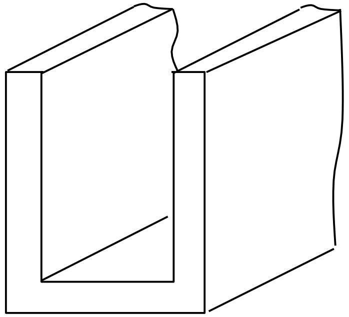
Plane Trusses



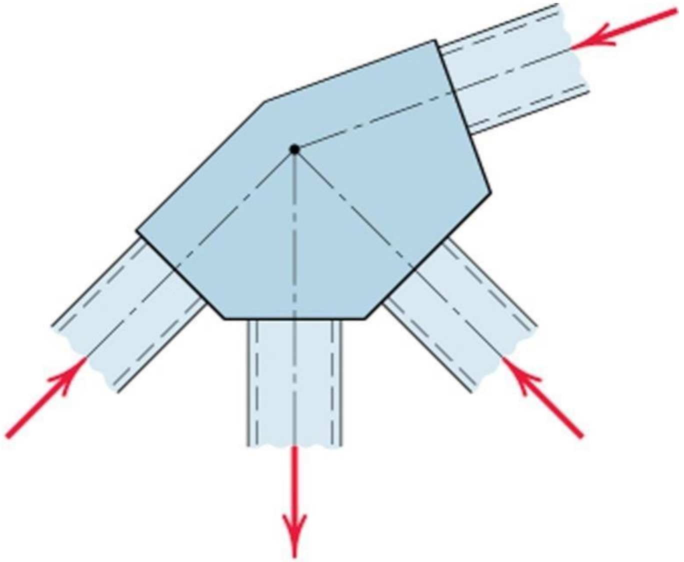

DR

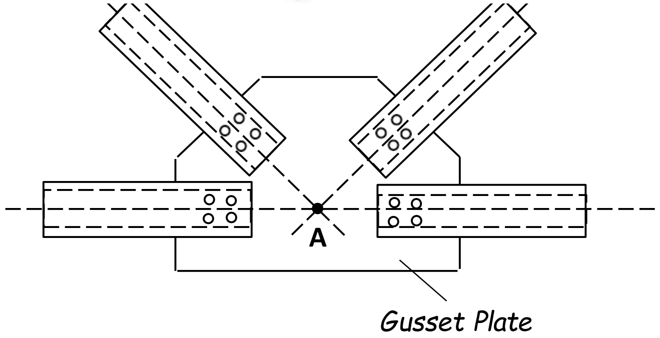



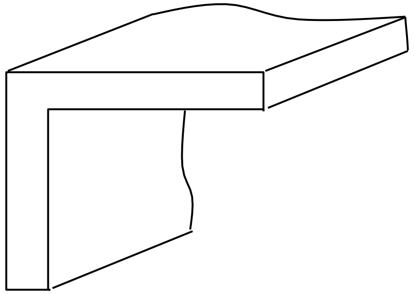
I-Beam



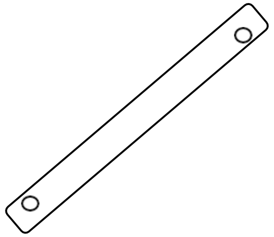
Channel Beam







Angled Beam

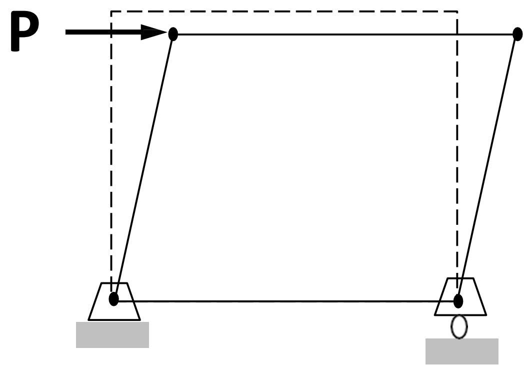


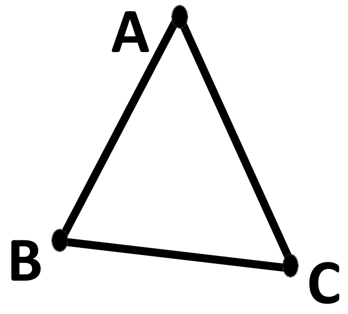
Bar

2. Simple Trusses

Simple Trusses

- The basic element of **a plane truss** is the **triangle**. **Three bars** joined by pins at their ends constitute a rigid frame. In planar trusses all bars and external forces acting on the system lie in a single plane.



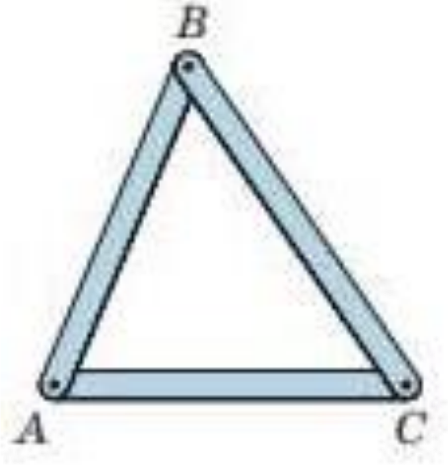


Simple Truss

- The basic element of a plane truss is the triangle
- Three bars joined by pins at their ends constitute a rigid frame
- The term rigid is used to mean no collapsible and also to mean that deformation of the members due to induced internal strains is negligible
- Structures built from a basic triangle in the manner described are known as simple trusses

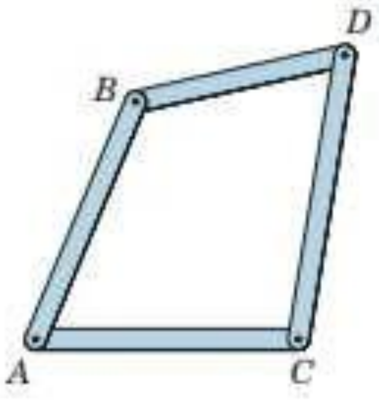
Plane Trusses

a) Stable



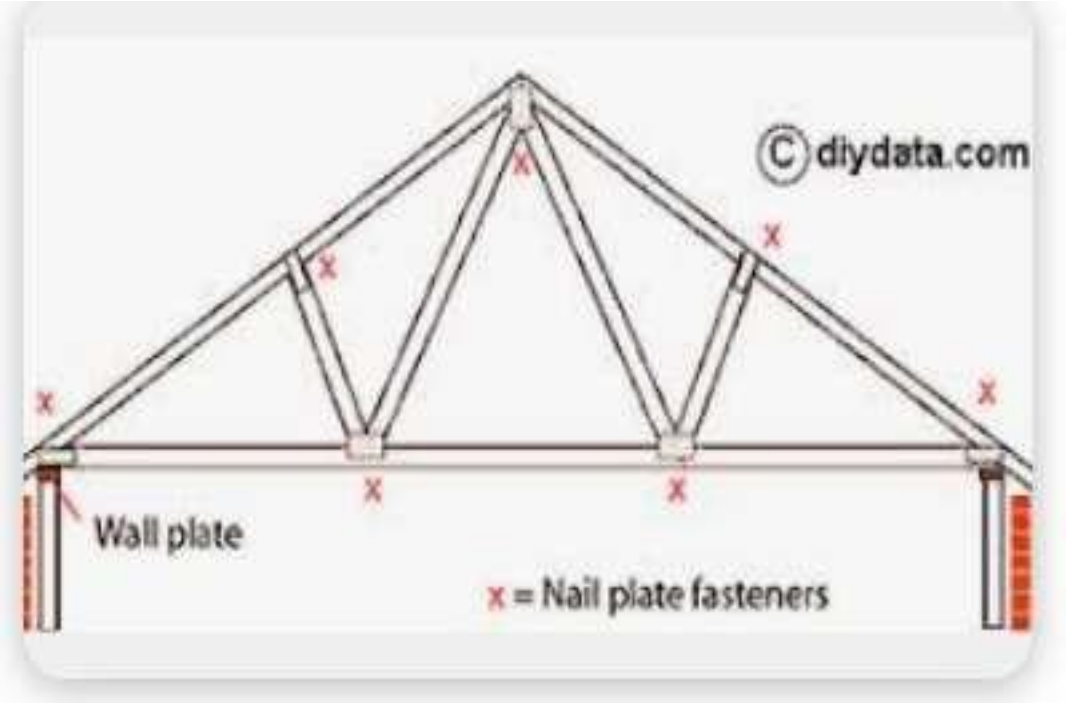
(a)

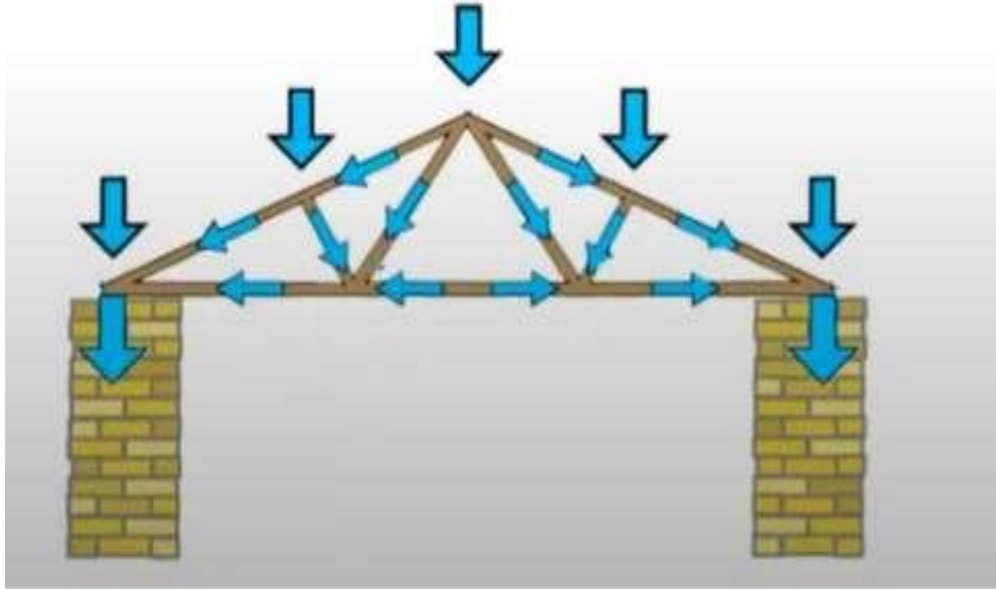
b)Unstable

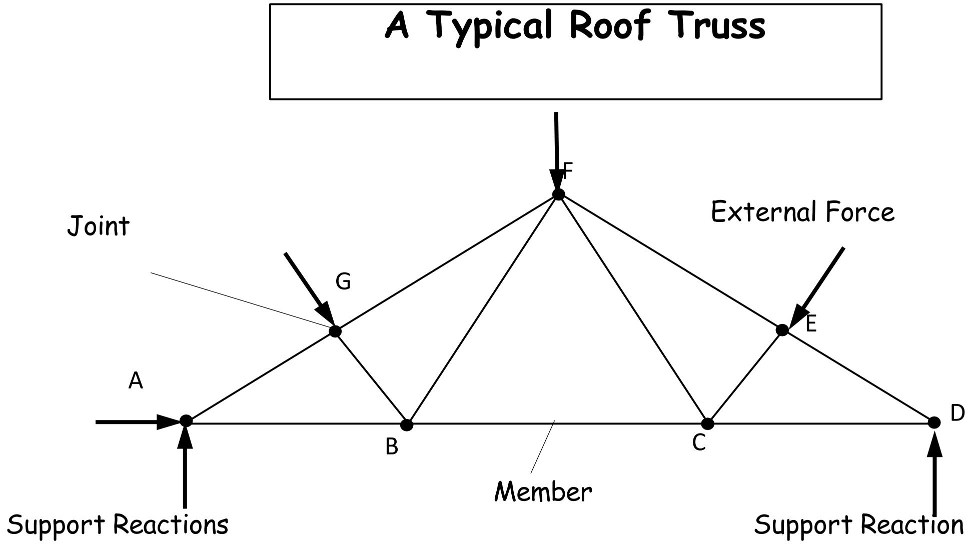


(b)

Simple Truss







- A truss can be extended by **adding extra triangles** to the system. Such trusses comprising only of triangles are called "**simple trusses** ".
- In a simple truss it is possible **to check the rigidity** of the truss.

m = 2j – 3 …… should be satisfied for rigidity

where, **m** is number of members

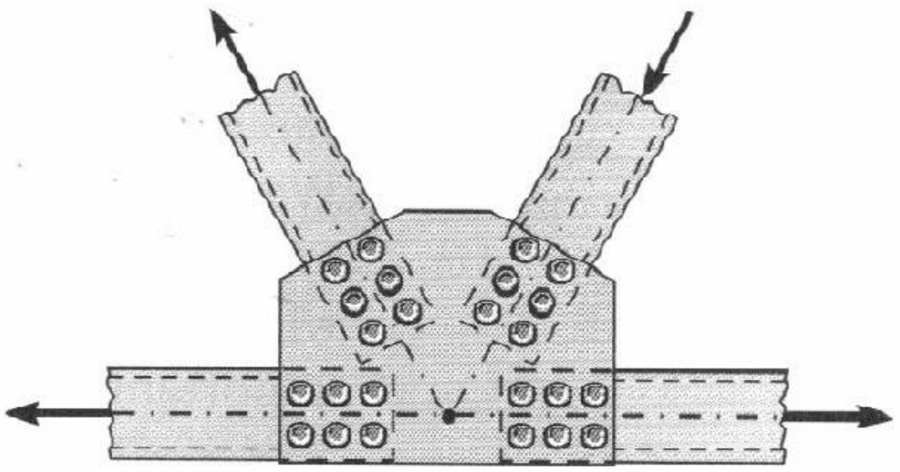
j is number of joints

3. Assumptions for Truss Design

Assumptions for Truss Design

1. **Members are joined together by smooth pins**

   - Center lines of joining members are concurrent at a point
   - In this case the joint does not support any moment since it allows for the rotation of the members.

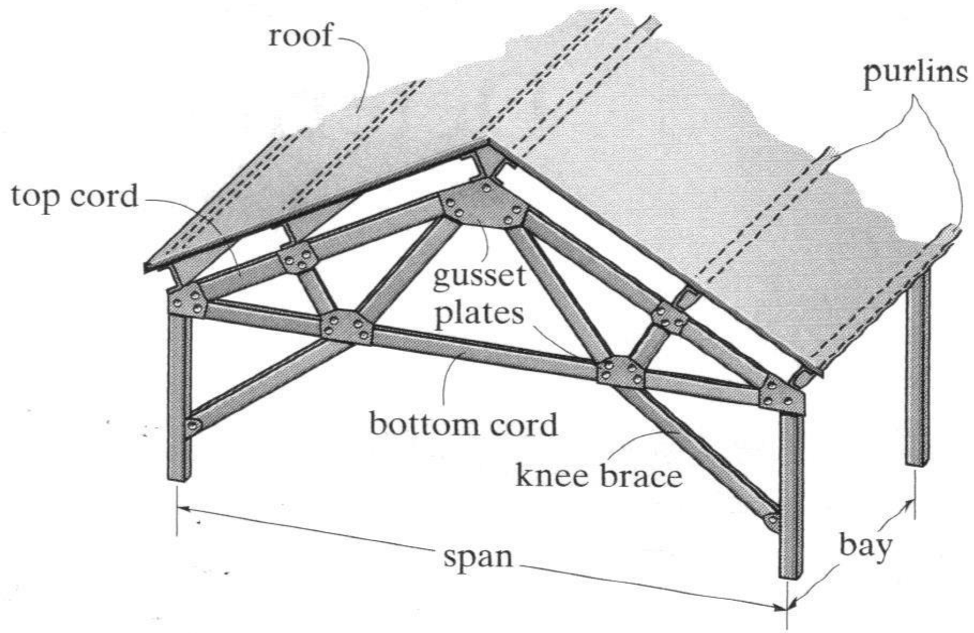


gusset plate

Assumptions for Truss Design

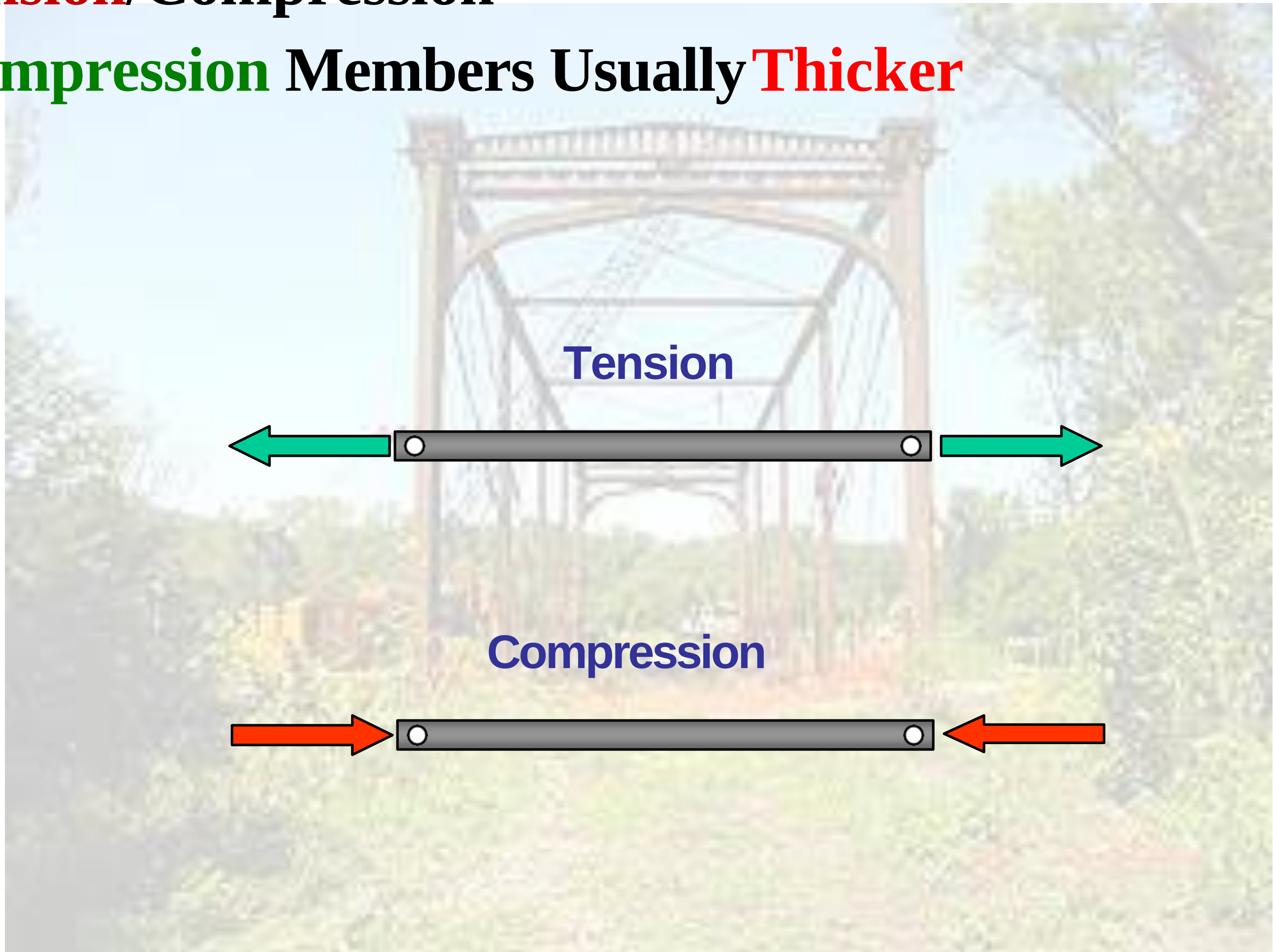
2. All loads are applied at joints (at pin connection**).**

- Self weight is neglected if **small compared** to forces they are supporting.

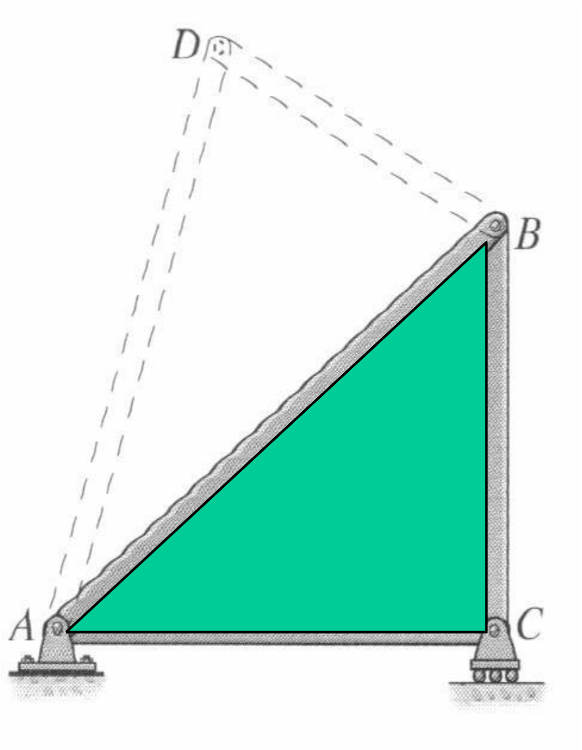


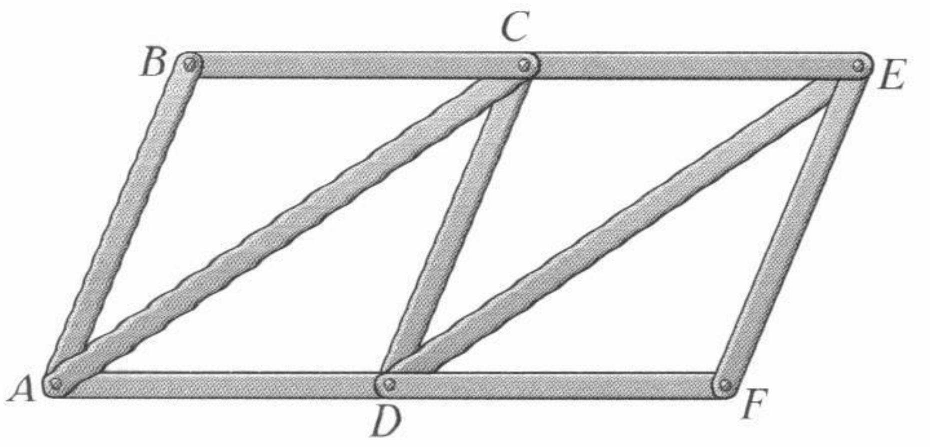
Assumptions for Truss Design

3. **Axial Force Members (all bars are two forces members).**
   - **Tension/Compression**
   - **Compression Members Usually Thicker**



Classification of Coplanar Trusses

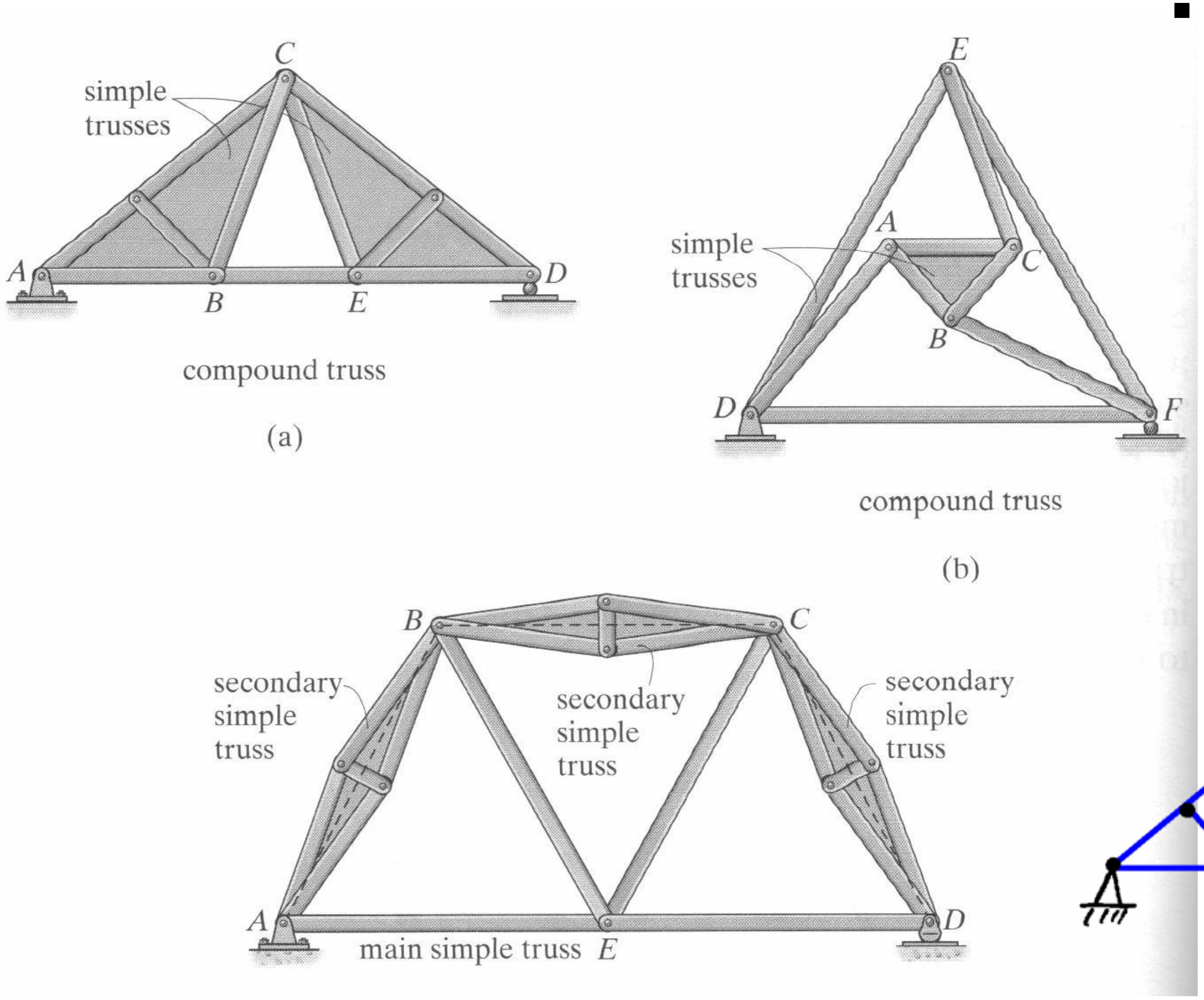




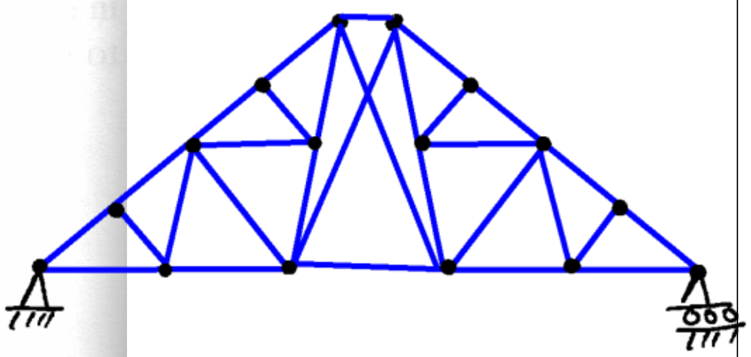
simple truss

SIMPLE TRUSS - Triangles

Classification of Coplanar Trusses

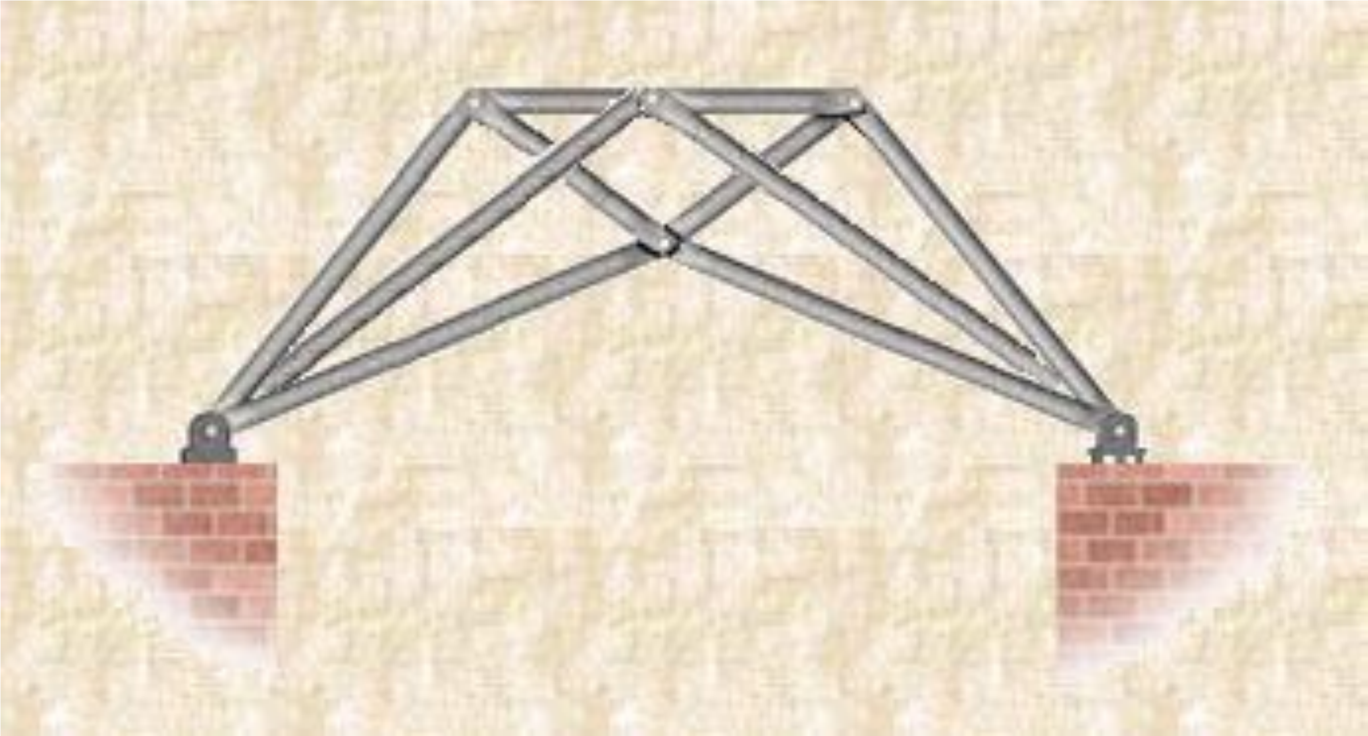


- Trusses made by joining two or more simple trusses rigidly are called **Compound Trusses**



Classification of Coplanar Trusses

Complex Truss: A complex truss uses a general layout of members different from that used in simple and compound trusses. It often incorporates **overlapping** members.



4. Methods of Truss Analysis

Methods of Truss Analysis

1) **Method of joints**

2) **Method of Sections**

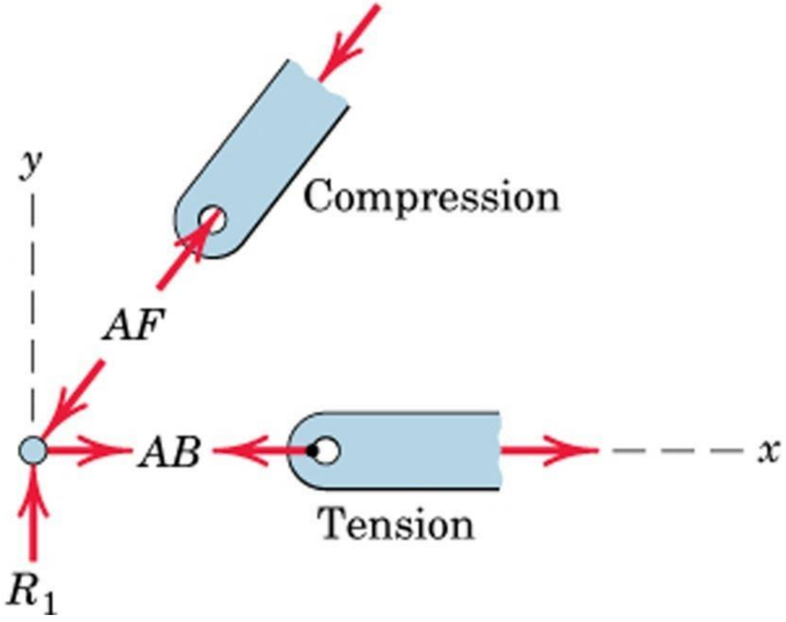
Method of joints

- This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint
- The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved

Force Analysis of plane trusses

1. Method of Joints

- This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint
- The method deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved ($\Sigma F_x = 0$, and $\Sigma F_y = 0$ for each joint)



Method of Joints - Procedure

- **Consider one joint at a time – Draw FBD**
  - **Condition:** At least one known force; at most two unknown forces
  - Determine support reactions
- **Establish sense of unknown force**
  - Assume **unknown** forces are in **tension**
- **Write equations of equilibrium of node**
  - **Select x-y coordinates** such that forces on FBD can **be easily resolved** into components
- **Take advantage of symmetries**
- **Identify zero force members**

Sign Convention!!!

- It is initially assumed that all the members work in tension. Therefore, when the FBDs of pins are being constructed, members are shown directed **away from the joint**
- After employing **the equations of equilibrium:**
  - if the result yields **a positive value** (+), it means that the member actually works in **tension (T),**
  - if the result yields a **negative value** (-), it means that the member works in **compression (C)**

Methods of Truss Analysis

1) Method of joints

Method of joints

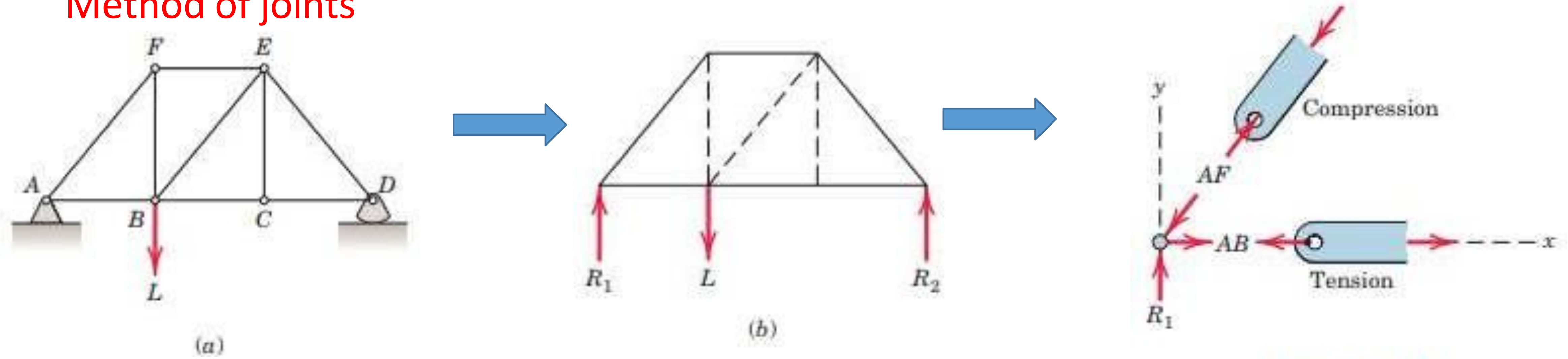


Figure 4/7

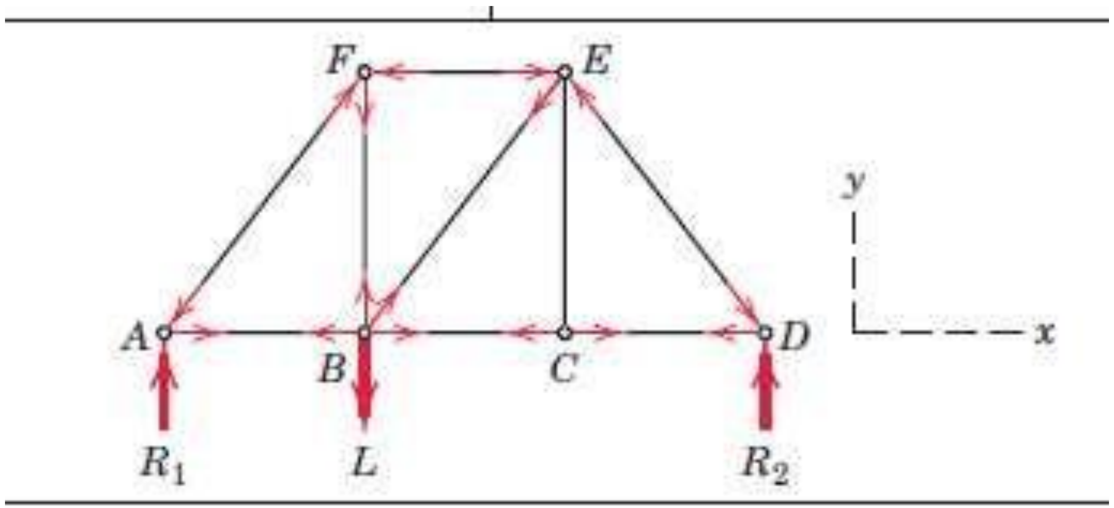
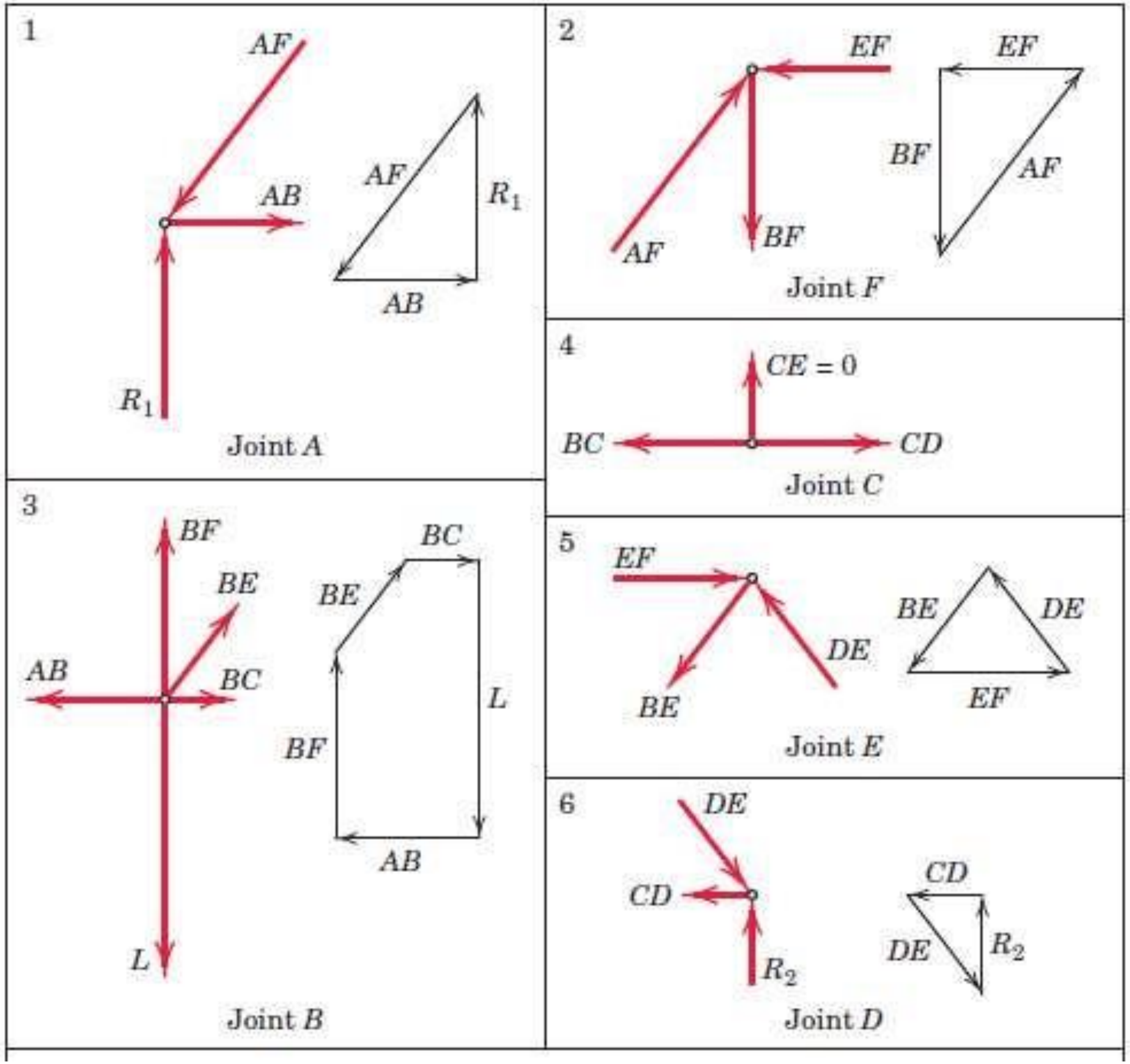

F
E
A
B
C
D
R1
L
R2
y
x

Figure 4/8

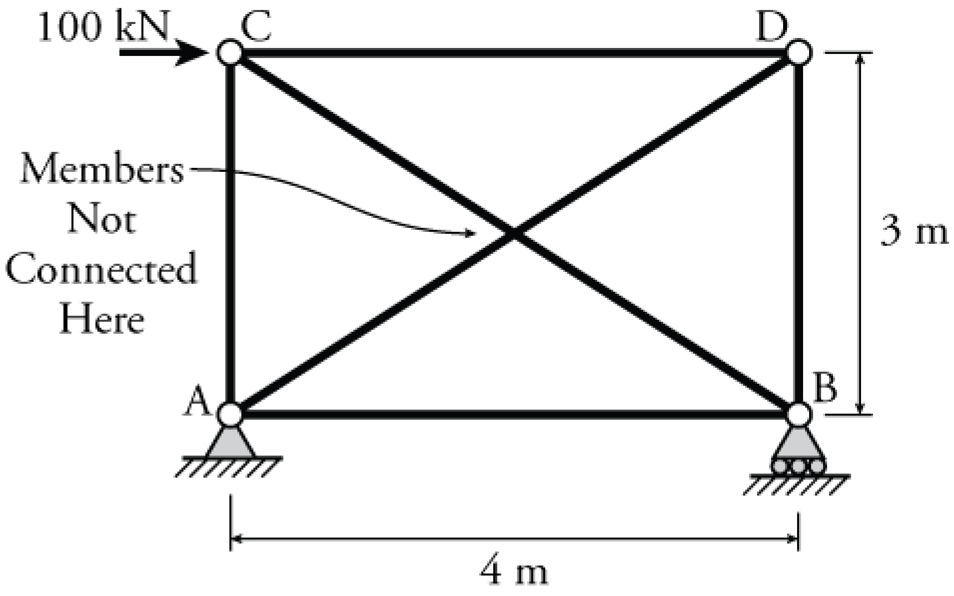

1
AF
AB
R1
Joint A
2
EF
BF
AF
Joint F
3
BF
BE
BC
AB
L
Joint B
4
CE = 0
BC
CD
Joint C
5
EF
BE
DE
Joint E
6
DE
CD
R2
Joint D

$m + 3 = 2j$

Where;

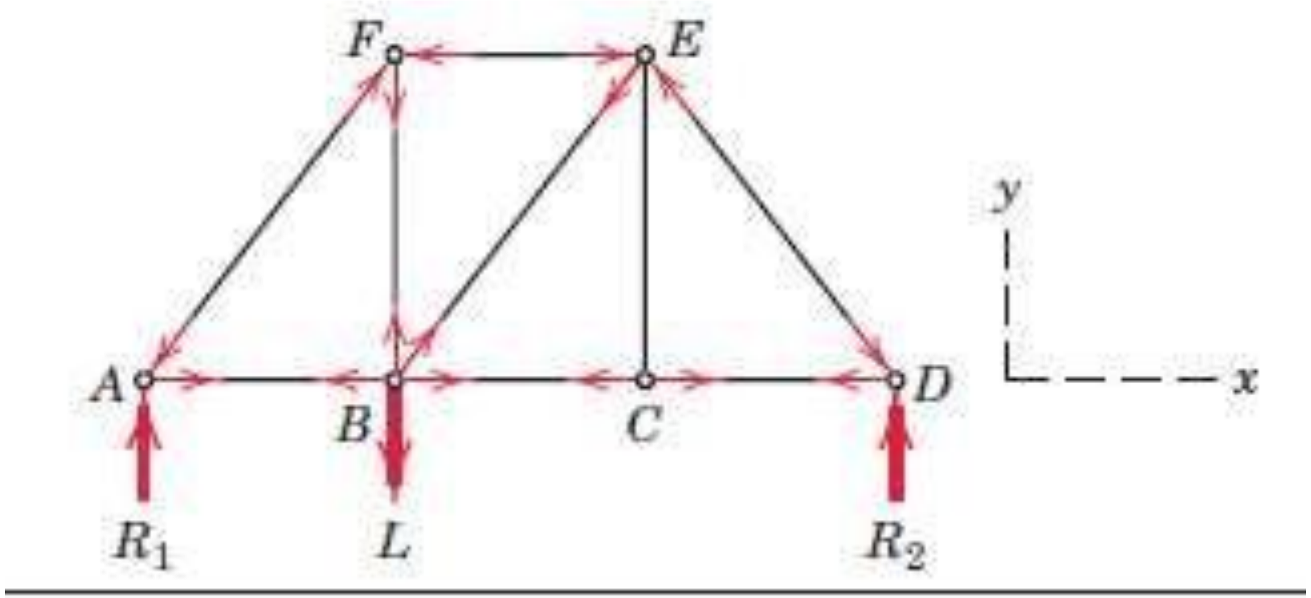
m + 3 = tension or compression forces, and three reactions

j = number of joints



If $m + 3 > 2j$, there are more members than independent equations, and the truss is statically indeterminate internally with redundant members present

If $m + 3 < 2j$, there is a deficiency of internal members, and the truss is unstable and will collapse under load.

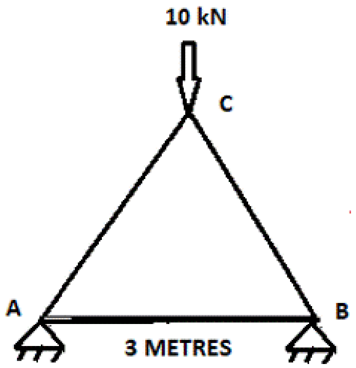


m = 9, number of joints = 6

m + 3 = 9 + 3 = 12 and 2 x j = 2 x 6= 12

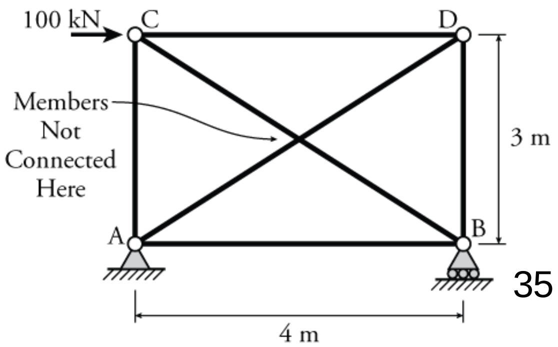
Both sides are equal → stable and determinate structure

Internal and External Redundancy



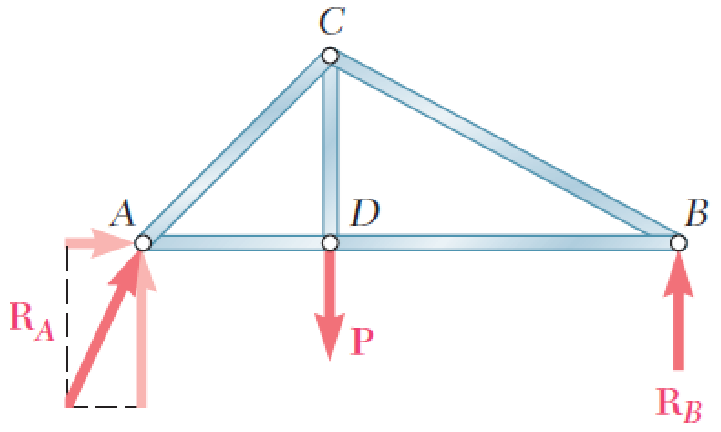
If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute external redundancy.

If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute internal redundancy and the truss is again statically indeterminate.

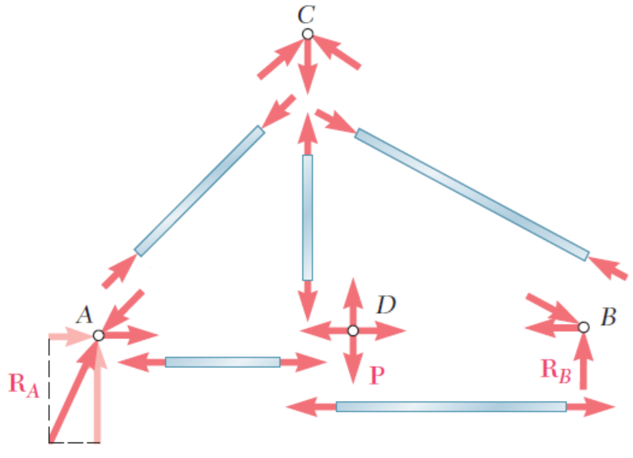


Example: Method of Joints

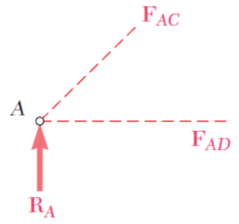
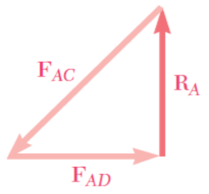
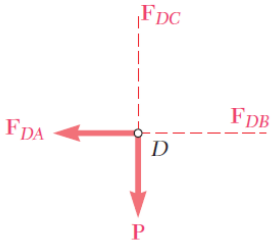
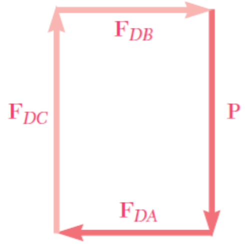
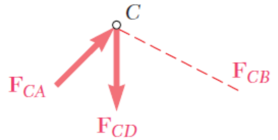
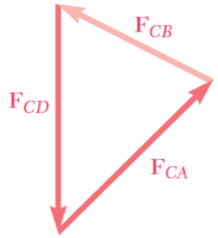
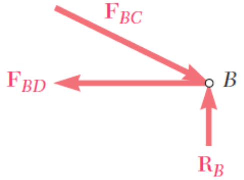
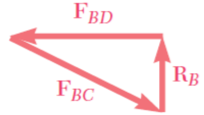
Truss in Equilibrium => Each Joint in Equilibrium



a)



b)

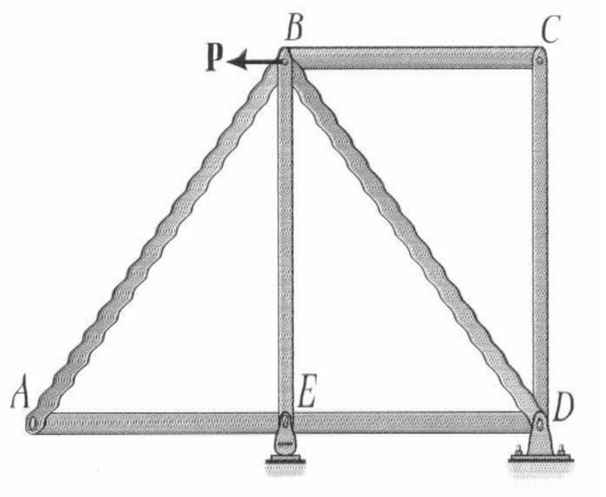
	Free Body Diagram	Force Polygon
Node A		
Node D		
Node C		
Node B		

5. Zero-Force Members

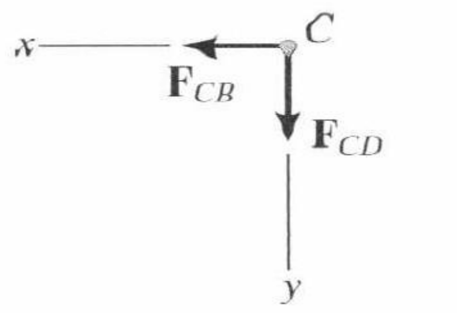
Zero-Force Members

- Truss analysis using the **method of joints** is **greatly simplified if we can first identify those members which support no loading.** It is **used to** increas the stability of the truss during construction and to provide added support if the loading is changed.

- Condition-1: If only **two members form a truss joint and no external load or support reaction** is applied to the joint, the two members must be **zero-force** Members



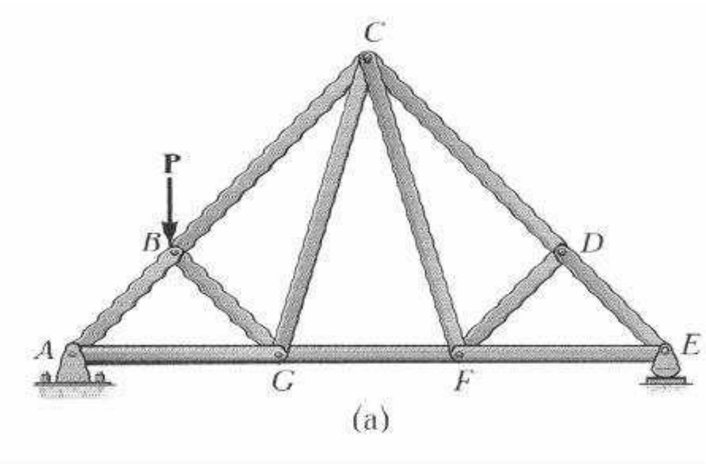
(a)



$$\overset{+}{\leftarrow} \Sigma F_x = 0;\ F_{CB} = 0$$

$$+\downarrow \Sigma F_y = 0;\ F_{CD} = 0$$

- Condition-2: If three members form a truss joint for which two of the members are **collinear**, the third member is a zero-force member provided **no external force or support reaction** is applied to the joint.



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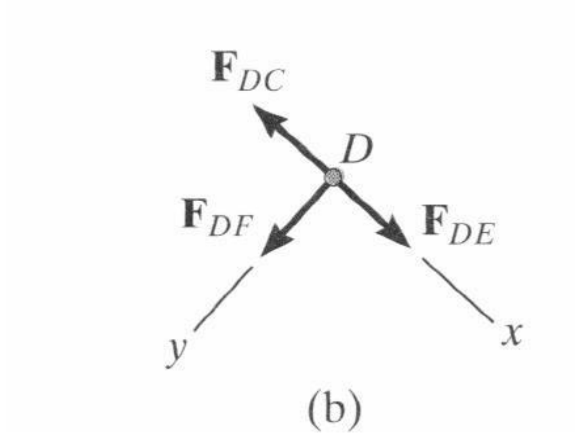

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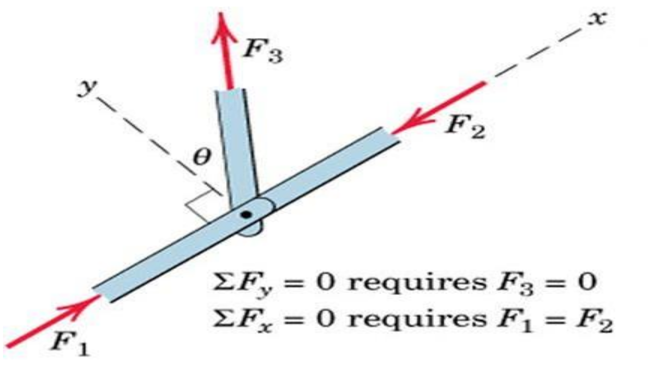
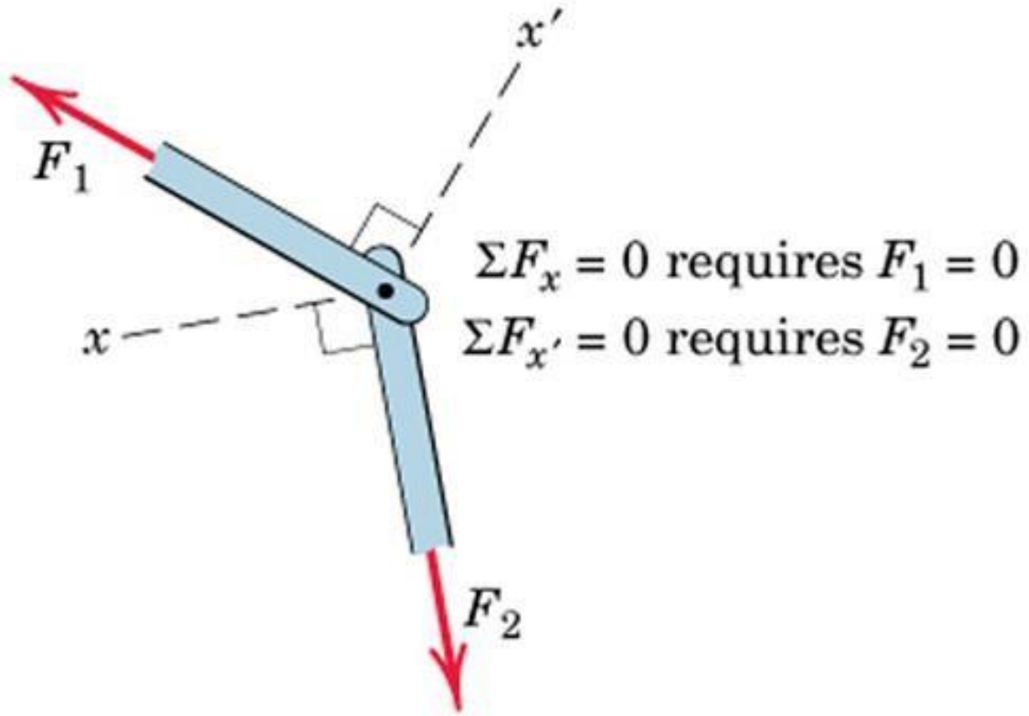
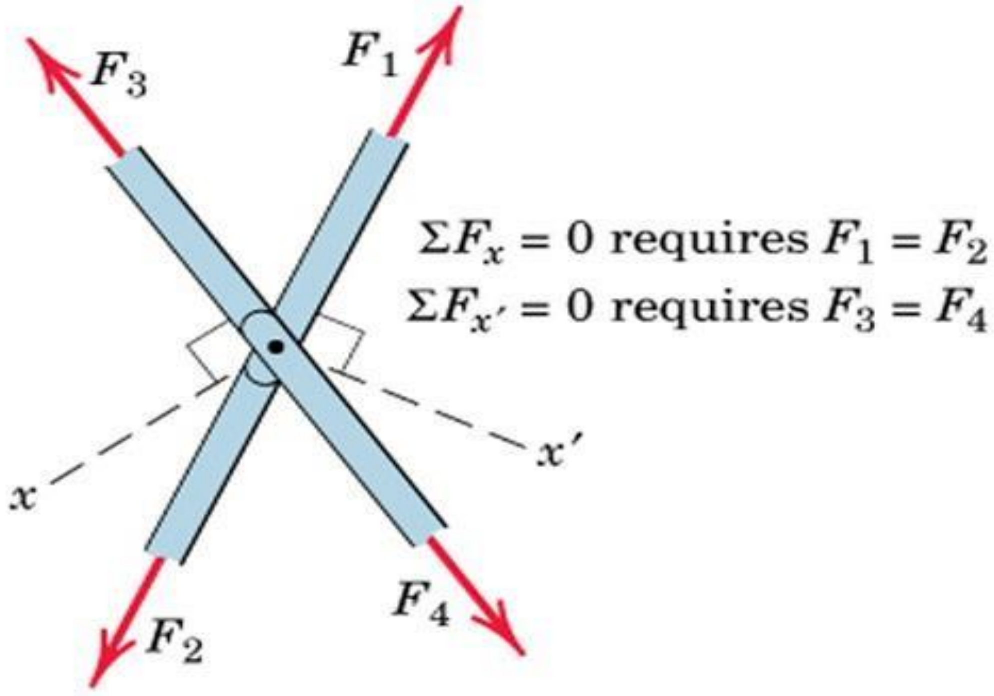


- Condition-3: When two **non-collinear** members are joined as shown, then in the absence of an externally load at this joint, the forces in both members must be zero.



- Condition-4: **Equal Force Members.** When two pairs of collinear members are joined as shown, the forces in each pair **must be equal** and **opposite.**



6. Additional Exercises

Compute the force in each member of the loaded cantilever truss by the method of joints.

SAMPLE PROBLEM 4/1

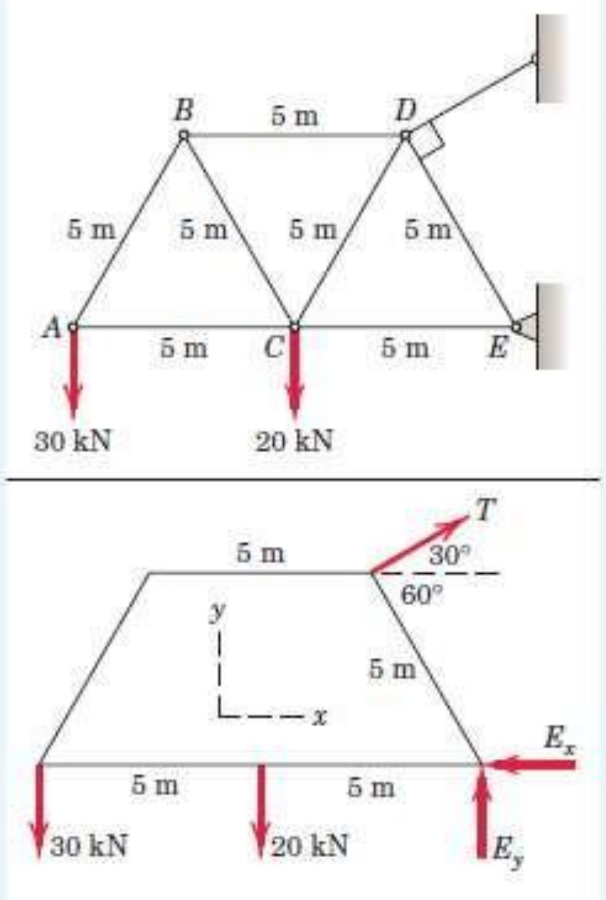
Compute the force in each member of the loaded cantilever truss by the method of joints.

Solution. If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80$ kN
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3$ kN
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10$ kN

Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A. Equilibrium requires

$[\Sigma F_y = 0]$	$0.866AB - 30 = 0$	$AB = 34.6$ kN T	*Ans.*
$[\Sigma F_x = 0]$	$AC - 0.5(34.6) = 0$	$AC = 17.32$ kN C	*Ans.*



① where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C. The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$[\Sigma F_y = 0]$ $\quad 0.866BC - 0.866(34.6) = 0 \qquad BC = 34.6 \text{ kN } C$ *Ans.*

$[\Sigma F_x = 0]$ $\quad BD - 2(0.5)(34.6) = 0 \qquad BD = 34.6 \text{ kN } T$ *Ans.*

Joint C now contains only two unknowns, and these are found in the same way as before:

$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$ *Ans.*

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

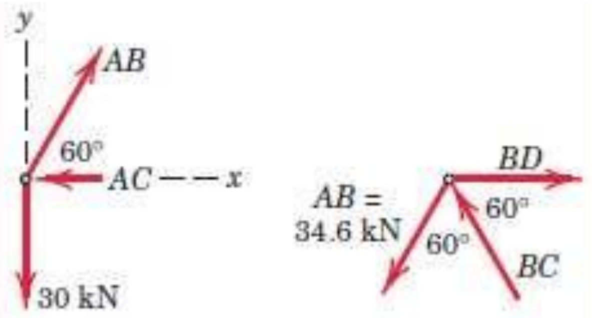
$CE = 63.5 \text{ kN } C$ *Ans.*

Finally, from joint E there results

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \qquad DE = 11.55 \text{ kN } C$ *Ans.*

and the equation $\Sigma F_x = 0$ checks.

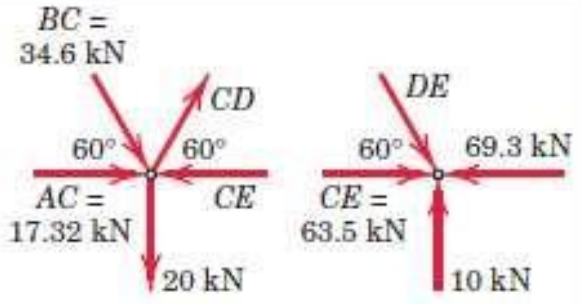
Note that the weights of the truss members have been neglected in comparison with the external loads.



Joint A Joint B

Helpful Hint

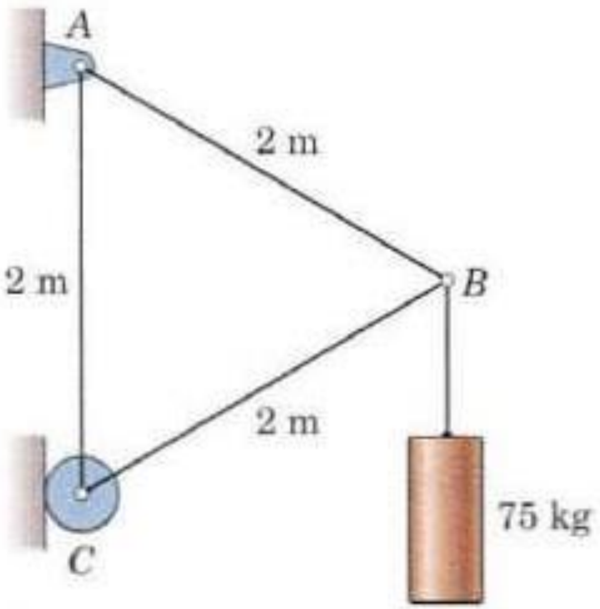
① It should be stressed that the tension/compression designation refers to the *member*, not the joint. Note that we draw the force arrow on the same side of the joint as the member which exerts the force. In this way tension (arrow away from the joint) is distinguished from compression (arrow toward the joint).



Joint C Joint E

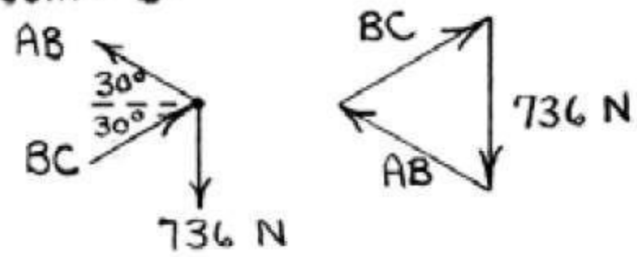
4/1 Determine the force in each member of the simple equilateral truss.

Ans. $AB = 736$ N T, $AC = 368$ N T, $BC = 736$ N C



4/1   Load = 75(9.81) = 736 N

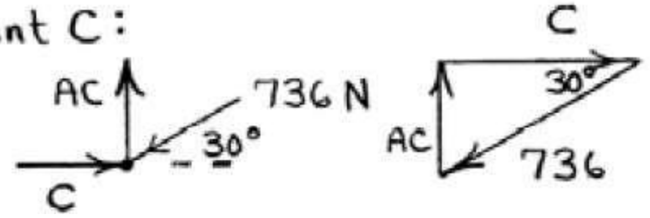
Joint B:



AB = 736 N T

BC = 736 N C

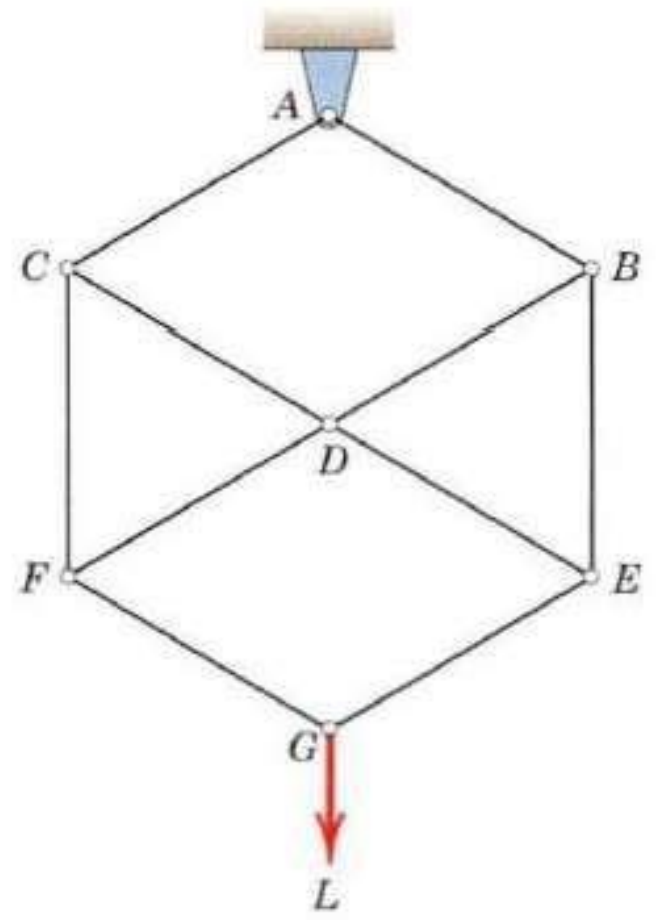
Joint C:



$AC = 736\left(\frac{1}{2}\right)$

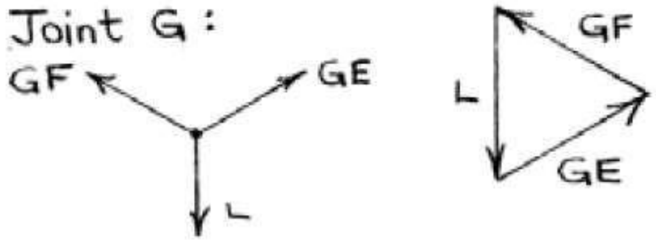
$= 368$ N T

4/10 Solve for the forces in members *BE* and *BD* of the truss which supports the load *L*. All interior angles are 60° or 120°.



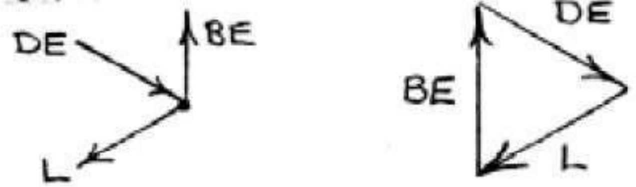
4/10

Joint G:



GE = GF = L C

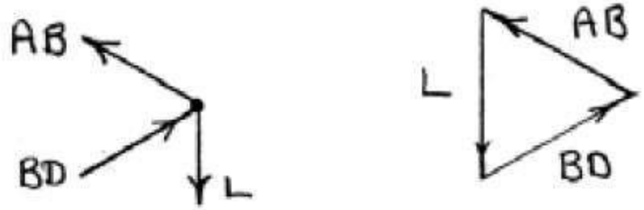
Joint E:



BE = L T

DE = L C

Joint B:

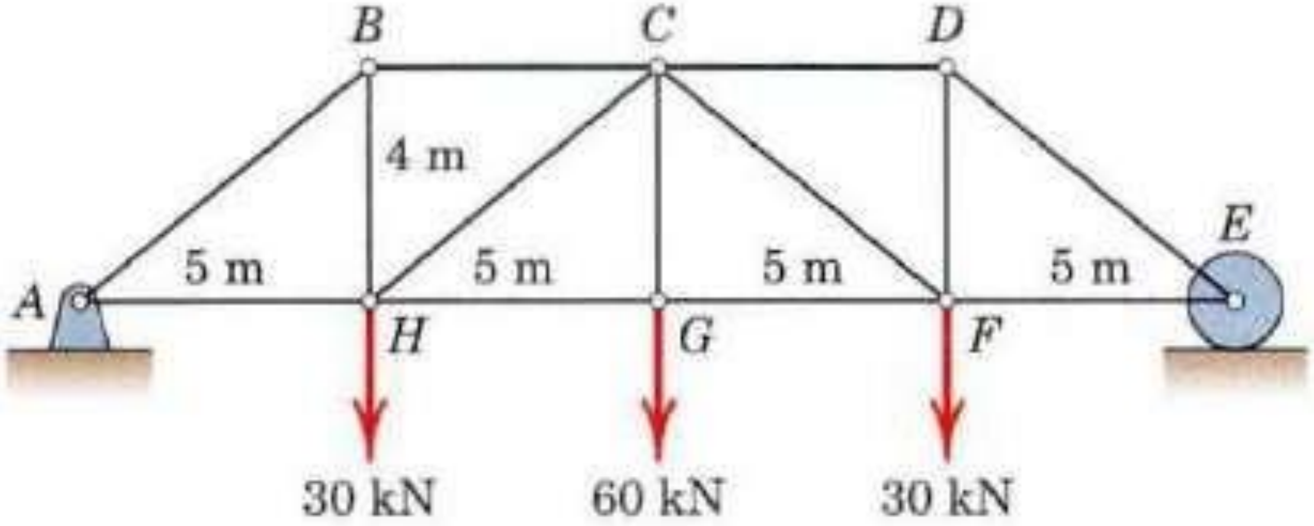


BD = L C

AB = L T

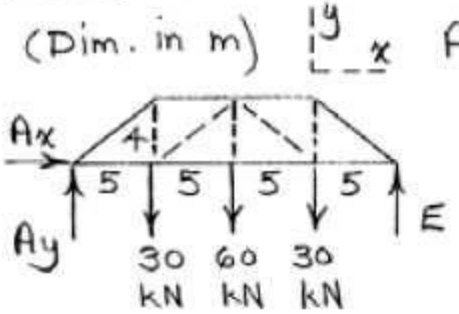
4/7 Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading.

Ans. $AB = DE = 96.0$ kN C
$AH = EF = 75$ kN T, $BC = CD = 75$ kN C
$BH = CG = DF = 60$ kN T
$CH = CF = 48.0$ kN C, $GH = FG = 112.5$ kN T

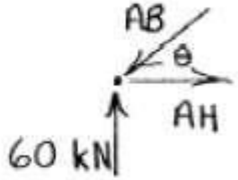


4/7 As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

(Dim. in m) $A_y = E = 60$ kN by $\Sigma F_y = 0$ and symmetry.



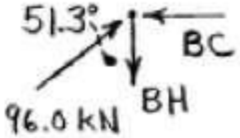
Joint A: $\left(\theta = \tan^{-1}\left(\frac{4}{5}\right) = 38.7°\right)$



$\Sigma F_y = 0$: $60 - AB\sin\theta = 0$, $\underline{AB = 96.0 \text{ kN } C}$

$\Sigma F_x = 0$: $AH - 96.0\cos\theta = 0$, $\underline{AH = 75 \text{ kN } T}$

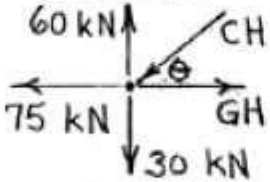
Joint B:



$\Sigma F_x = 0$: $-BC + 96.0\sin 51.3° = 0$, $\underline{BC = 75 \text{ kN } C}$

$\Sigma F_y = 0$: $-BH + 96.0\cos 51.3° = 0$, $\underline{BH = 60 \text{ kN } T}$

Joint H:

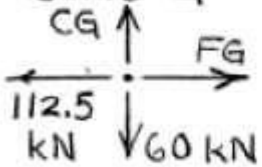


$\Sigma F_y = 0$: $-CH\sin\theta + 30 = 0$, $\underline{CH = 48.0 \text{ kN } C}$

$\Sigma F_x = 0$: $-48.0\cos\theta + GH - 75 = 0$

$\underline{GH = 112.5 \text{ kN } T}$

Joint G:

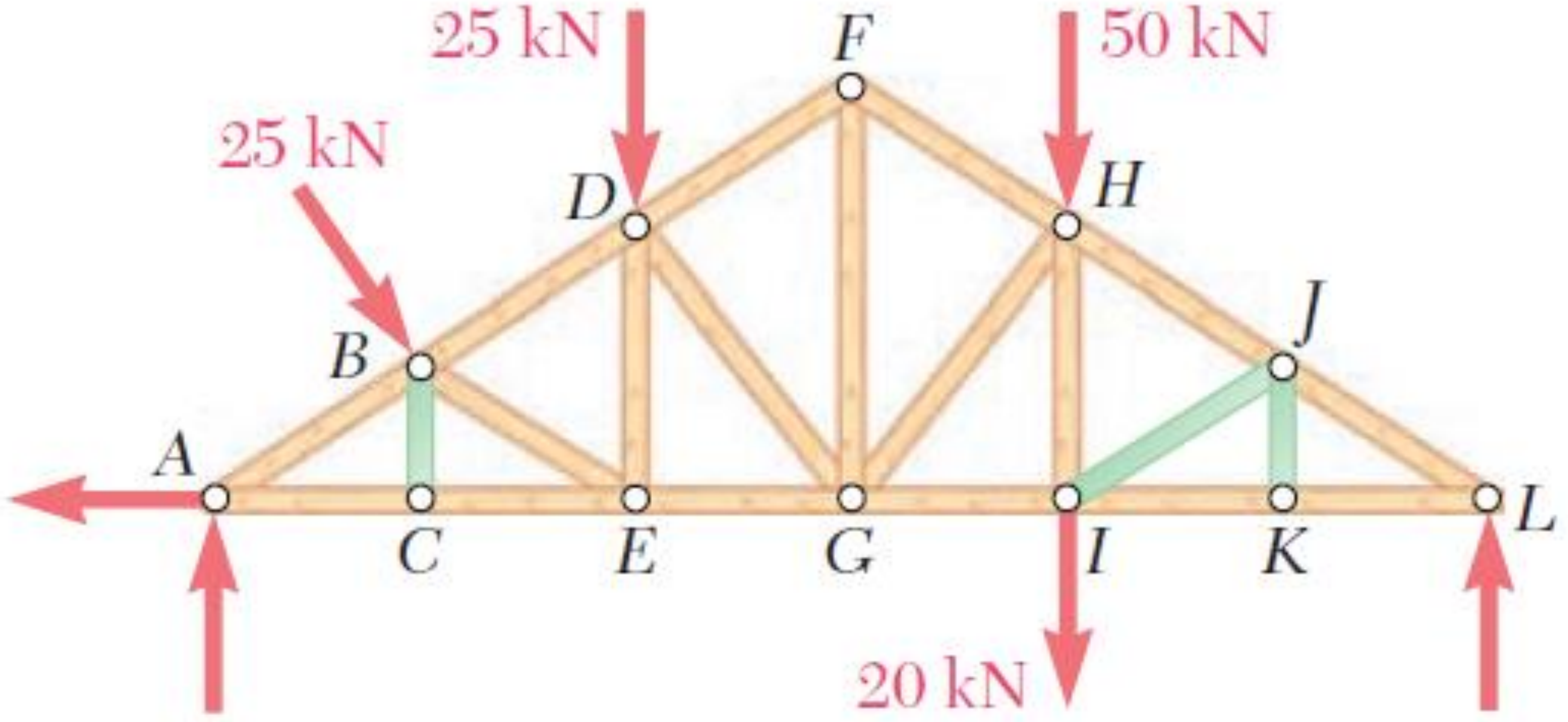


$\Sigma F_y = 0 \Rightarrow \underline{CG = 60 \text{ kN } T}$

By symmetry:
FG = 112.5 kN T, CF 48.0 kN C
CD = 75 kN C, DF = 60 kN T
EF = 75 kN T, DE = 96.0 kN C

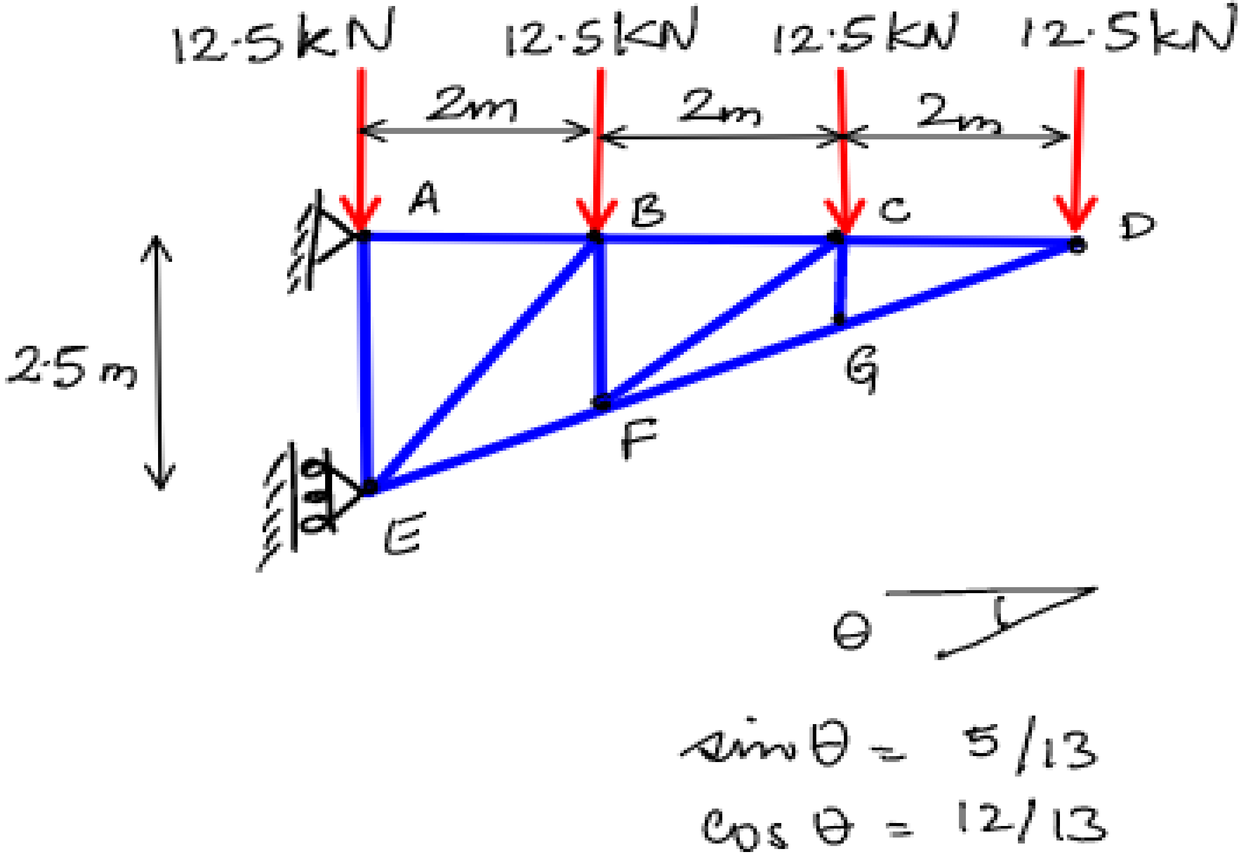
Example 1

- Using the method of joints, determine all zero force members of the Fink roof truss shown in fig. Assume all joints are pin connections.



Example 2

Determine the force in each member of the loaded truss.



Example 3

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.