Chapter 4 – Kinematics of Rigid Bodies

1. Translation
2. Rotation about a fixed axis
3. General plane motion

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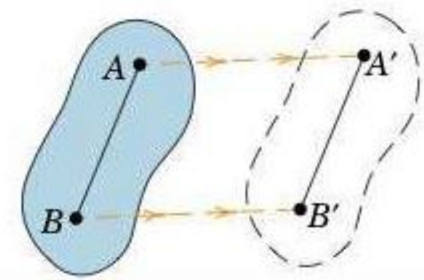
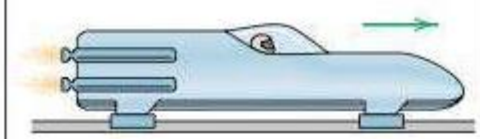
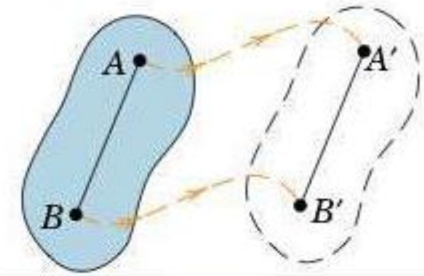
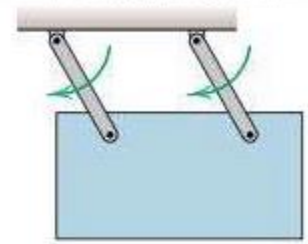
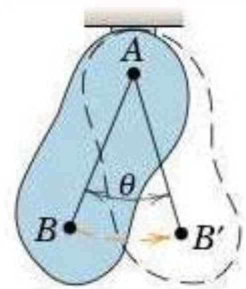
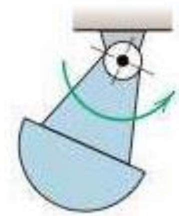
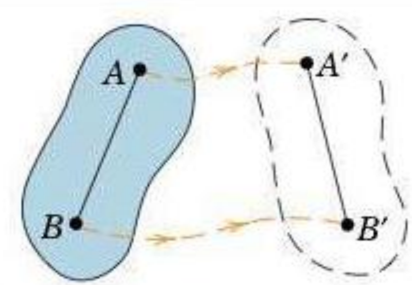
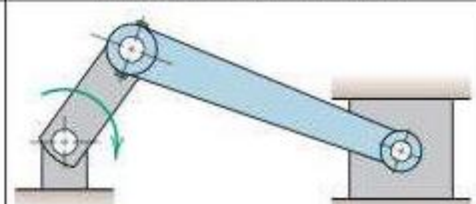
Kinematics of Rigid Bodies

Relates the motions of various particles forming a rigid body w/o ref. to forces causing the motion.

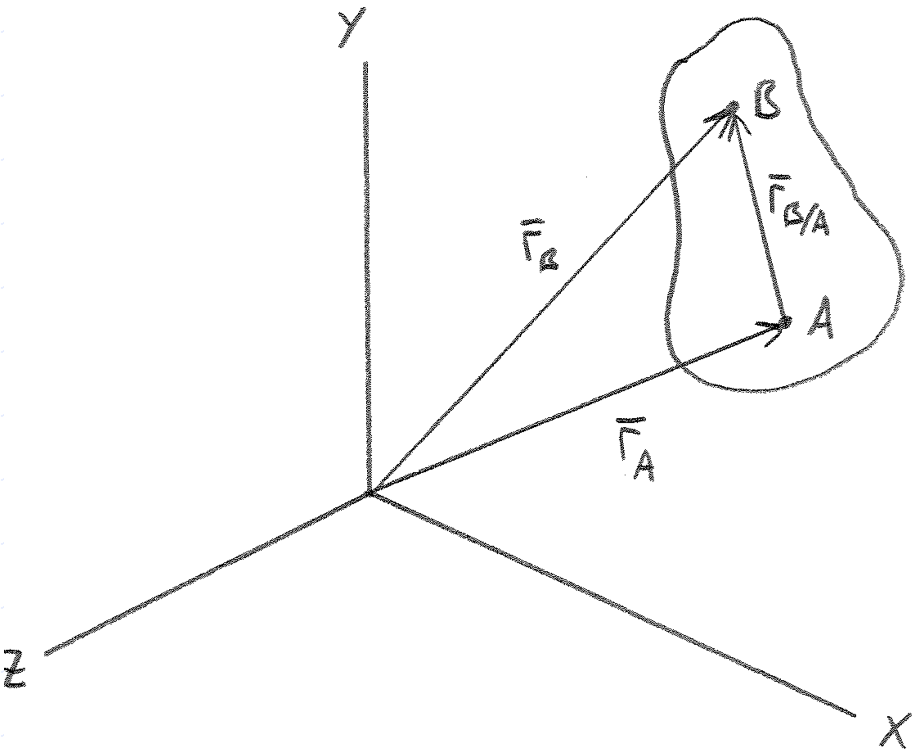
Types of motion:

1. Translation
2. Rotation about a fixed axis
3. General Plane motion

	Type of Rigid-Body Plane Motion	Example
(*a*) Rectilinear translation		 Rocket test sled
(*b*) Curvilinear translation		 Parallel-link swinging plate
(*c*) Fixed-axis rotation		 Compound pendulum
(*d*) General plane motion		 Connecting rod in a reciprocating engine

1. Translation

Translation



$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

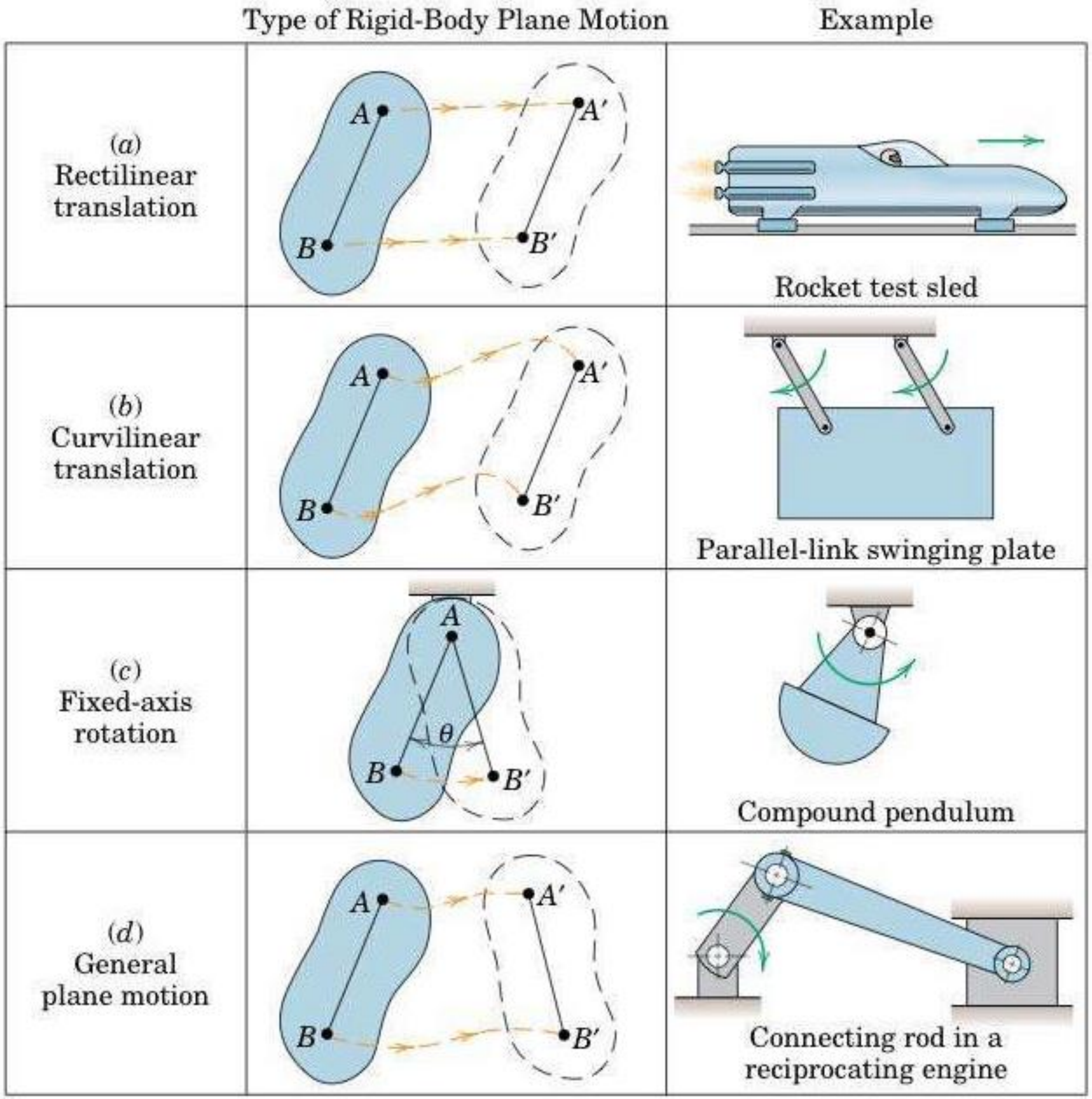
const. in mag. & dir.

$$\bar{v}_B = \underbrace{\dot{\bar{r}}_A}_{\bar{v}_A} + \underbrace{\dot{\bar{r}}_{B/A}}_{\bar{0}}$$

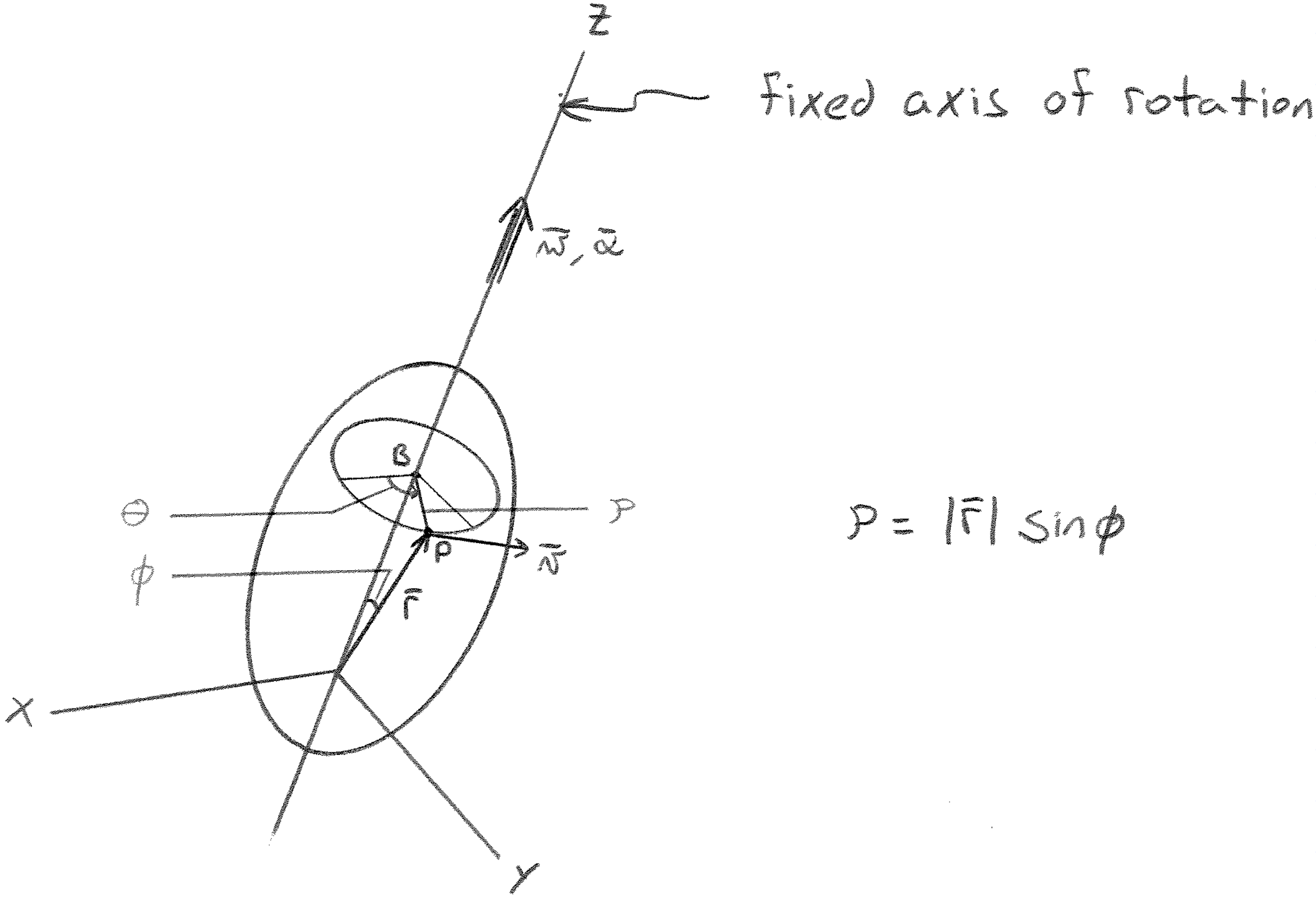
$$\Rightarrow \quad \bar{v}_B = \bar{v}_A$$

$$\Rightarrow \quad \bar{a}_B = \bar{a}_A$$

2. Rotation about a fixed axis


Type of Rigid-Body Plane Motion
Example
(a)
Rectilinear translation
A
B
A′
B′
Rocket test sled
(b)
Curvilinear translation
A
B
A′
B′
Parallel-link swinging plate
(c)
Fixed-axis rotation
A
B
B′
θ
Compound pendulum
(d)
General plane motion
A
B
A′
B′
Connecting rod in a reciprocating engine

Rotation about a Fixed Axis

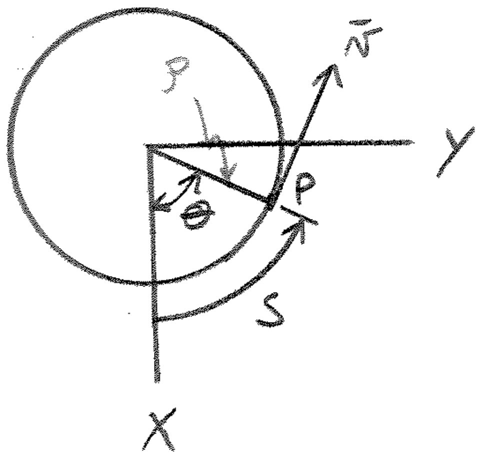


$$\rho = |\bar{r}| \sin\phi$$

$\bar{\omega} = \omega \hat{k}$, $\omega = \frac{d\theta}{dt}$ angular vel.

$\bar{\alpha} = \alpha \hat{k}$, $\alpha = \frac{d\omega}{dt}$ angular acc.

Top View:



$$s = \rho\theta = |\bar{r}| \sin\phi \, \theta$$

$$\Rightarrow \frac{ds}{dt} = |\bar{r}| \sin\phi \, \frac{d\theta}{dt}$$

$v = |\bar{v}|$ $\omega = |\bar{\omega}|$

$$\Rightarrow \quad |\bar{v}| = |\bar{\omega}||\bar{r}|\sin\phi$$

$|\bar{\omega} \times \bar{r}| = |\bar{r} \times \bar{\omega}|$

by definition

$$\Rightarrow \quad |\bar{v}| = |\bar{\omega} \times \bar{r}| = |\bar{r} \times \bar{\omega}|$$

magnitude of $\bar{v}$

For appropriate direction, must have

$$\boxed{\bar{v} = \bar{\omega} \times \bar{r}}$$

$$\boxed{\bar{v} = \bar{\omega} \times \bar{r}}$$

$$\bar{a} = \frac{d}{dt}(\bar{\omega} \times \bar{r}) = \underbrace{\frac{d\bar{\omega}}{dt}}_{\bar{\alpha}} \times \bar{r} + \bar{\omega} \times \underbrace{\frac{d\bar{r}}{dt}}_{\bar{v} = \bar{\omega} \times \bar{r}}$$

$$\therefore \quad \boxed{\bar{a} = \bar{\alpha} \times \bar{r} + \bar{\omega} \times (\bar{\omega} \times \bar{r})}$$

Here, $\bar{a}_t = \bar{\alpha} \times \bar{r}$ = tangential acc.

$\bar{a}_n = \bar{\omega} \times (\bar{\omega} \times \bar{r})$ = normal acc.

Equations Defining the Rotation of a Rigid Body about a Fixed Axis

Recall $$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

Are "rotational analogs" of

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

For rectilinear motion of a particle

Cases:

1. $\alpha = \alpha(t)$
2. $\alpha = \alpha(\theta)$
3. $\alpha = \alpha(\omega)$
4. $\alpha = 0$ (uniform rotation)
5. $\alpha =$ const. (uniformly accelerated rotation)

Cases ①, ②, ③: Integrate directly

Cases ④, ⑤: Can derive "rotational kinematic equations"

$$\left.\theta = \theta_0 + \omega t\right\} \quad \alpha = 0$$

$$\left.\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta - \theta_0 &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 - \omega_0^2 &= 2\alpha(\theta - \theta_0) \end{aligned}\right\} \quad \alpha = \text{const.}$$

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

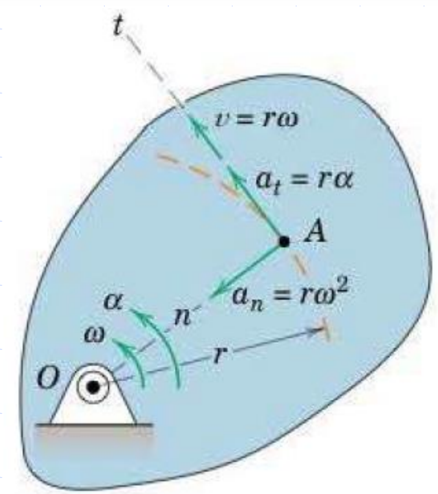
$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad \text{or} \quad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega\, d\omega = \alpha\, d\theta \quad \text{or} \quad \dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta$$

$$v = r\omega$$

$$a_n = r\omega^2 = v^2/r = v\omega$$

$$a_t = r\alpha$$



$$\omega = \omega_0 + \alpha t \qquad\qquad v = v_0 + at$$

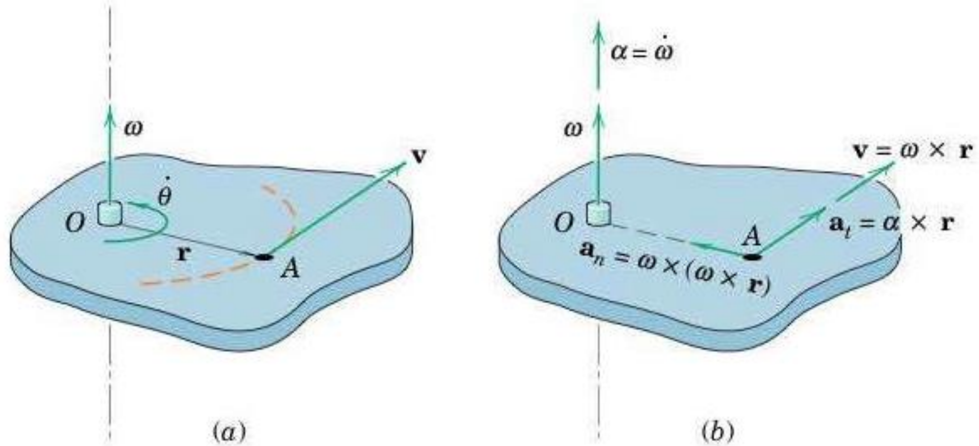
$$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0) \quad \equiv \quad v^2 = v_0{}^2 + 2a(s - s_0)$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad\qquad s = s_0 + v_0 t + \tfrac{1}{2}at^2$$

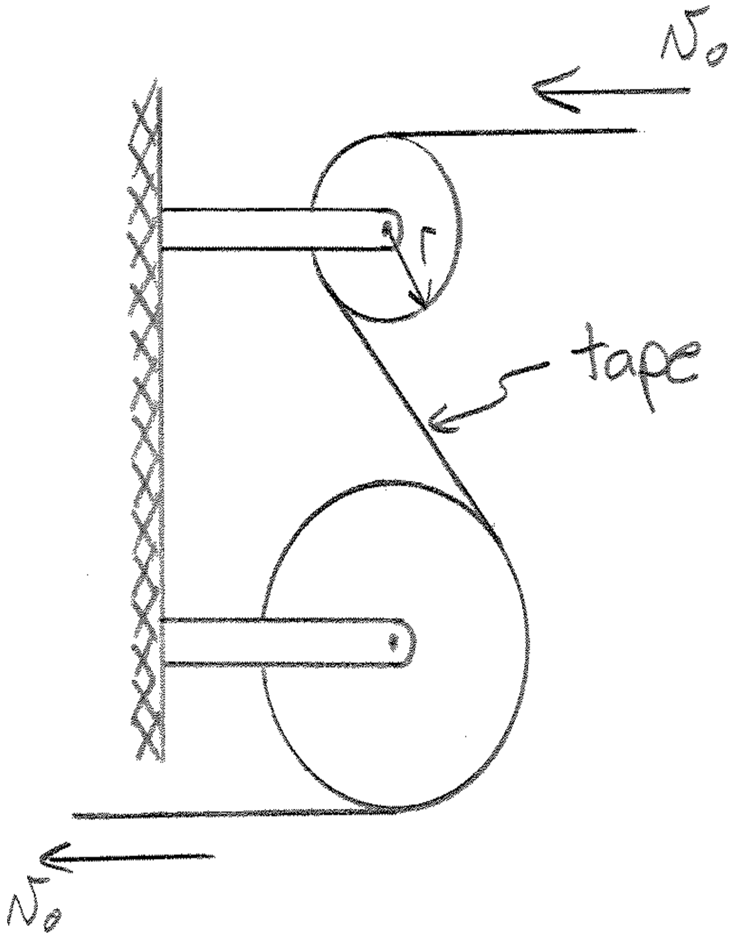
$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

$$\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$$



Example:



Data: $v_0 = 2\ ft/s$

$r = 0.9\ in$

Tape speed is uniformly increased from v_0 to $v = 6\ ft/s$ in $\Delta t = 4s$.

Find, for top drum during Δt:

(A) ang. acc.

(B) # revolutions

Ang. velocities

$$v_0 = r\omega_0$$

$$\Rightarrow \quad \omega_0 = \frac{v_0}{r} = \frac{(2 \text{ ft/s})}{(9/12 \text{ ft})} = 26.667 \text{ rad/s}$$

Similarly,

$$\omega = \frac{v}{r} = \frac{(6 \text{ ft/s})}{(9/12 \text{ ft})} = 80.0 \text{ rad/s}$$

PART A:

α = const, $\therefore$ can use:

$$\omega = \omega_0 + \alpha t \quad (1)$$

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2 \quad (2)$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0) \quad (3)$$

Use (1):

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha = \frac{(80.0\ \text{rad/s}) - (26.667\ \text{rad/s})}{(4\,\text{s})} = 13.333\ \text{rad/s}^2$$

$$\therefore \quad \bar{\alpha} = 13.3\ \text{rad/s}^2 \circlearrowleft$$

PART B:

Use 2 or 3.

From 3,

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$= \frac{\left(80.0\ \mathrm{rad/s}\right)^2 - \left(26.667\ \mathrm{rad/s}\right)^2}{2\left(13.3\ \mathrm{rad/s^2}\right)}$$

$$= \left(213.333\ \mathrm{rad}\right)\left(\frac{1\ \mathrm{rev}}{2\pi\ \mathrm{rad}}\right) = 33.95\ \mathrm{rev}$$

$$\therefore \quad \Delta\theta = 34.0\ \mathrm{rev}$$

Sample Problem 5/1

A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4t$ rad/s^2, where t is the time in seconds during which the torque is applied. Determine (*a*) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (*b*) the time required for the flywheel to reverse its direction of rotation, and (*c*) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

Solution. The counterclockwise direction will be taken arbitrarily as positive.

(a) Since α is a known function of the time, we may integrate it to obtain angular
1 velocity. With the initial angular velocity of $-1800(2\pi)/60 = -60\pi$ rad/s, we have

$$[d\omega = \alpha\, dt] \qquad \int_{-60\pi}^{\omega} d\omega = \int_0^t 4t\, dt \qquad \omega = -60\pi + 2t^2$$

Substituting the clockwise angular speed of 900 rev/min or $\omega = -900(2\pi)/60 = -30\pi$ rad/s gives

$$-30\pi = -60\pi + 2t^2 \qquad t^2 = 15\pi \qquad t = 6.86 \text{ s} \qquad \textit{Ans.}$$

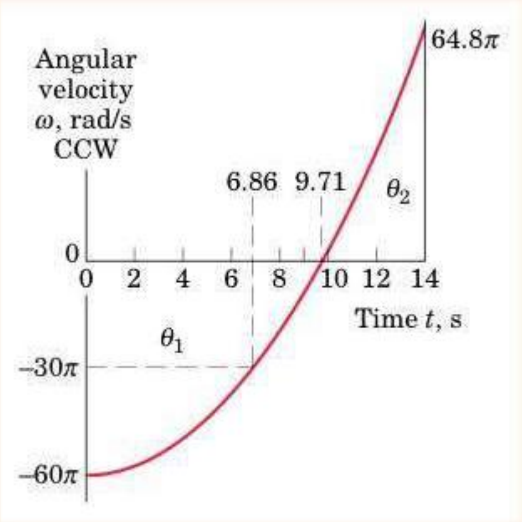
(b) The flywheel changes direction when its angular velocity is momentarily zero. Thus,

$$-30\pi = -60\pi + 2t^2 \qquad t^2 = 15\pi \qquad t = 6.86 \text{ s} \qquad \textit{Ans.}$$

(b) The flywheel changes direction when its angular velocity is momentarily zero. Thus,

$$0 = -60\pi + 2t^2 \qquad t^2 = 30\pi \qquad t = 9.71 \text{ s} \qquad \textit{Ans.}$$

(c) The total number of revolutions through which the flywheel turns during 14 seconds is the number of clockwise turns N_1 during the first 9.71 seconds, plus the number of counterclockwise turns N_2 during the remainder of the interval. Integrating the expression for ω in terms of t gives us the angular displacement in radians. Thus, for the first interval



$$[d\theta = \omega\, dt] \qquad \int_0^{\theta_1} d\theta = \int_0^{9.71} (-60\pi + 2t^2)\, dt$$

② $$\theta_1 = [-60\pi t + \tfrac{2}{3}t^3]_0^{9.71} = -1220 \text{ rad}$$

or $N_1 = 1220/2\pi = 194.2$ revolutions clockwise.

For the second interval

$$\int_0^{\theta_2} d\theta = \int_{9.71}^{14} (-60\pi + 2t^2)\, dt$$

③ $$\theta_2 = [-60\pi t + \tfrac{2}{3}t^3]_{9.71}^{14} = 410 \text{ rad}$$

or $N_2 = 410/2\pi = 65.3$ revolutions counterclockwise. Thus, the total number of revolutions turned during the 14 seconds is

$$N = N_1 + N_2 = 194.2 + 65.3 = 259 \text{ rev} \qquad \textit{Ans.}$$

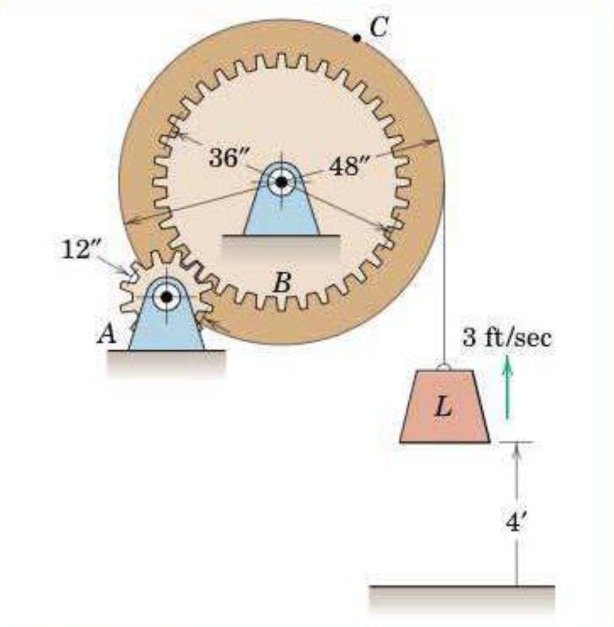
We have plotted ω versus t and we see that θ_1 is represented by the negative area and θ_2 by the positive area. If we had integrated over the entire interval in one step, we would have obtained $|\theta_2| - |\theta_1|$.

② Again note that the minus sign signifies clockwise in this problem.

③ We could have converted the original expression for α into the units of rev/s^2, in which case our integrals would have come out directly in revolutions.

Sample Problem 5/2

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 3 ft/sec in a vertical rise of 4 ft with constant acceleration. As the load passes this position, compute (*a*) the acceleration of point C on the cable in contact with the drum and (*b*) the angular velocity and angular acceleration of the pinion A.



Solution. ***(a)*** If the cable does not slip on the drum, the vertical velocity and acceleration of the load L are, of necessity, the same as the tangential velocity v and tangential acceleration a_t of point C. For the rectilinear motion of L with constant acceleration, the n- and t-components of the acceleration of C become

$[v^2 = 2as]$ $\quad a = a_t = v^2/2s = 3^2/[2(4)] = 1.125 \text{ ft/sec}^2$

① $[a_n = v^2/r]$ $\quad a_n = 3^2/(24/12) = 4.5 \text{ ft/sec}^2$

$[a = \sqrt{a_n{}^2 + a_t{}^2}]$ $\quad a_C = \sqrt{(4.5)^2 + (1.125)^2} = 4.64 \text{ ft/sec}^2$ *Ans.*

(b) The angular motion of gear A is determined from the angular motion of gear B by the velocity v_1 and tangential acceleration a_1 of their common point of contact. First, the angular motion of gear B is determined from the motion of point C on the attached drum. Thus,

$[v = r\omega]$ $\quad \omega_B = v/r = 3/(24/12) = 1.5 \text{ rad/sec}$

$[a_t = r\alpha]$ $\quad \alpha_B = a_t/r = 1.125/(24/12) = 0.562 \text{ rad/sec}^2$

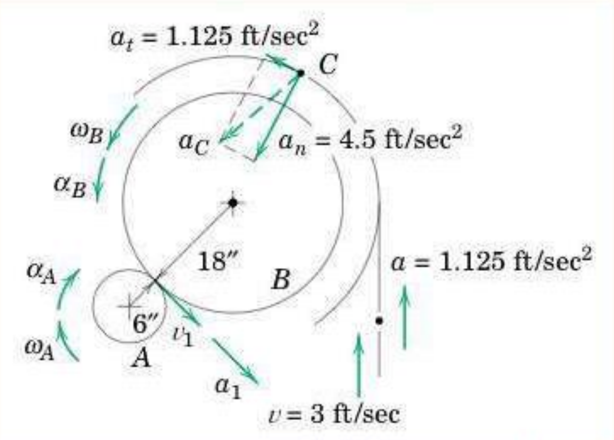
Then from $v_1 = r_A\omega_A = r_B\omega_B$ and $a_1 = r_A\alpha_A = r_B\alpha_B$, we have

$$\omega_A = \frac{r_B}{r_A}\omega_B = \frac{18/12}{6/12}1.5 = 4.5 \text{ rad/sec CW} \qquad \textit{Ans.}$$

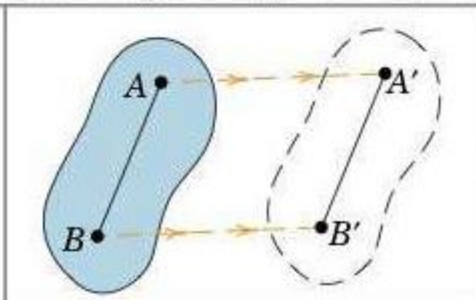
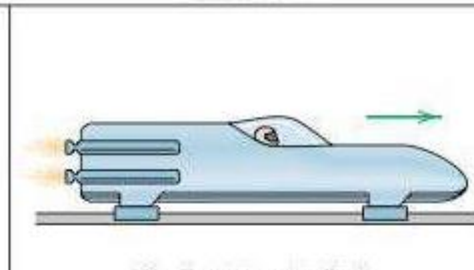
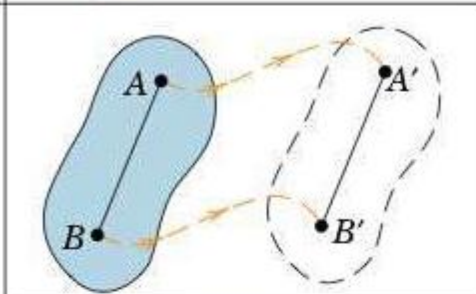
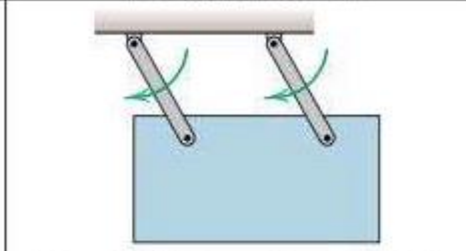
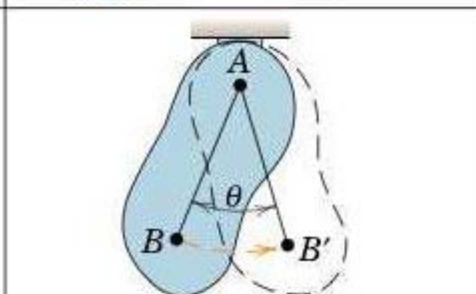
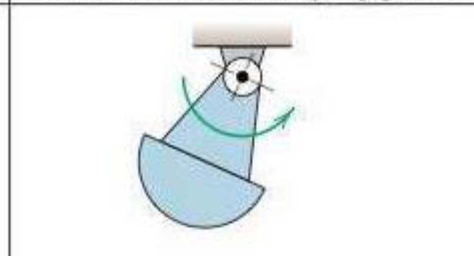
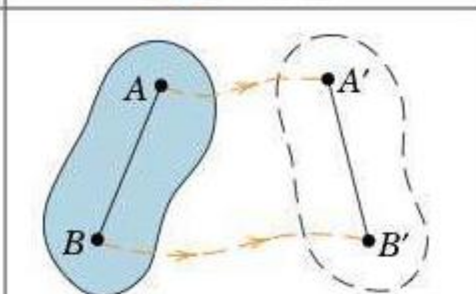
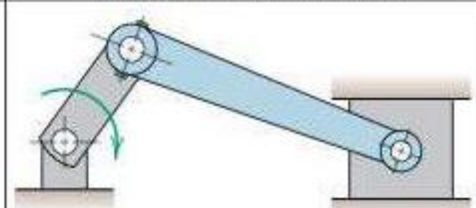
$$\alpha_A = \frac{r_B}{r_A}\alpha_B = \frac{18/12}{6/12}0.562 = 1.688 \text{ rad/sec}^2 \text{ CW} \qquad \textit{Ans.}$$

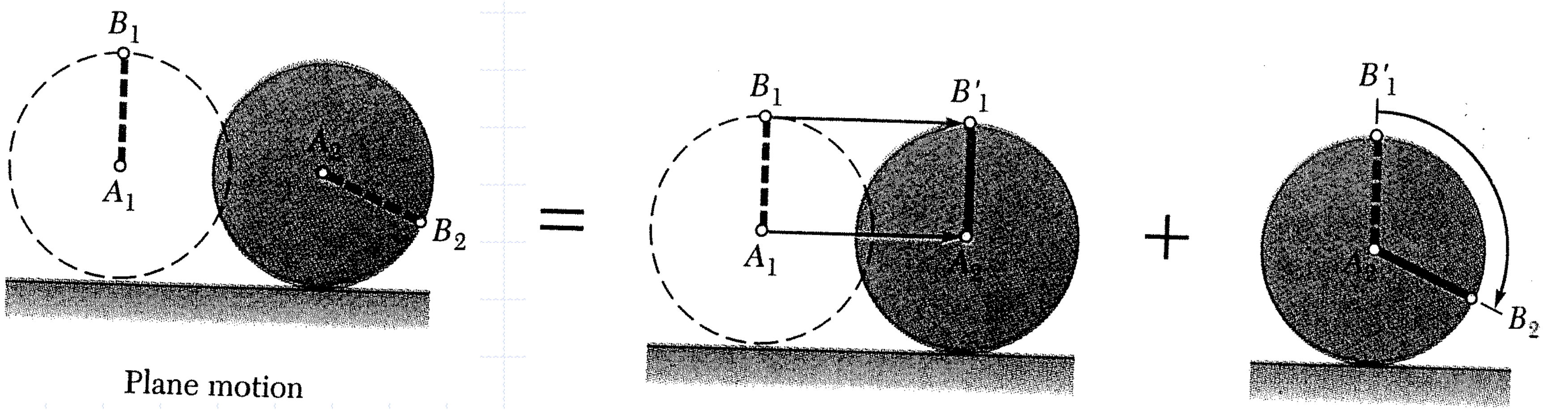
Helpful Hint

① Recognize that a point on the cable changes the direction of its velocity after it contacts the drum and acquires a normal component of acceleration.

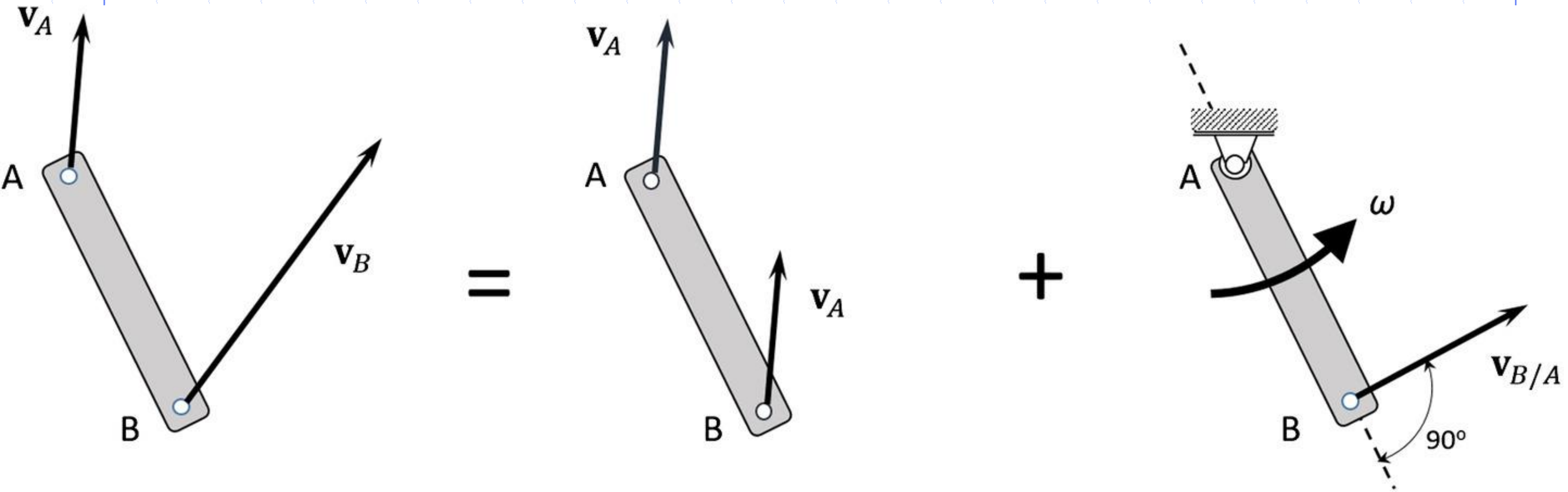


3. General plane motion

	Type of Rigid-Body Plane Motion	Example
(*a*) Rectilinear translation		 Rocket test sled
(*b*) Curvilinear translation		 Parallel-link swinging plate
(*c*) Fixed-axis rotation		 Compound pendulum
(*d*) General plane motion		 Connecting rod in a reciprocating engine



Plane motion = Translation with A + Rotation about A

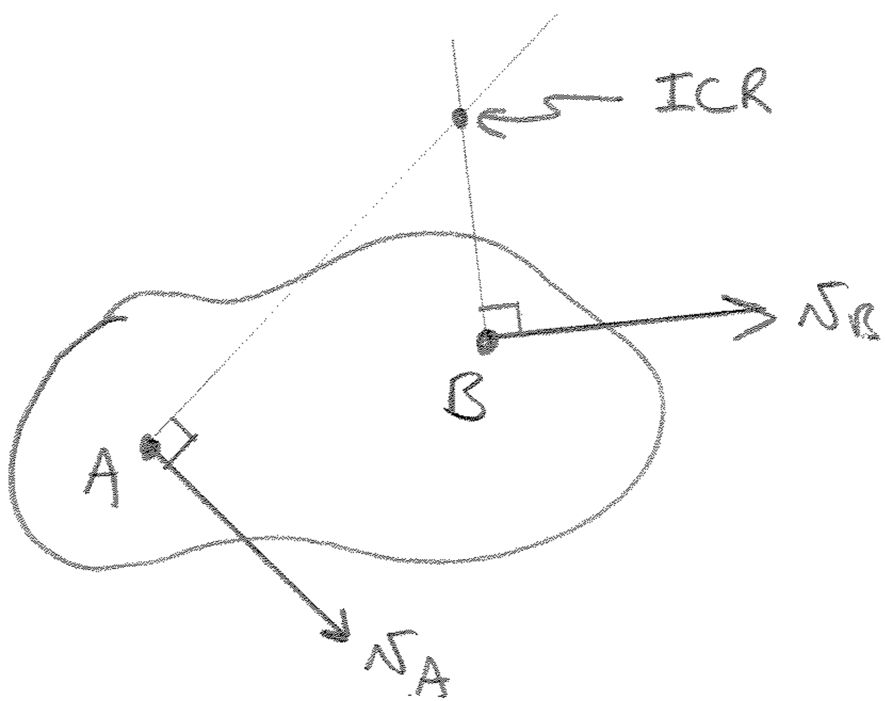


General Plane Motion = Translation with velocity of point A + Rotation about point A

Instantaneous Center of Rotation

Locating the ICR

If the dir. of $\bar{v}_A, \bar{v}_B$ are known:



Dynamics of Rigid Bodies in Plane Motion

In Statics,

$$\bar{F} = \bar{0}$$

$$\bar{M}_P = \bar{0}$$

In Dynamics, we'll show that

$$\bar{F} = m\bar{a}_c$$

$$\bar{M}_P = m\bar{r}_{c/P} \times \bar{a}_P + \dot{\bar{H}}_P$$

$\downarrow$ some restrictions

$$\bar{M}_P = \dot{\bar{H}}_P$$

$\downarrow$ more restrictions

$$\bar{M}_P = I_P \bar{\alpha}$$

$$\boxed{\begin{array}{c} \bar{F} = m\bar{a}_c \\ \bar{M}_P = I_P \bar{\alpha} \end{array}}$$

Main Results to show

$\bar{M}_P$ = moment about P of all external forces acting on system

$$\bar{M}_P = I_P \bar{\alpha}$$

I_P = mass moment of inertia about P

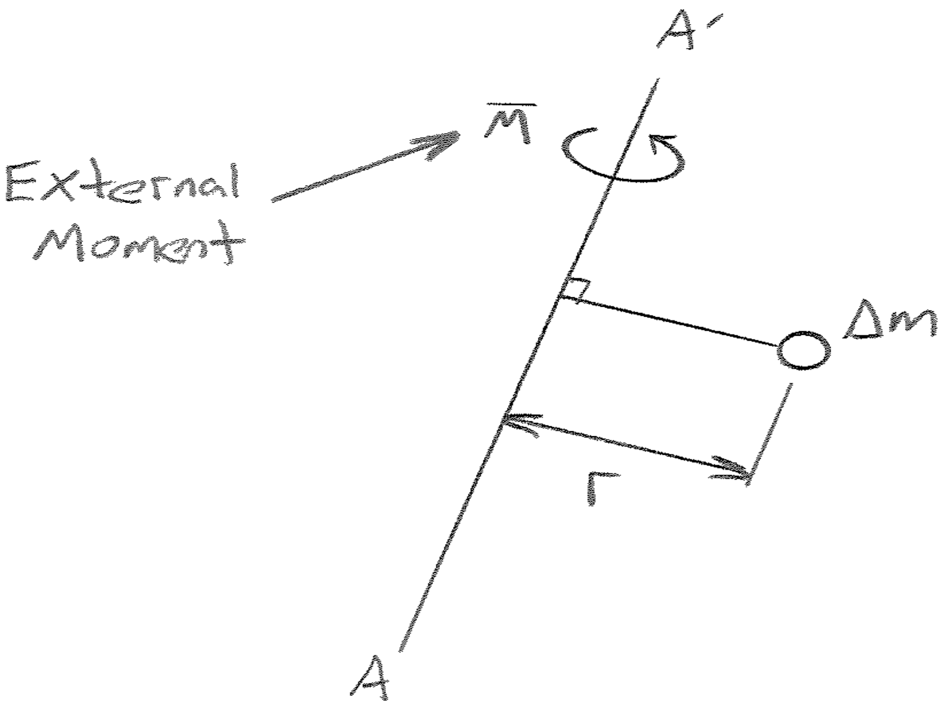
$\bar{a}_c$ = absolute acceleration of system mass center c

m = total system mass

$\bar{H}_P$ = angular momentum of system about P

$$\bar{H}_P = I_P \bar{\omega}$$

Mass Moments of Inertia (MOI)

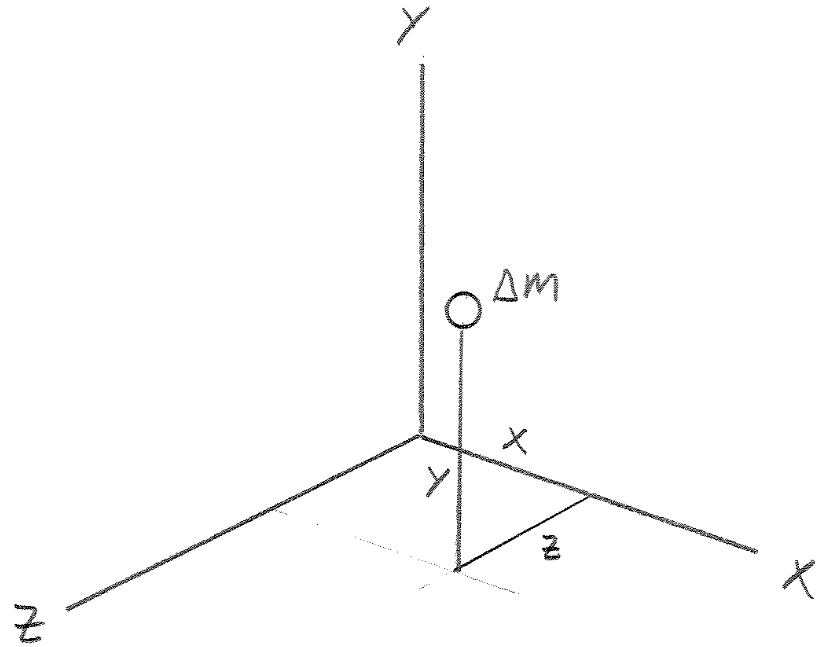


We define

$$\Delta I = r^2 \Delta m$$

$$\boxed{I = \int r^2\, dm}$$

$$I_z = \int (x^2 + y^2)\, dm$$



Radius of Gyration

Defined such that $I = k^2 m$

$$\therefore \quad \boxed{k = \sqrt{\frac{I}{m}}}$$

Parallel Axis Theorem

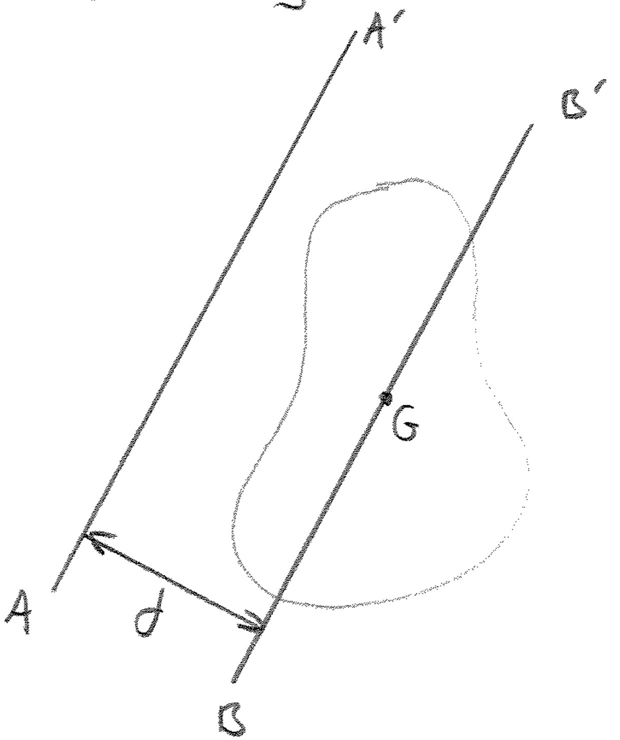
I = MOI about an axis AA'

$\bar{I}$ = MOI about an axis BB' (AA' // BB')

m = total mass of RB

d = dist. separating AA' & BB'

$$\boxed{I = \bar{I} + md^2}$$



Equations of Motion

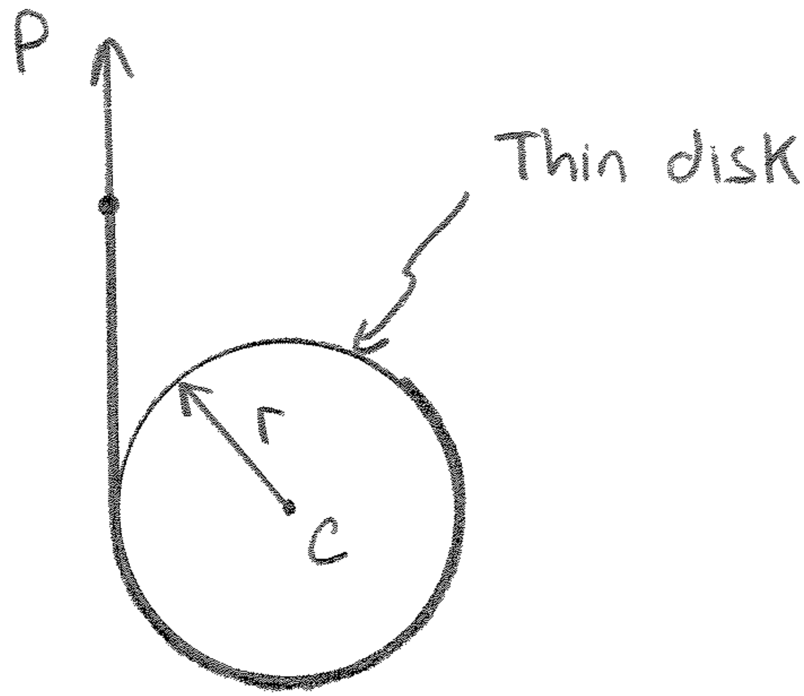
$$\boxed{\begin{aligned} \bar{F} &= m\bar{a}_c \\ \bar{M}_P &= I_P\bar{\alpha} \end{aligned}}$$

EOM

Limitations:

1. RB of constant mass
2. Plane motion
3. P is mass center or a fixed point

Example:



Data: $r = 0.5\,m$

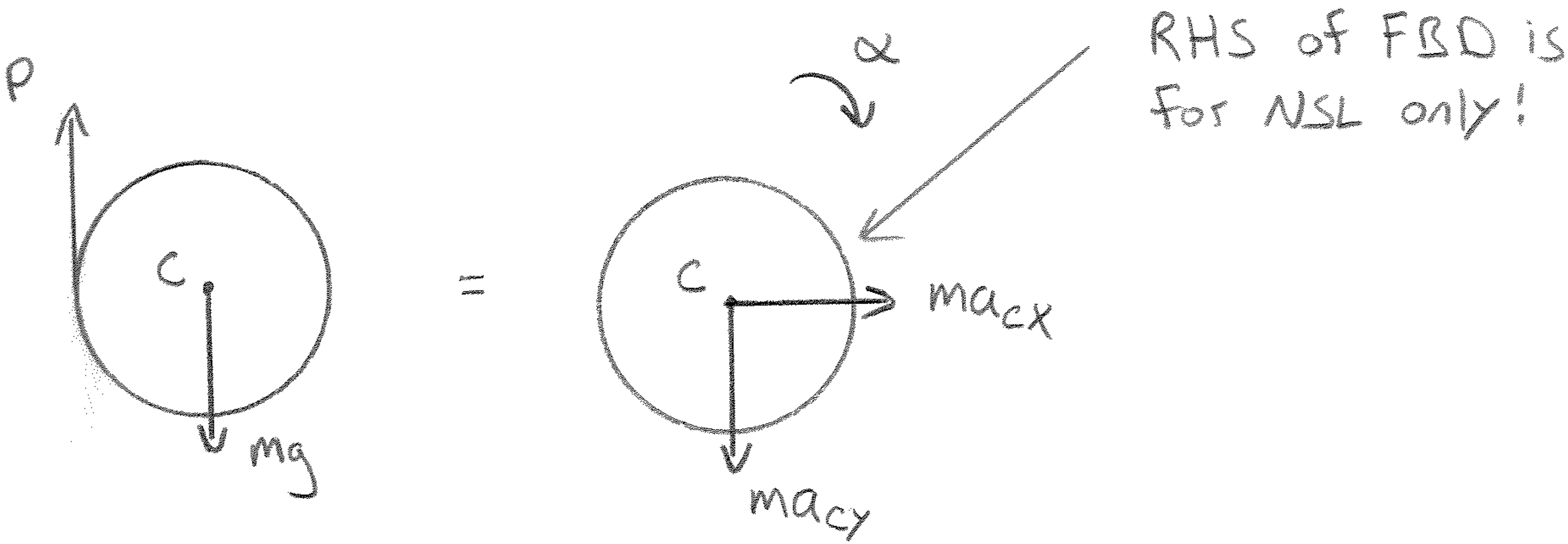
$m = 15\,Kg$

$P = 180\,N$

Find (A) $\bar{a}_c$

(B) $\bar{\alpha}$

FBD of Disk



$\overset{+}{\rightarrow}\ \Sigma F:\quad 0 = ma_{cx} \quad \Rightarrow \quad a_{cx} = 0$

$+\downarrow\ \Sigma F:\quad mg - P = ma_{cy}$ ——— $(*)$

$\circlearrowright\ \Sigma M_c:\quad rP = I_P\alpha$ ——— $(**)$

where $I_P = \frac{1}{2}mr^2$

From (*),

$$a_{cy} = \frac{mg - P}{m} = g - \frac{P}{m}$$

$$= \frac{(15\,\text{Kg})(9.81\,\text{m/s}^2) - (180\,\text{N})}{(15\,\text{Kg})}$$

$$= -2.19\ \text{m/s}^2$$

$$\therefore\quad \bar{a}_c = 2.19\ \text{m/s}^2 \uparrow$$

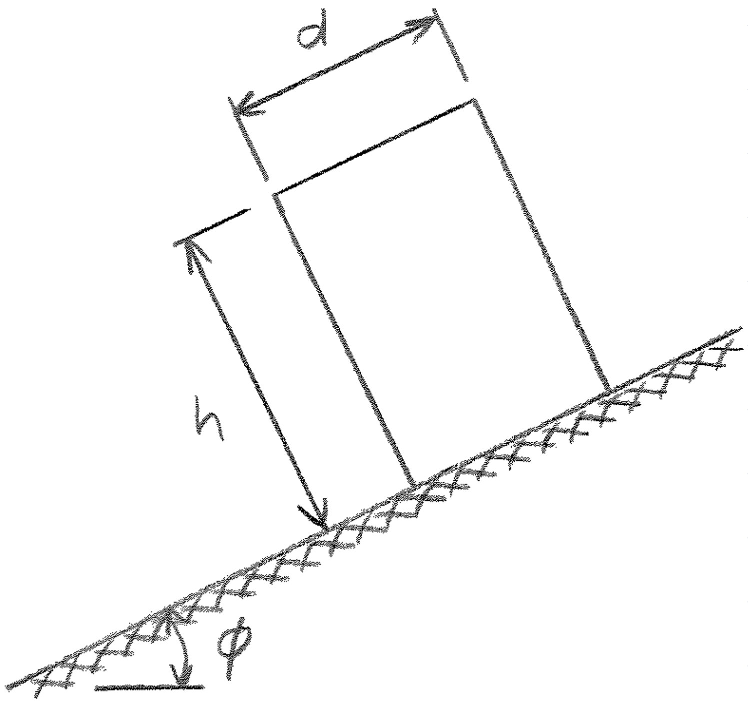
From (**),

$$\cancel{r}P = \frac{1}{2}mr^{\cancel{2}}\alpha \quad \Rightarrow \quad \alpha = \frac{2P}{mr}$$

$$\alpha = \frac{2(180N)}{(15\,kg)(0.5\,m)} = 48.0 \ rad/s^2$$

$$\therefore \quad \alpha = 48 \ rad/s^2 \ \circlearrowright$$

Example:

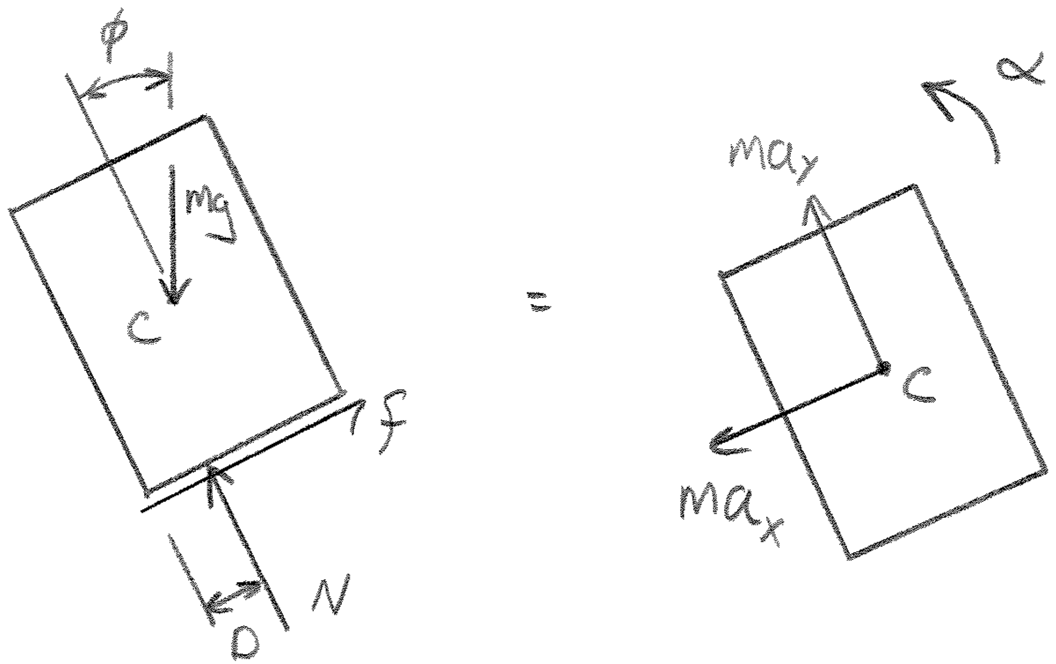


Data: μ_K, ϕ

Assume slip.

Find $(d/h)_{min}$ such that the cylinder does not tip while sliding down the inclined plane.

FBD of cylinder,



At minimum ratio of d to h, tip is impending.

Set $D = 0$

$\alpha = 0 \Rightarrow a_y = 0$

Rotational dynamics:

$$+\circlearrowleft \sum_i M_c: \quad f\cdot\frac{h}{2} - N\cdot\frac{d}{2} = I_c \cancel{\alpha}^{\,0} \quad \text{——} \quad (*)$$

Friction Law:

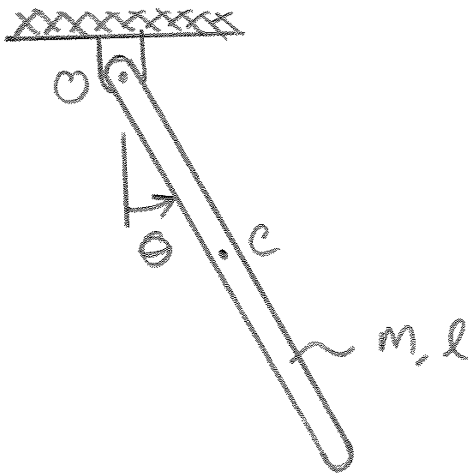
$$f = \mu_K N \quad \text{————————} \quad (**)$$

$(**) \rightarrow (*)$:

$$\mu_K \cancel{N}\cdot\frac{h}{\cancel{2}} - \cancel{N}\cdot\frac{d}{\cancel{2}} = 0$$

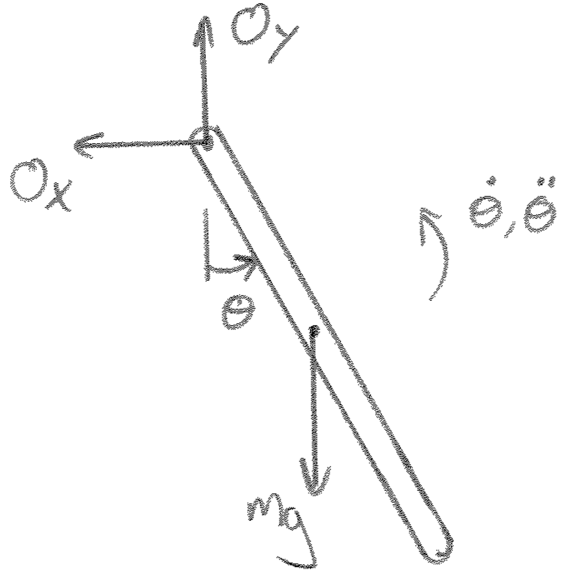
$$\Longrightarrow \quad \frac{d}{h} = \mu_K \qquad \therefore \quad \left(\frac{d}{h}\right)_{min} = \mu_K$$

Example:



Derive the diff. Eq. of motion

FBD:



$$+\circlearrowleft \sum M_O: \quad -mg\frac{\ell}{2}\sin\theta = I_O\ddot{\theta}$$

$$I_O = \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2$$

$$= \frac{1}{3}m\ell^2$$

$$\frac{1}{3}\cancel{m}\ell^{\cancel{2}}\ddot{\theta} + \cancel{m}g\frac{\cancel{\ell}}{2}\sin\theta = 0$$

$$\Rightarrow \quad \ddot{\theta} + \frac{3g}{2\ell}\sin\theta = 0$$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS

(m = mass of body shown)

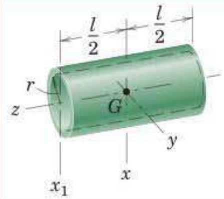
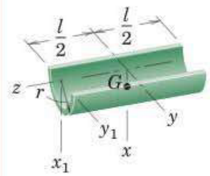
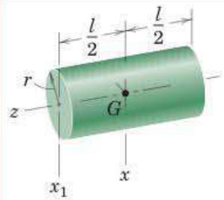
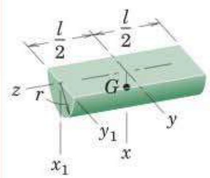
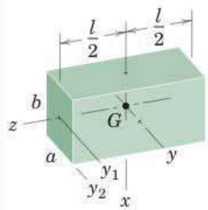
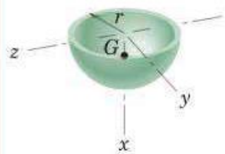
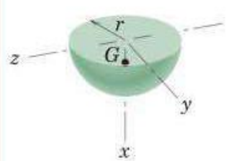
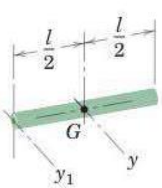
BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Circular Cylindrical Shell	—	$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$
	Half Cylindrical Shell	$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$ $\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right)mr^2$
	Circular Cylinder	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$
	Semicylinder	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Rectangular Parallelepiped	—	$I_{xx} = \frac{1}{12}m(a^2 + l^2)$ $I_{yy} = \frac{1}{12}m(b^2 + l^2)$ $I_{zz} = \frac{1}{12}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_2y_2} = \frac{1}{3}m(b^2 + l^2)$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

(m = mass of body shown)

BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Spherical Shell	—	$I_{zz} = \frac{2}{3}mr^2$
	Hemispherical Shell	$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{5}{12}mr^2$
	Sphere	—	$I_{zz} = \frac{2}{5}mr^2$
	Hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320}mr^2$
	Uniform Slender Rod	—	$I_{yy} = \frac{1}{12}ml^2$ $I_{y_1y_1} = \frac{1}{3}ml^2$

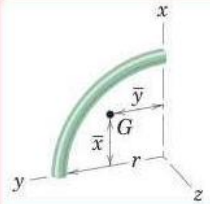
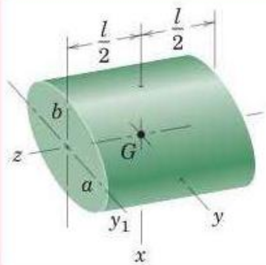
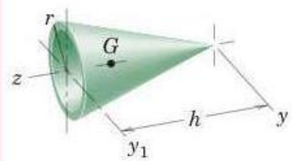
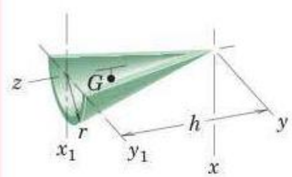
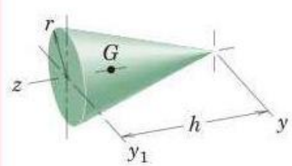
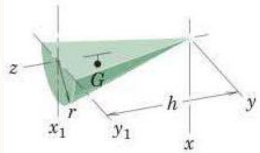
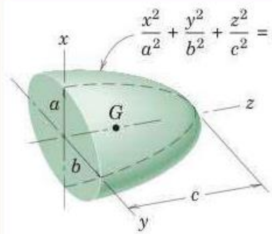
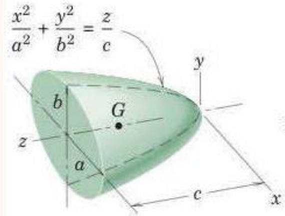
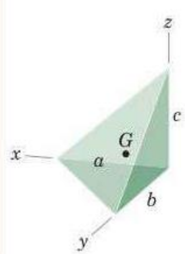
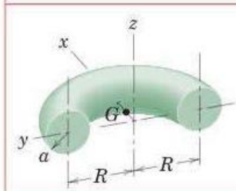
 Quarter-Circular Rod	$\bar{x} = \bar{y}$ $= \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_{zz} = mr^2$
 Elliptical Cylinder	—	$I_{xx} = \frac{1}{4}ma^2 + \frac{1}{12}ml^2$ $I_{yy} = \frac{1}{4}mb^2 + \frac{1}{12}ml^2$ $I_{zz} = \frac{1}{4}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{4}mb^2 + \frac{1}{3}ml^2$
 Conical Shell	$\bar{z} = \frac{2h}{3}$	$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{yy} = \frac{1}{4}mr^2 + \frac{1}{18}mh^2$
 Half Conical Shell	$\bar{x} = \frac{4r}{3\pi}$ $\bar{z} = \frac{2h}{3}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
 Right-Circular Cone	$\bar{z} = \frac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS ***Continued***

(m = mass of body shown)

BODY	MASS CENTER	MASS MOMENTS OF INERTIA
 Half Cone	$\bar{x} = \frac{r}{\pi}$ $\bar{z} = \frac{3h}{4}$	$I_{xx} = I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{zz} = \left(\frac{3}{10} - \frac{1}{\pi^2}\right)mr^2$
 Semiellipsoid	$\bar{z} = \frac{3c}{8}$	$I_{xx} = \frac{1}{5}m(b^2 + c^2)$ $I_{yy} = \frac{1}{5}m(a^2 + c^2)$ $I_{zz} = \frac{1}{5}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{5}m(b^2 + \frac{19}{64}c^2)$ $\bar{I}_{yy} = \frac{1}{5}m(a^2 + \frac{19}{64}c^2)$
 Elliptic Paraboloid	$\bar{z} = \frac{2c}{3}$	$I_{xx} = \frac{1}{6}mb^2 + \frac{1}{2}mc^2$ $I_{yy} = \frac{1}{6}ma^2 + \frac{1}{2}mc^2$ $I_{zz} = \frac{1}{6}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{6}m(b^2 + \frac{1}{3}c^2)$ $\bar{I}_{yy} = \frac{1}{6}m(a^2 + \frac{1}{3}c^2)$
 Rectangular Tetrahedron	$\bar{x} = \frac{a}{4}$ $\bar{y} = \frac{b}{4}$ $\bar{z} = \frac{c}{4}$	$I_{xx} = \frac{1}{10}m(b^2 + c^2)$ $I_{yy} = \frac{1}{10}m(a^2 + c^2)$ $I_{zz} = \frac{1}{10}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{3}{80}m(b^2 + c^2)$ $\bar{I}_{yy} = \frac{3}{80}m(a^2 + c^2)$ $\bar{I}_{zz} = \frac{3}{80}m(a^2 + b^2)$
 Half Torus	$\bar{x} = \frac{a^2 + 4R^2}{2\pi R}$	$I_{xx} = I_{yy} = \frac{1}{2}mR^2 + \frac{5}{8}ma^2$ $I_{zz} = mR^2 + \frac{3}{4}ma^2$