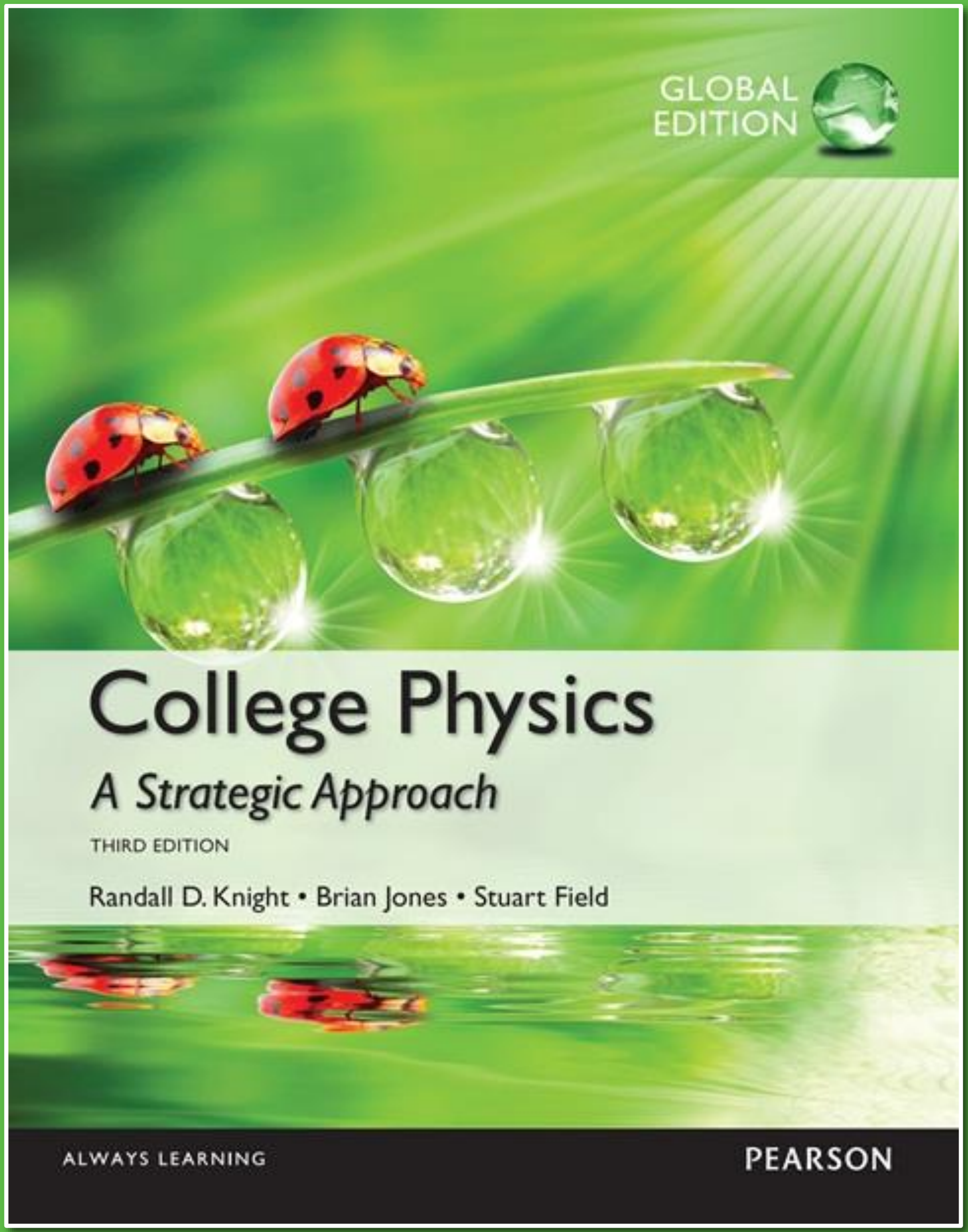


Lecture Presentation

Chapter 3

Vectors and Motion in Two Dimensions

Chapter 3 Vectors and Motion in Two Dimensions

Chapter 3 Preview
Looking Ahead

Vectors and Components

The dark green vector is the ball's initial velocity. The light green **component vectors** show initial horizontal and vertical velocity.

You'll learn how to find components of vectors and how to use these components to solve problems.

Physical Quantities

Scalar

- Magnitude
- No direction

Examples:

Length, mass, time, electric current, temperature, area, distance, speed, energy, density, pressure, power

Vector

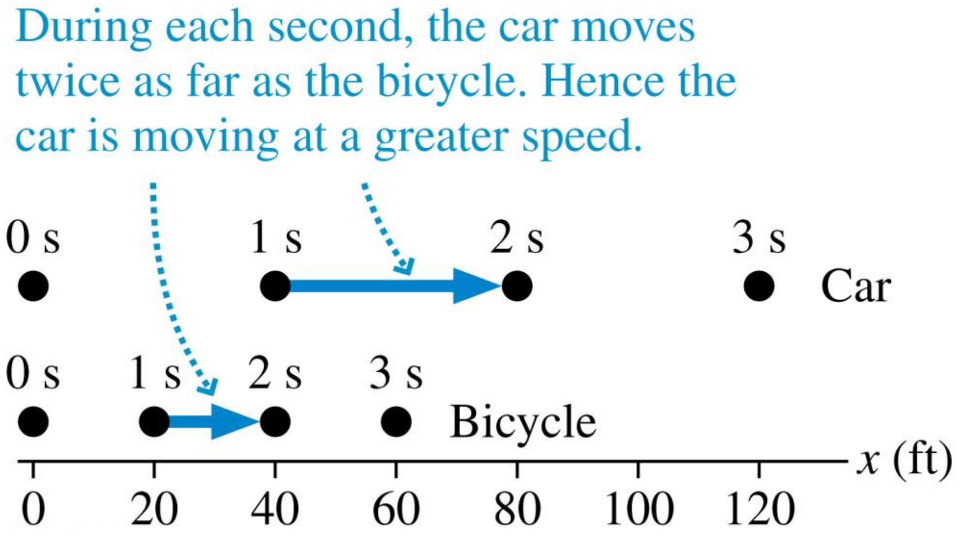
- Magnitude
- Direction

Examples:

Displacement, velocity, force, acceleration, momentum, weight impulse

Velocity and Speed

- Motion at a constant speed in a straight line is called **uniform motion**.

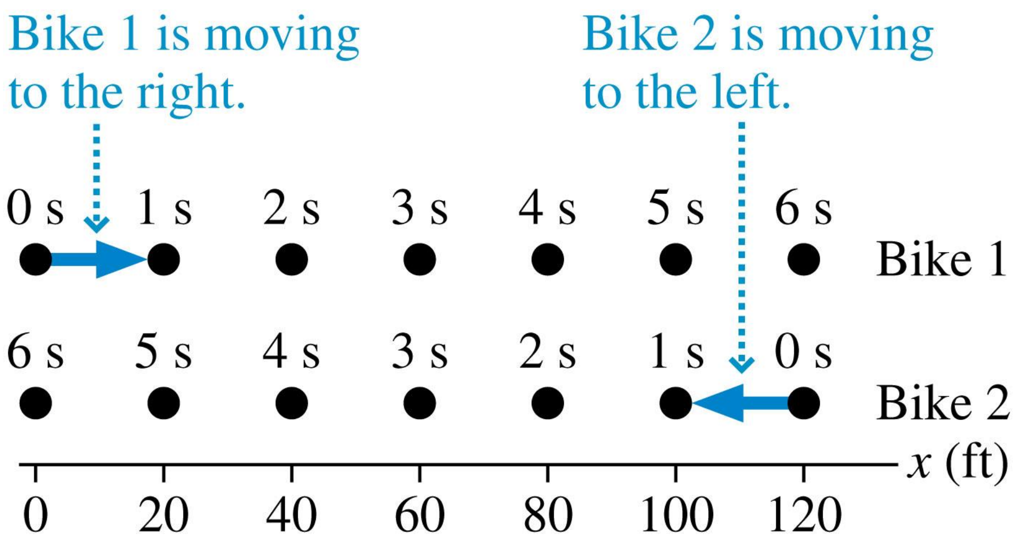


$$\text{speed} = \frac{\text{distance traveled in a given time interval}}{\text{time interval}}$$

Speed of an object in uniform motion

Velocity and Speed

- **Speed measures only how fast an object moves, but velocity tells us both an object's speed *and its direction*.**



$$\text{velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{\Delta x}{\Delta t}$$

Velocity of a moving object

- This velocity is called the *average* velocity.

Reading Question

If Samir walks 100 m to the right, then 200 m to the left, his net displacement vector

A. Points to the right.
B. Points to the left.
C. Has zero length.
D. Cannot tell without more information.

Reading Question

What is the difference between speed and velocity?

A. Speed is an average quantity while velocity is not.
B. Velocity contains information about the direction of motion while speed does not.
C. Speed is measured in mph, while velocity is measured in m/s.
D. The concept of speed applies only to objects that are neither speeding up nor slowing down, while velocity applies to every kind of motion.
E. Speed is used to measure how fast an object is moving in a straight line, while velocity is used for objects moving along curved paths.

Reading Question

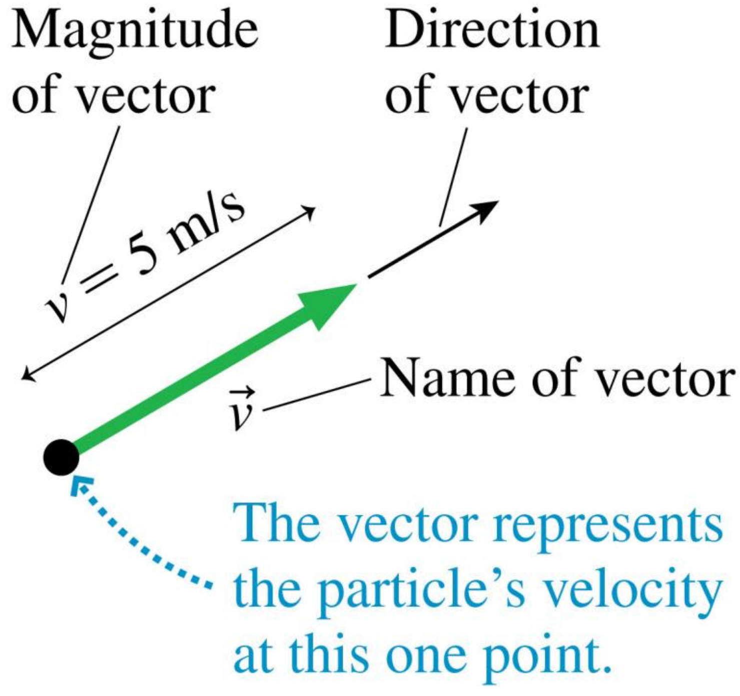
Velocity vectors point

A. In the same direction as displacement vectors.
B. In the opposite direction as displacement vectors.
C. Perpendicular to displacement vectors.
D. In the same direction as acceleration vectors.
E. Velocity is not represented by a vector.

Section 3.1 Using Vectors

Using Vectors

- A vector is a quantity with both a size (magnitude) and a direction. **Two vectors are equal if they have the same magnitude and direction.**
- The figure shows how to represent a particle's velocity as a vector $\vec{v}$.
- The particle's speed at this point is 5 m/s and it is moving in the direction indicated by the arrow.
- The magnitude of a vector, a *scalar* quantity (cannot be a negative number)



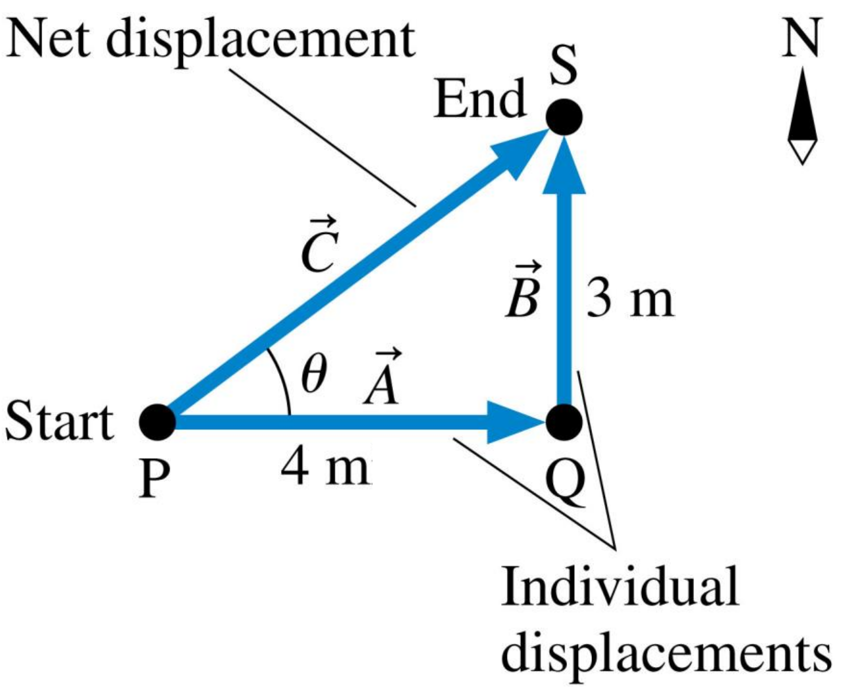
Vector Addition

- $\vec{C}$ is the *net displacement* because it describes the net result of the hiker's having first displacement $\vec{A}$, then displacement $\vec{B}$.
- The net displacement $\vec{C}$ is an initial displacement $\vec{A}$ *plus* a second displacement $\vec{B}$:

$$\vec{C} = \vec{A} + \vec{B}$$

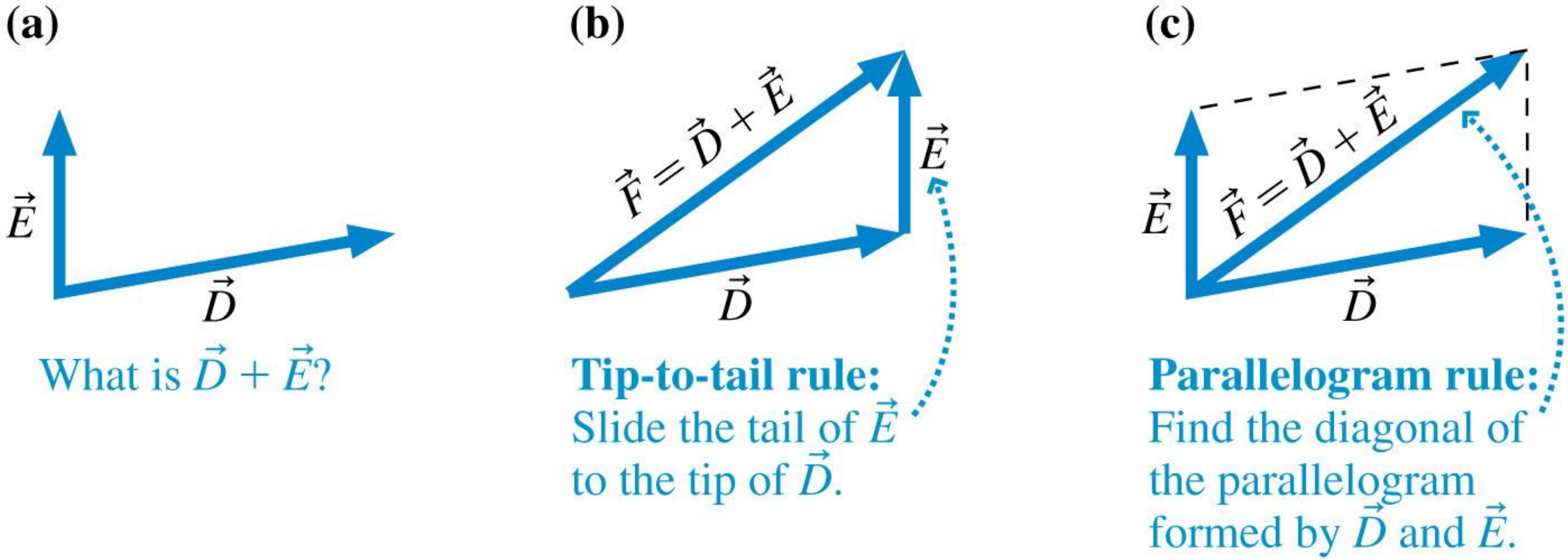
- The sum of the two vectors is called the resultant vector. Vector addition is commutative:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



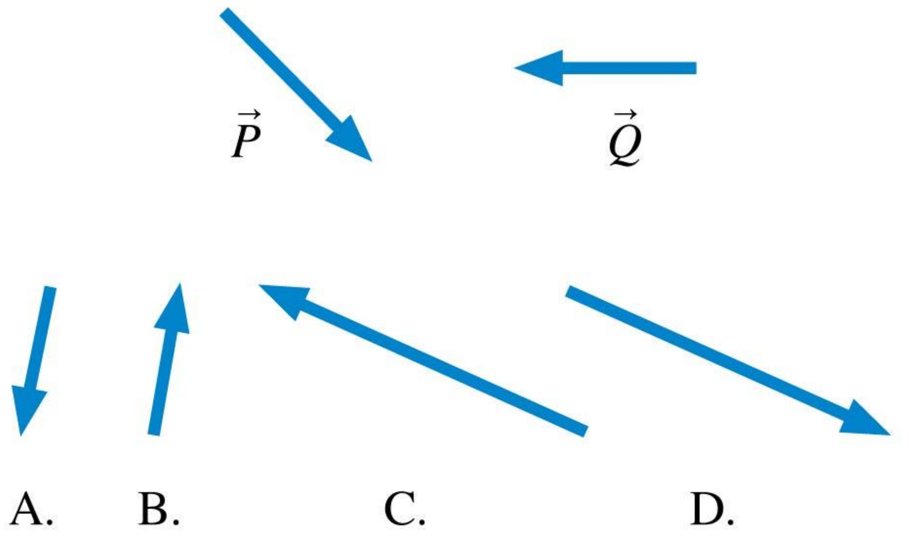
Vector Addition

- The figure shows the *tip-to-tail rule* of vector addition and the *parallelogram rule* of vector addition.



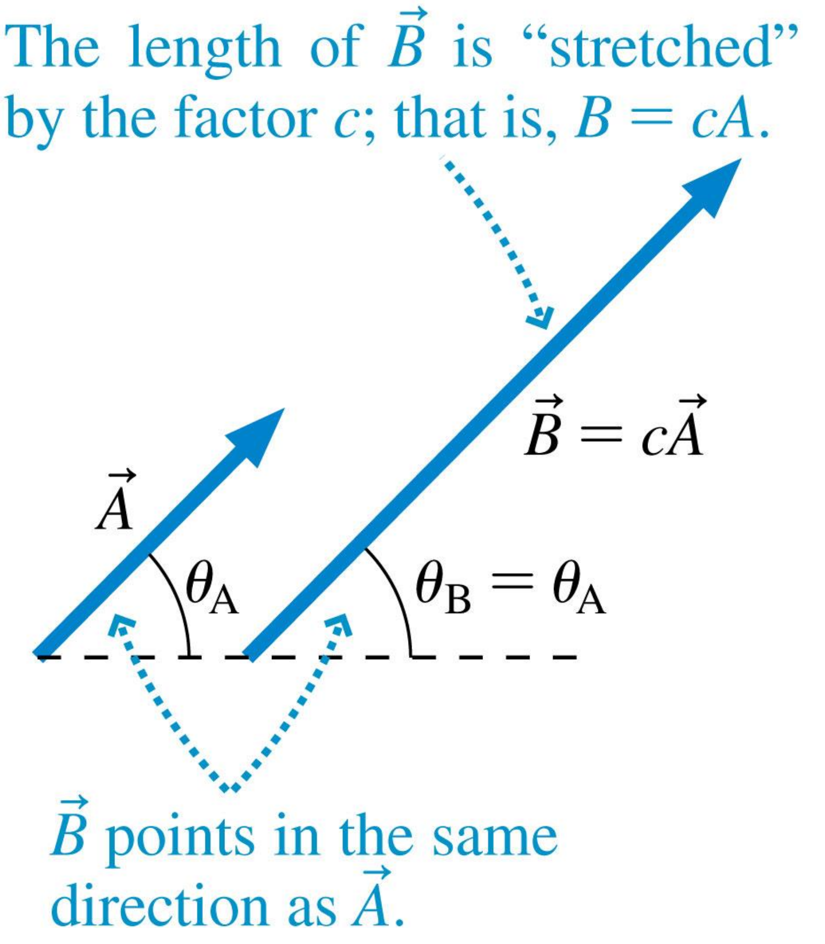
QuickCheck 3.1

- Given vectors $\vec{P}$ and $\vec{Q}$, what is $\vec{P}+\vec{Q}$?



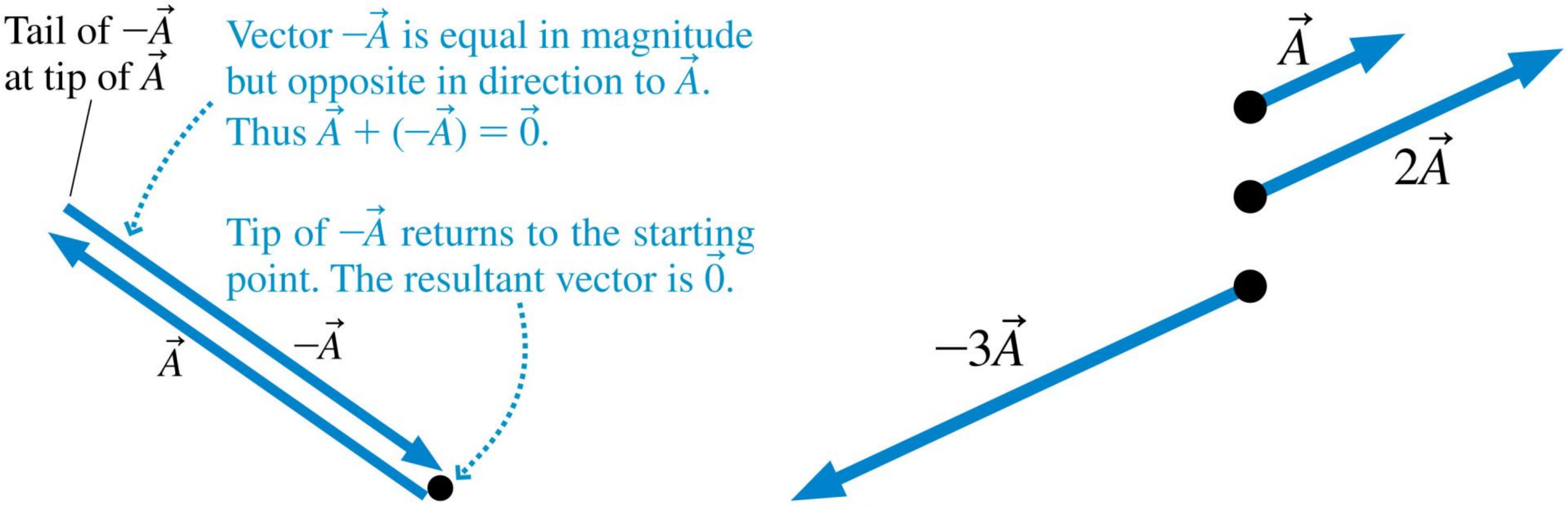
Multiplication by a Scalar

- **Multiplying a vector by a positive scalar gives another vector of *different magnitude* but pointing in the *same direction*.**
- If we multiply a vector by zero the product is a vector having zero length. The vector is known as the **zero vector**.



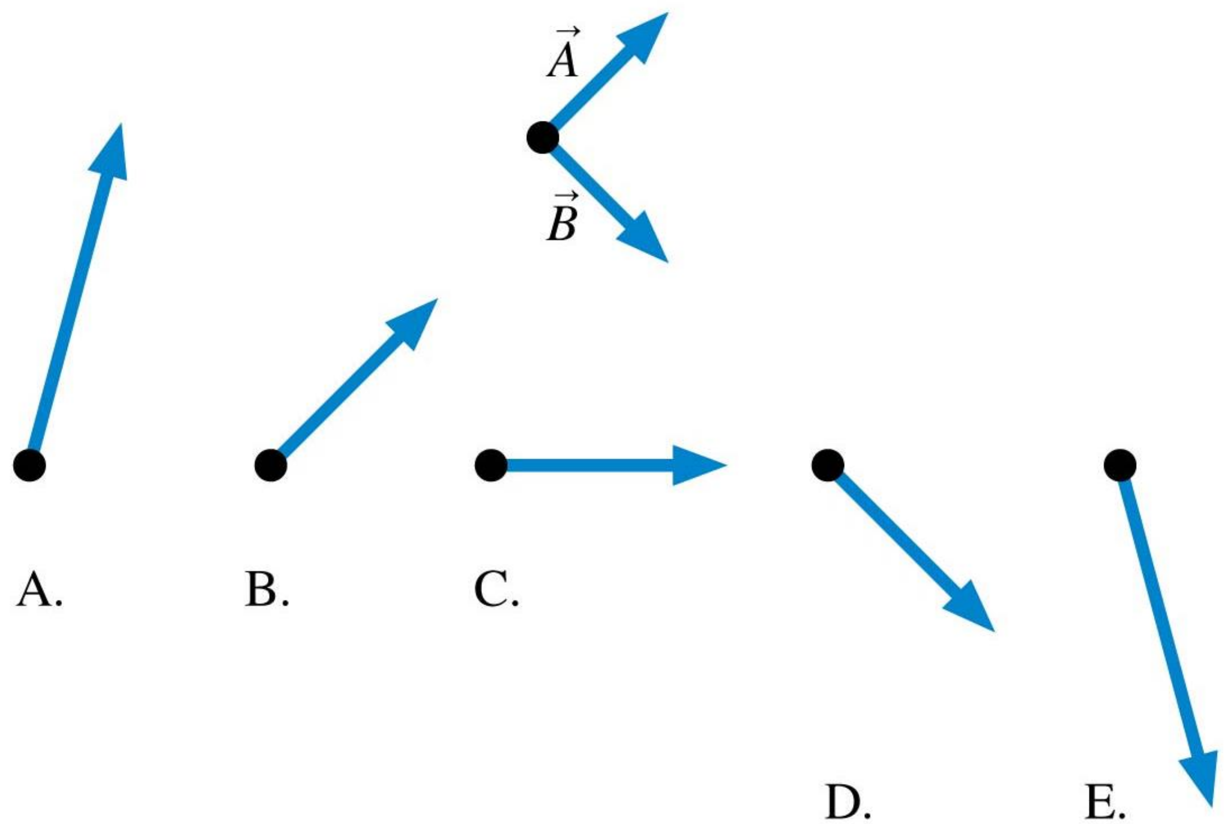
Multiplication by a Scalar

- **A vector cannot have a negative magnitude.**
- If we multiply a vector by a negative number we reverse its direction.
- Multiplying a vector by –1 reverses its direction without changing its length (magnitude).



QuickCheck 3.2

- Which of the vectors in the second row shows $\vec{A} + \vec{B}$?



Vector Subtraction

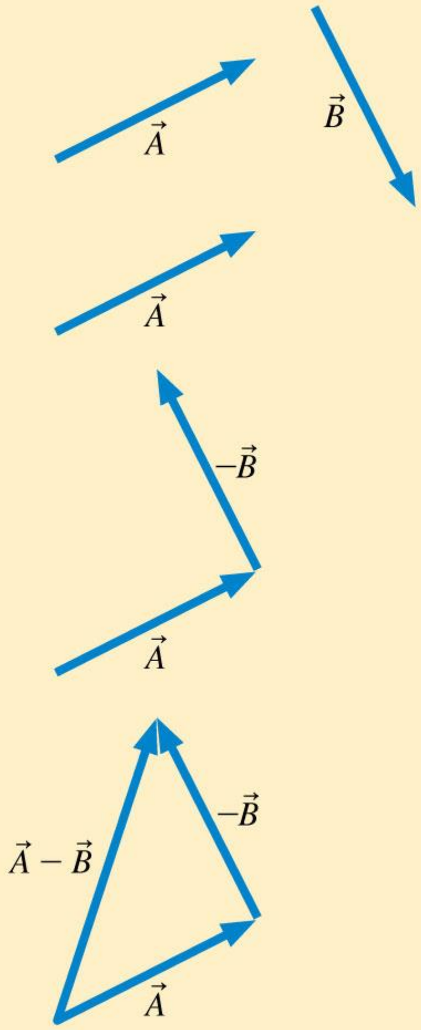
Subtracting vectors

To subtract $\vec{B}$ from $\vec{A}$:

❶ Draw $\vec{A}$.

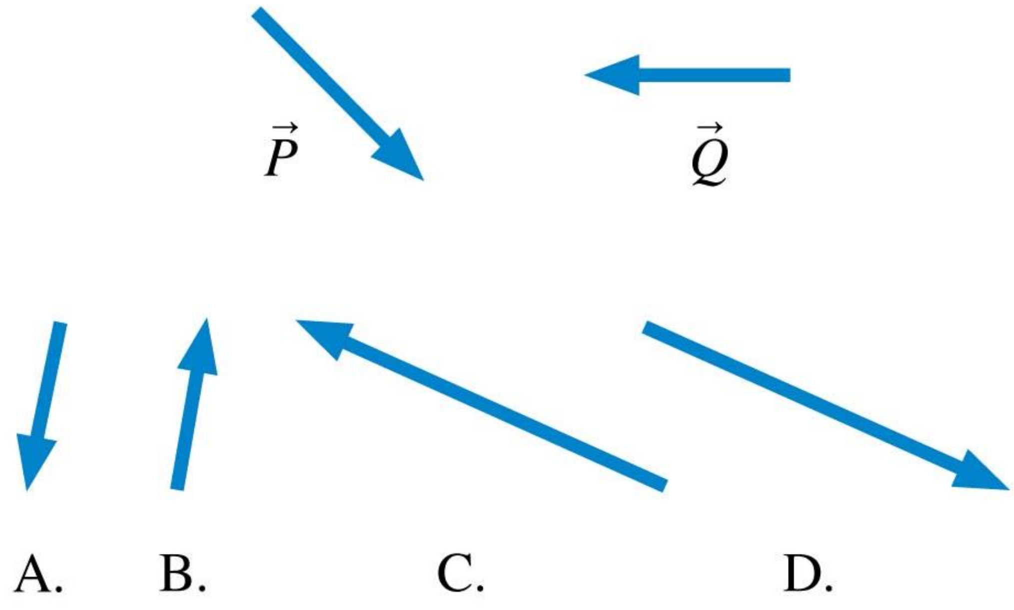
❷ Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.

❸ Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.



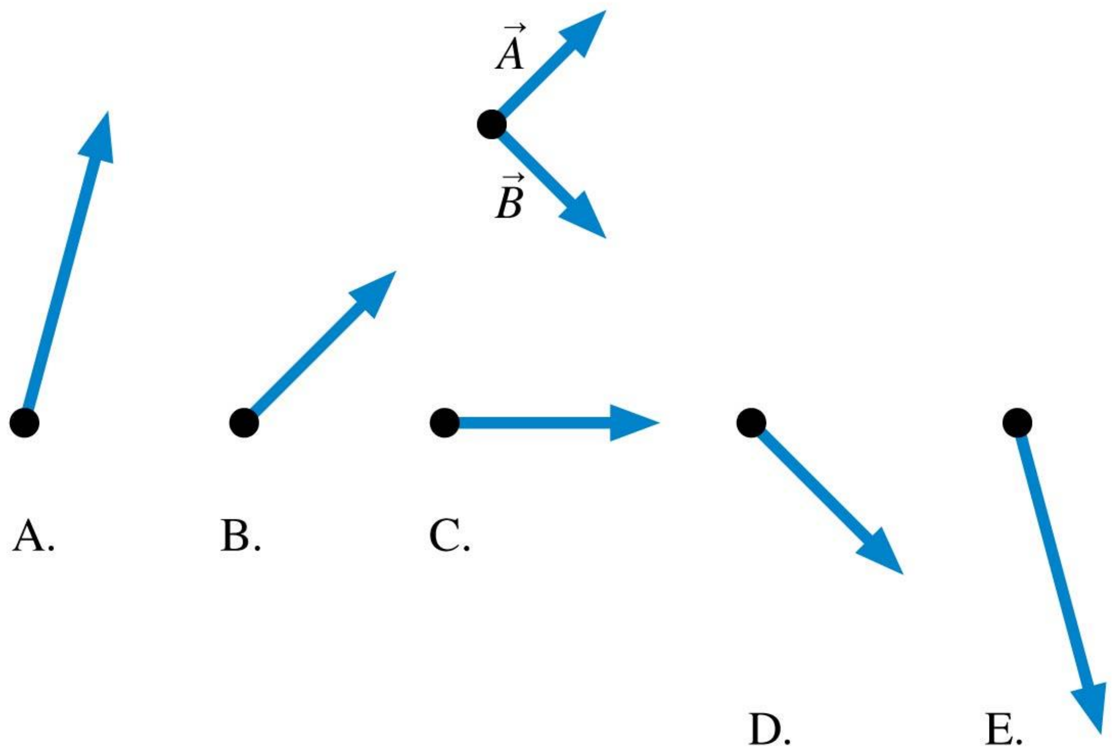
QuickCheck 3.3

- Given vectors $\vec{P}$ and $\vec{Q}$, what is $\vec{P} - \vec{Q}$?



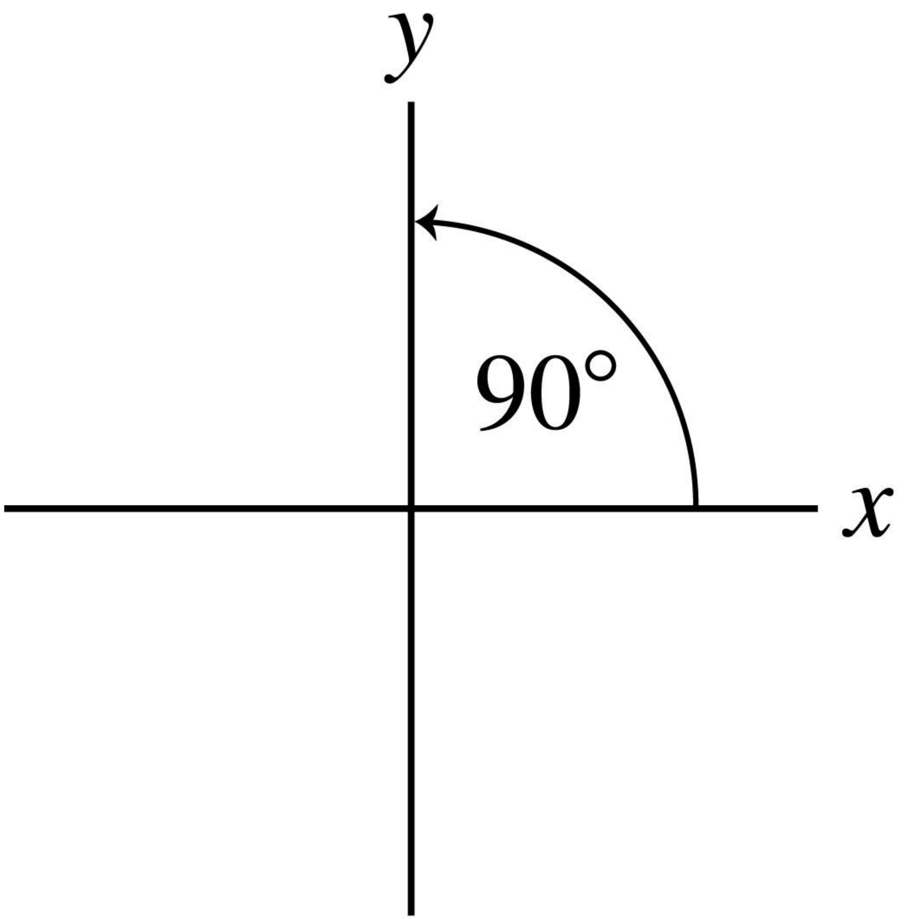
QuickCheck 3.4

- Which of the vectors in the second row shows $2\vec{A} - \vec{B}$?



Coordinate Systems

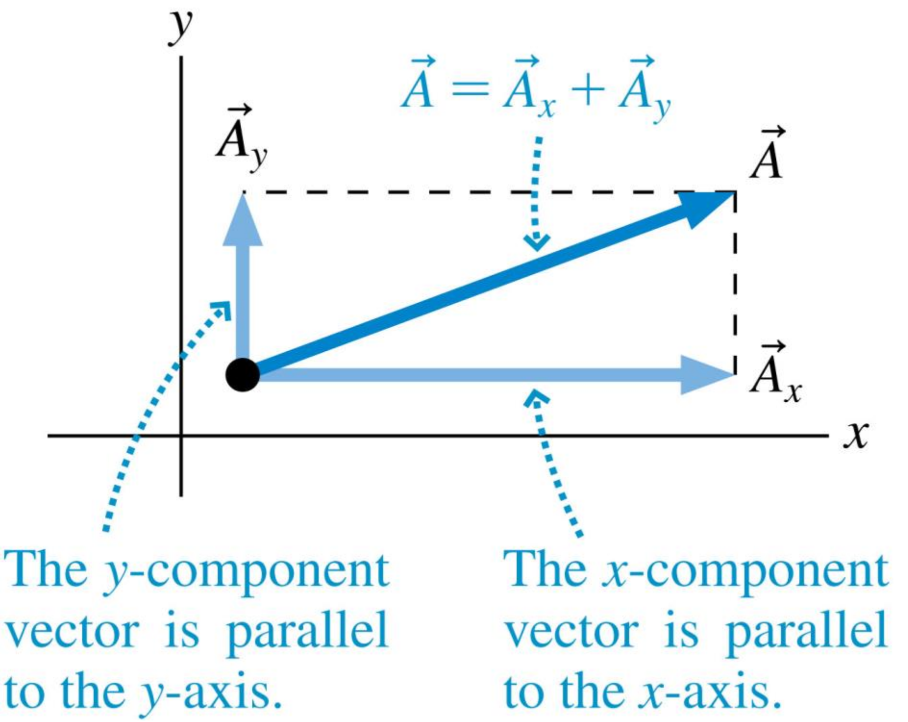
- A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements.
- We will generally use **Cartesian coordinates**.
- Coordinate axes have a positive end and a negative end, separated by a zero at the origin where the two axes cross.



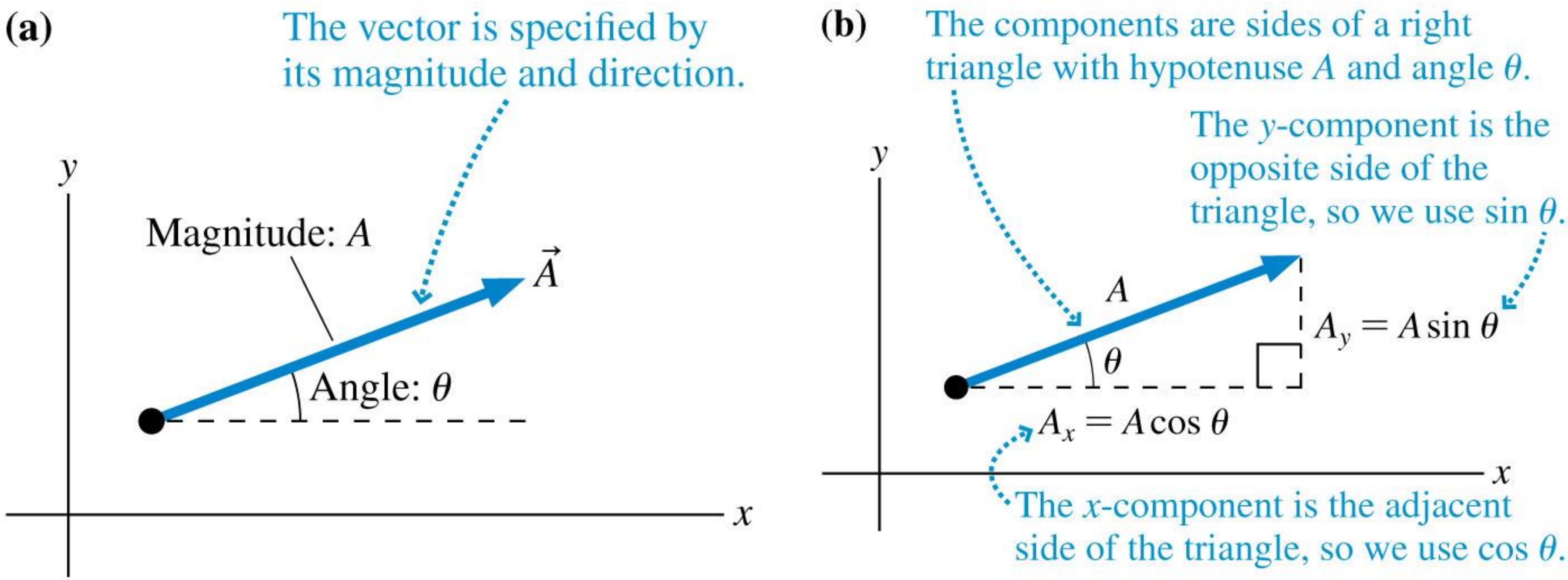
Component Vectors

- For a vector A and an *xy*-coordinate system we can define two new vectors parallel to the axes that we call the component vectors of $\vec{A}$.
- You can see, using the parallelogram rule, that $\vec{A}$ is the vector sum of the two component vectors:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

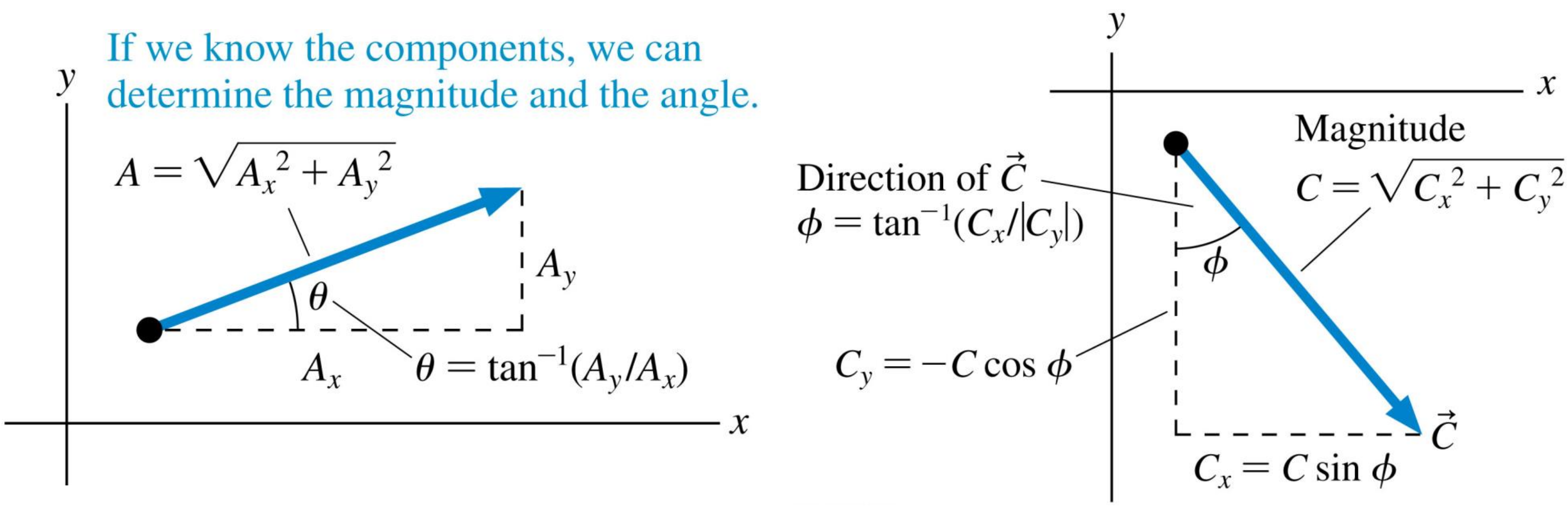


Components



(a)
The vector is specified by its magnitude and direction.
y
Magnitude: A
$\vec{A}$
Angle: θ
x
(b)
The components are sides of a right triangle with hypotenuse A and angle θ.
y
The y-component is the opposite side of the triangle, so we use sin θ.
A
θ
$A_y = A \sin\theta$
$A_x = A \cos\theta$
x
The x-component is the adjacent side of the triangle, so we use cos θ.

Components


If we know the components, we can determine the magnitude and the angle.
$A = \sqrt{A_x^2 + A_y^2}$
θ
A_y
A_x
$\theta = \tan^{-1}(A_y/A_x)$
x
y
Direction of $\vec{C}$
$\phi = \tan^{-1}(C_x/|C_y|)$
Magnitude
$C = \sqrt{C_x^2 + C_y^2}$
ϕ
$C_y = -C \cos \phi$
$C_x = C \sin \phi$
$\vec{C}$

QuickCheck 3.7

- What are the *x*- and *y*-components of this vector?

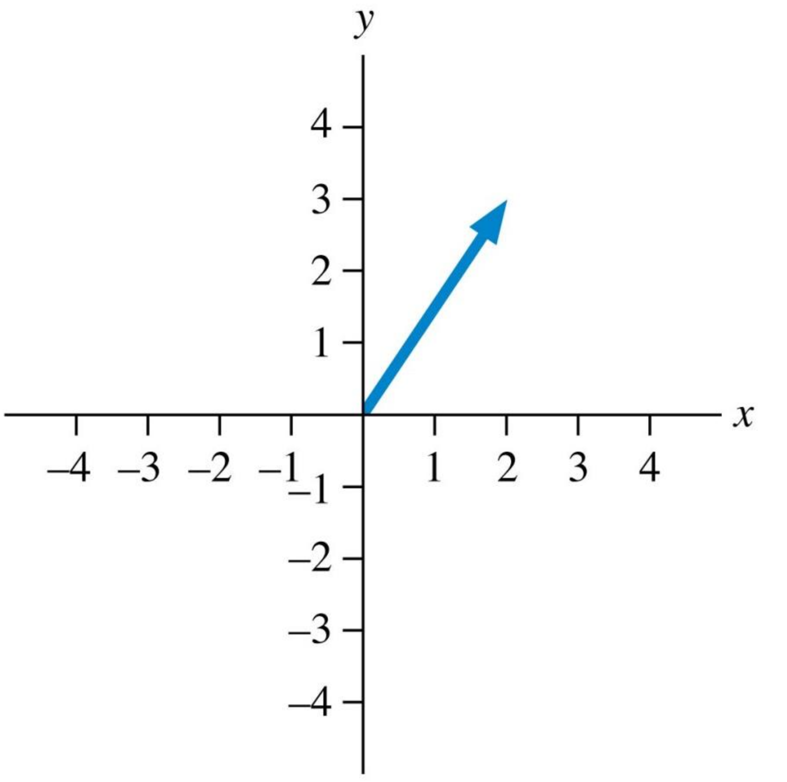
A. 3, 2

B. 2, 3

C. –3, 2

D. 2, –3

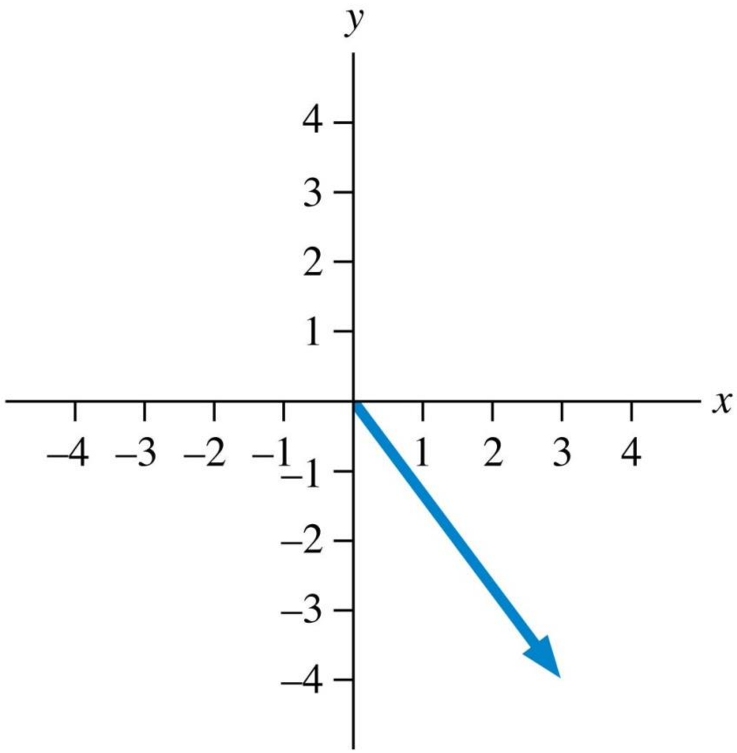
E. –3, –2



QuickCheck 3.8

- What are the *x*- and *y*-components of this vector?

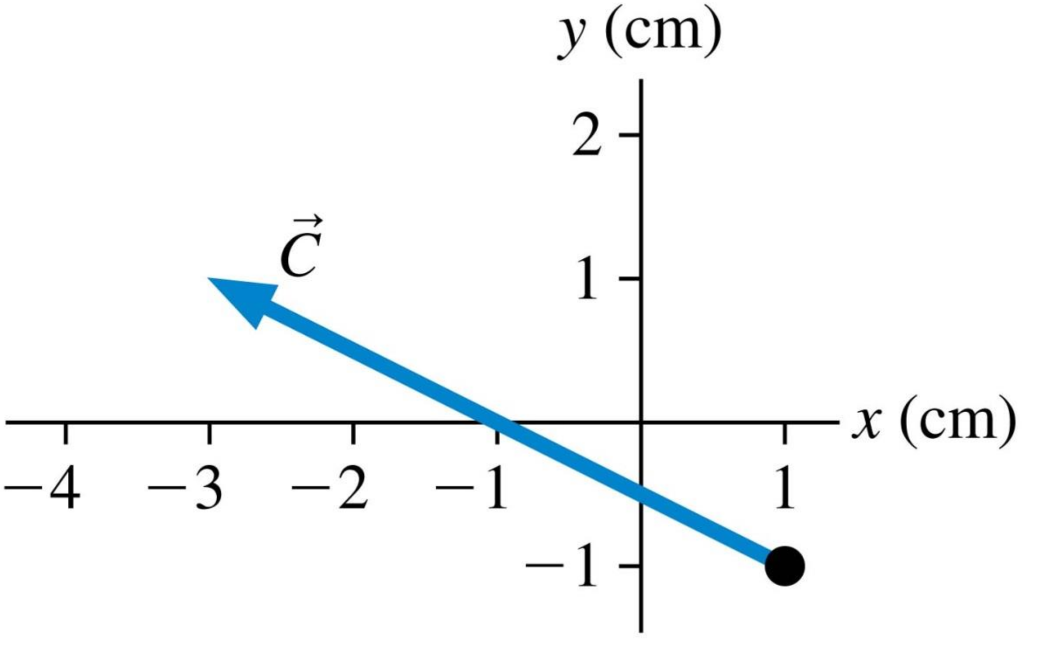
A. 3, 4
B. 4, 3
C. –3, 4
D. 4, –3
E. 3, –4



QuickCheck 3.9

- What are the x- and y-components of vector $\vec{C}$?

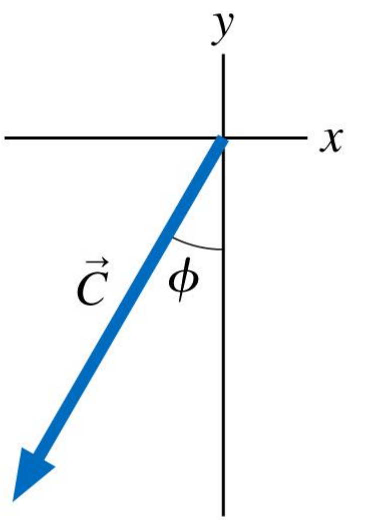
A. 1, –3
B. –3, 1
C. 1, –1
D. –4, 2
E. 2, –4



QuickCheck 3.10

- The angle Φ that specifies the direction of vector $\vec{C}$ is

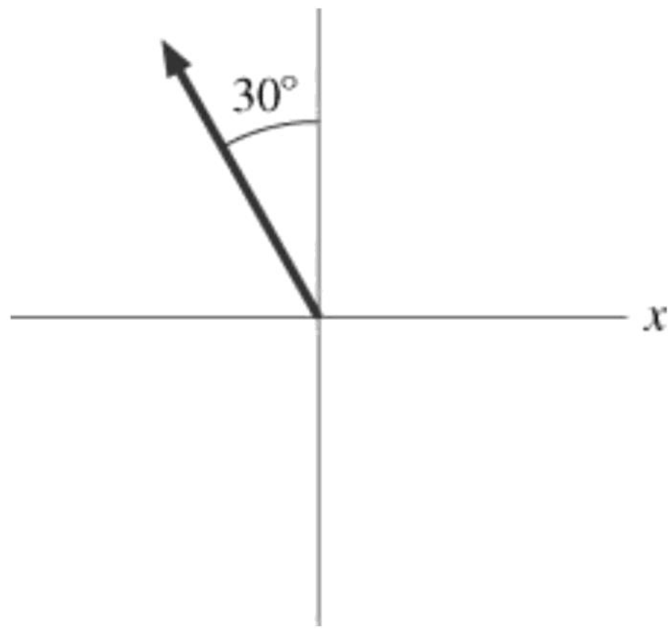
A. $\tan^{-1}(C_x/C_y)$
B. $\tan^{-1}(C_y/C_x)$
C. $\tan^{-1}(|C_x|/C_y)$
D. $\tan^{-1}(|C_x|/|C_y|)$
E. $\tan^{-1}(|C_y|/|C_x|)$



QuickCheck 3.11

- The following vector has length 4.0 units. What are the *x*- and *y*-components?

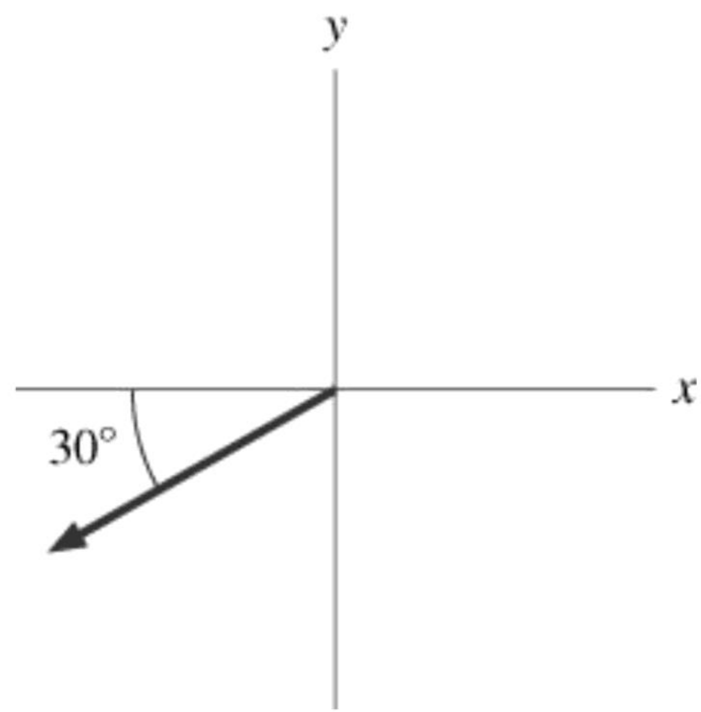
A. 3.5, 2.0
B. –2.0, 3.5
C. –3.5, 2.0
D. 2.0, –3.5
E. –3.5, –2.0



QuickCheck 3.12

- The following vector has length 4.0 units. What are the *x*- and *y*-components?

A. 3.5, 2.0
B. 2.0, 3.5
C. –3.5, 2.0
D. 2.0, –3.5
E. –3.5, –2.0

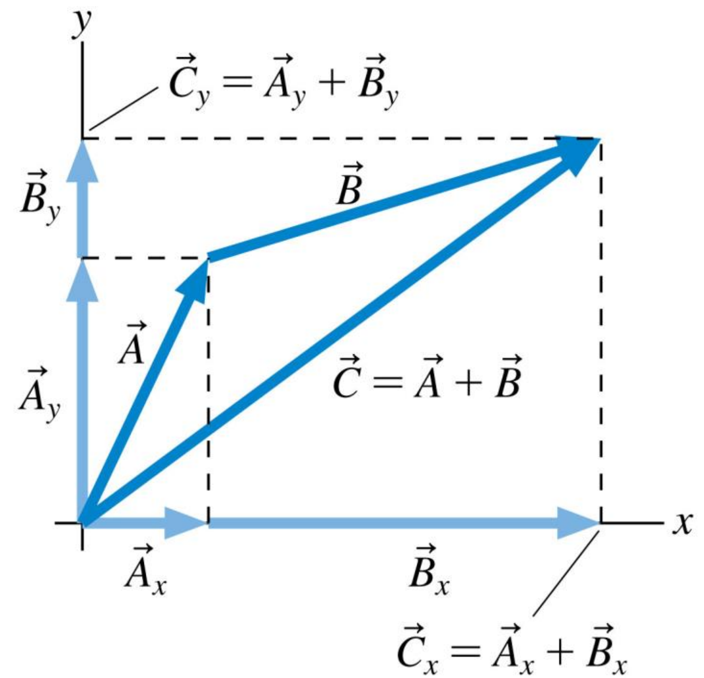


Working with Components

- We can add vectors using components.
- Let's look at the vector sum $\vec{C} = \vec{A} + \vec{B}$ for the vectors shown in the figure. You can see that the component vectors of $\vec{C}$ are the sums of the component vectors of $\vec{A}$ and $\vec{B}$. The same is true of the components: $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

$$D_x = A_x + B_x + C_x + \cdots$$

$$D_y = A_y + B_y + C_y + \cdots$$



Working with Components

$$\vec{F} = \vec{n} + \vec{w} + \vec{f}$$

- Equation 3.18 is really just a shorthand way of writing the two simultaneous equations:

$$F_x = n_x + w_x + f_x$$

$$F_y = n_y + w_y + f_y$$

- In other words, a vector equation is interpreted as meaning: Equate the *x*-components on both sides of the equals sign, then equate the *y*-components. Vector notation allows us to write these two equations in a more compact form.

QuickCheck 3.13

A_x is the ____________ of the vector $\vec{A}_x$.

A. Magnitude
B. *y*-component
C. Direction
D. Size
E. Displacement

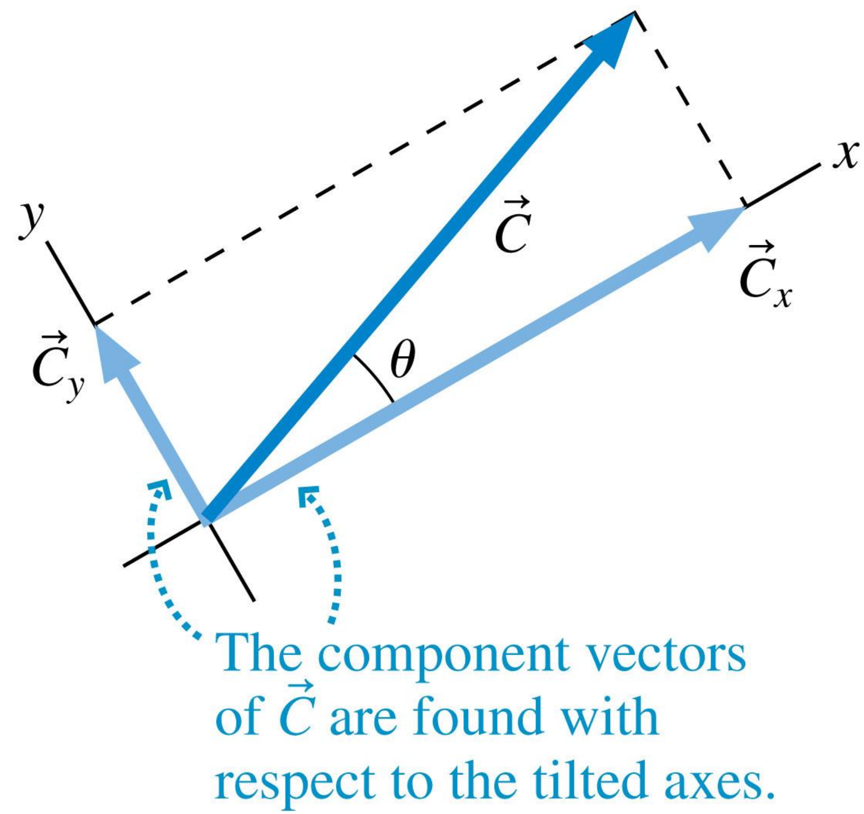
Reading Question 3.2

A_x is positive if $\vec{A}_x$ is directed ________; A_y is positive if $\vec{A}_y$ is directed ________.

A. Right, up
B. Left, up
C. Right, down
D. Left, down

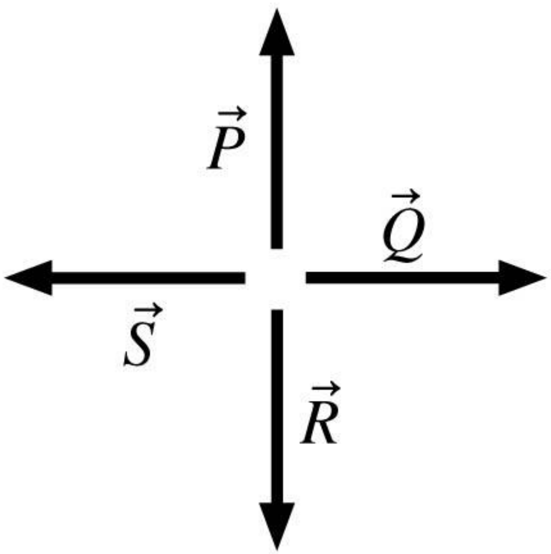
Tilted Axes

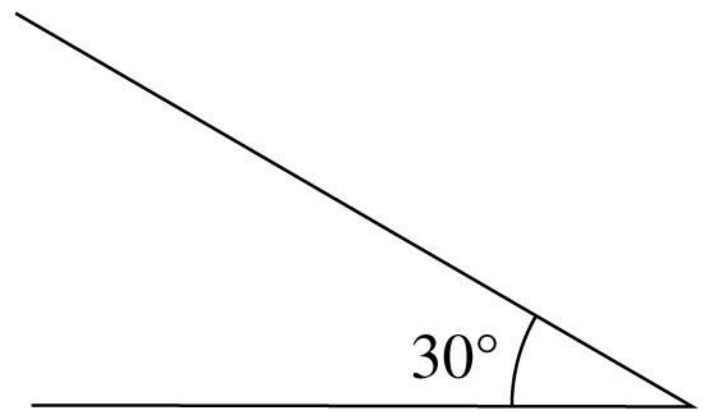
- For motion on a slope, it is often most convenient to put the *x*-axis along the slope.
- When we add the *y*-axis, this gives us a tilted coordinate system.
- Finding components with tilted axes is done the same way as with horizontal and vertical axes. The components are parallel to the tilted axes and the angles are measured from the tilted axes.



Example Problem

The following vectors have length 4.0 units. For each vector, what is the component **parallel** to the ramp?





Example Problem

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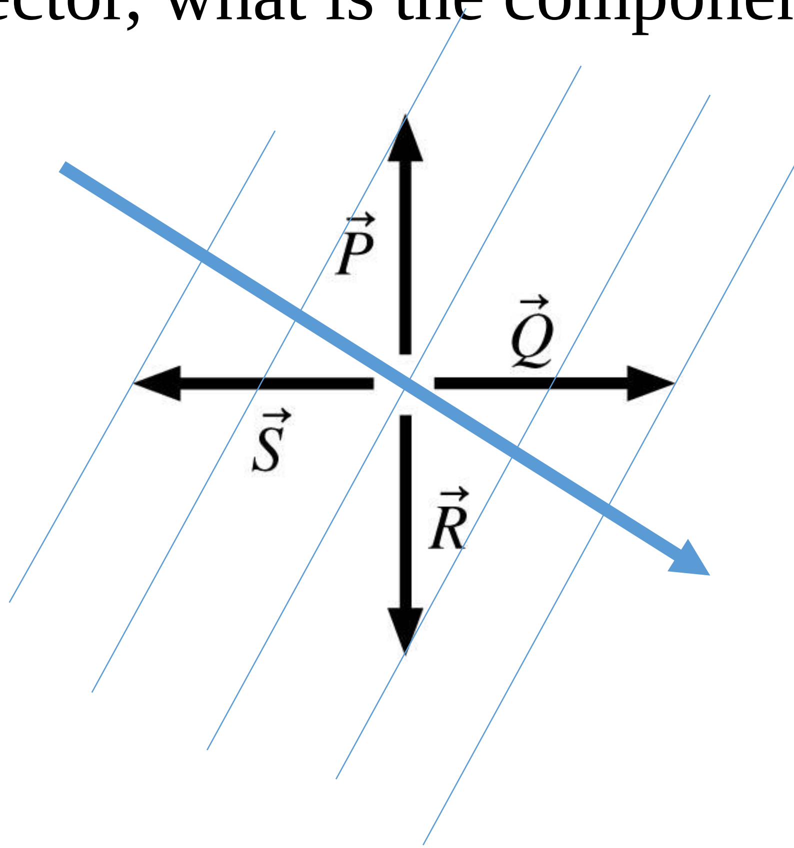
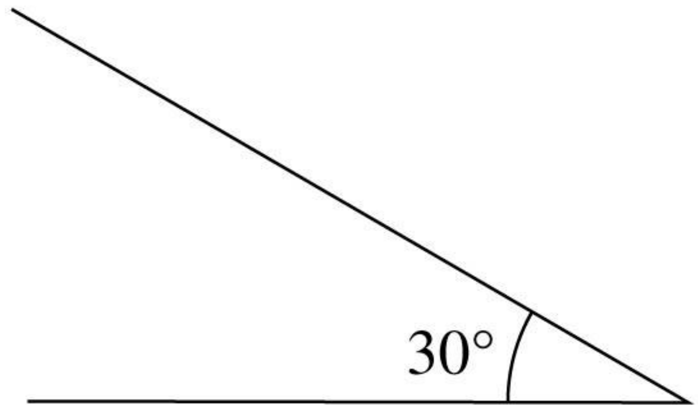




x(P) = - 4sin(30) = -2

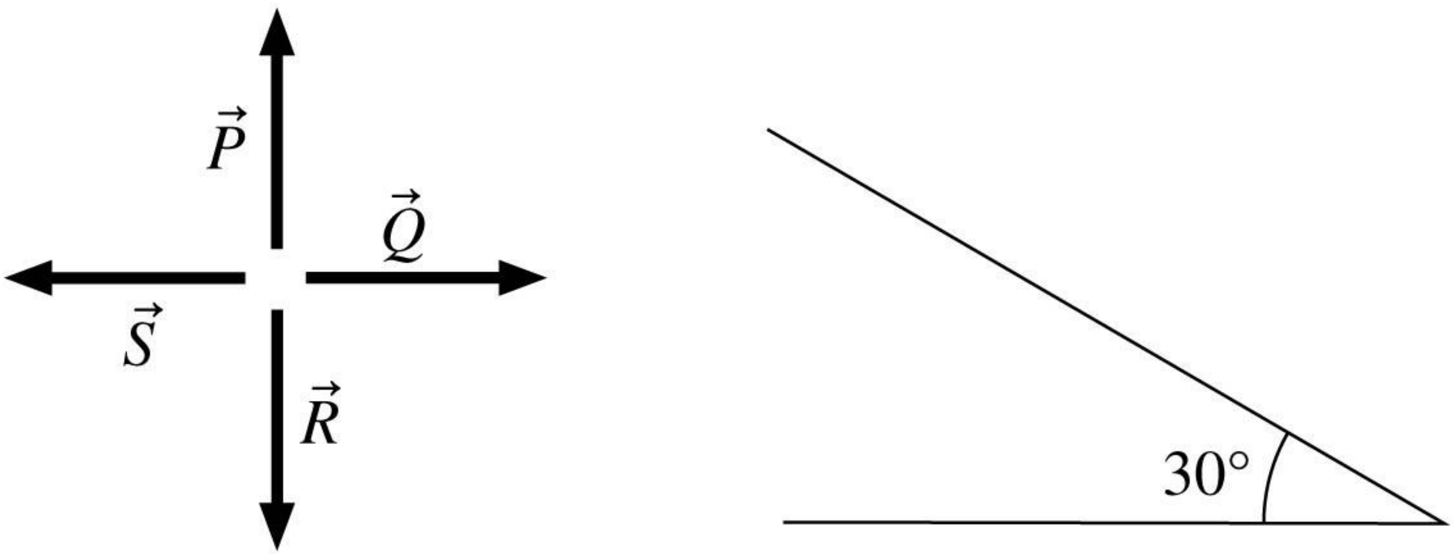
x(R) = 4sin(30) = 2

x(Q) = 4cos(30) = 2√3 ≈ 3.46

x(S) = - 4cos(30) = - 2√3 ≈ -3.46

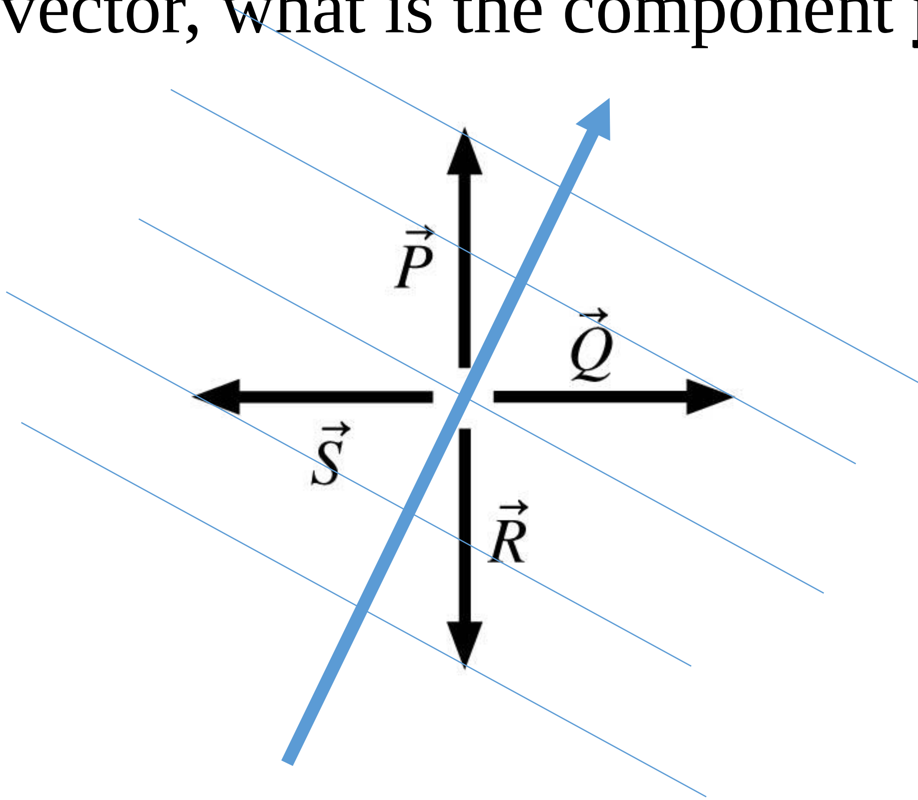
Example Problem

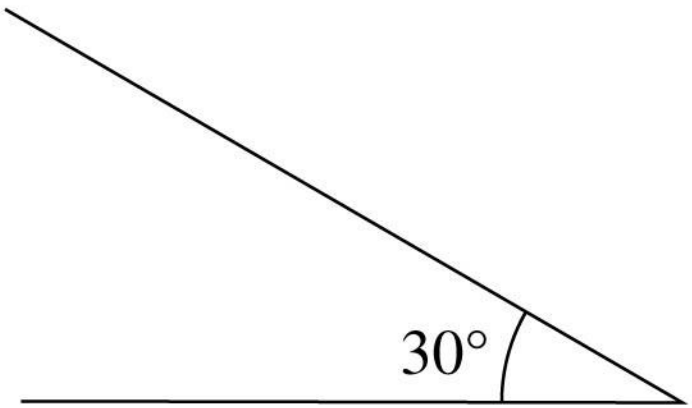
The following vectors have length 4.0 units. For each vector, what is the component **perpendicular** to the ramp?



Example Problem

The following vectors have length 4.0 units. For each vector, what is the component **perpendicular** to the ramp?





y(P) = 4cos(30) = $2\sqrt{3} \approx 3.46$

y(R) = - 4cos(30) = - $2\sqrt{3} \approx$ -3.46

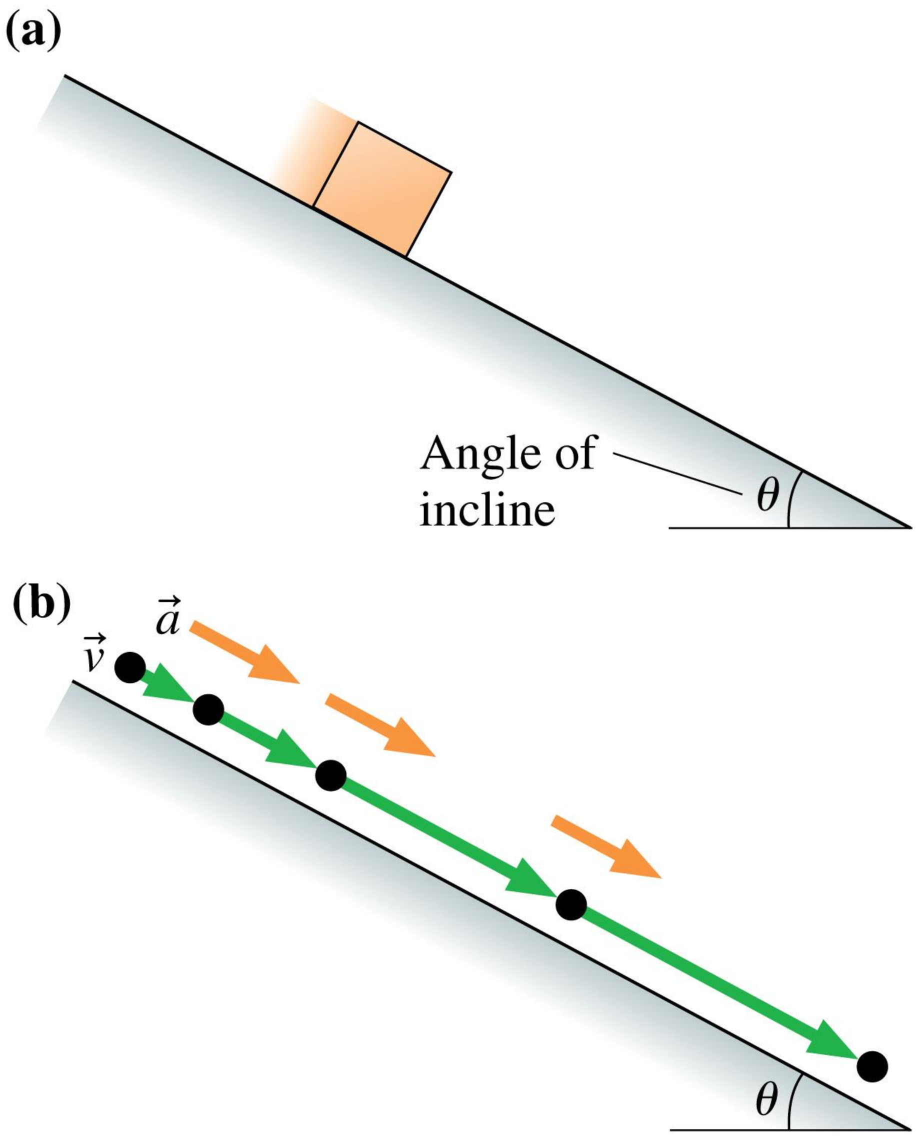
y(Q) = 4sin(30) = 2

y(S) = - 4sin(30) = -2

Section 3.4 Motion on a Ramp

Accelerated Motion on a Ramp

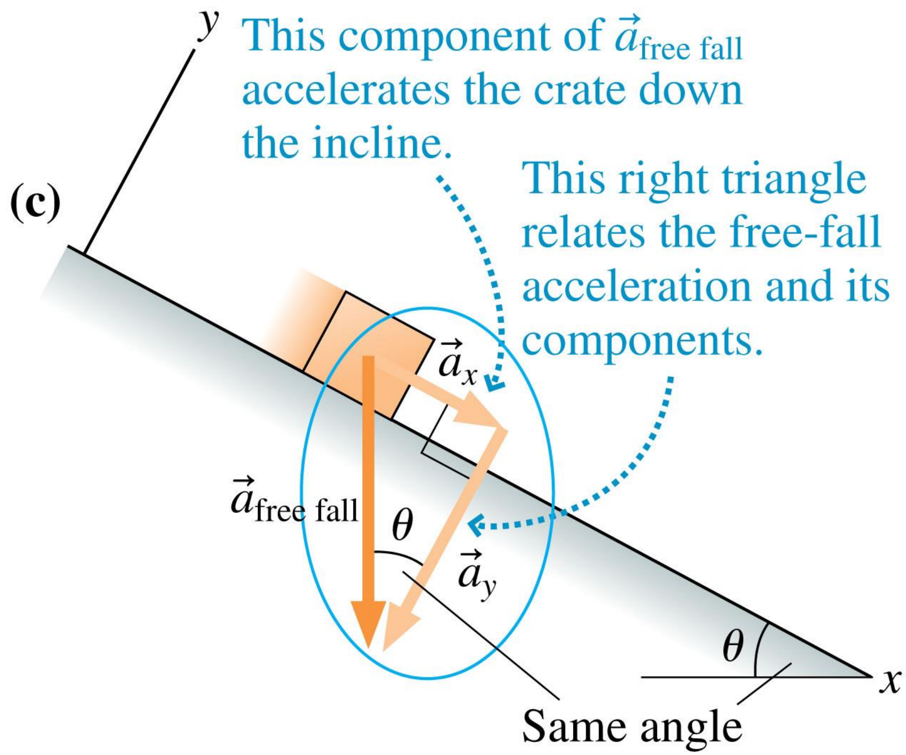
- A crate slides down a frictionless (i.e., smooth) ramp tilted at angle θ.
- The crate is constrained to accelerate parallel to the surface.
- Both the acceleration and velocity vectors are parallel to the ramp.



Accelerated Motion on a Ramp

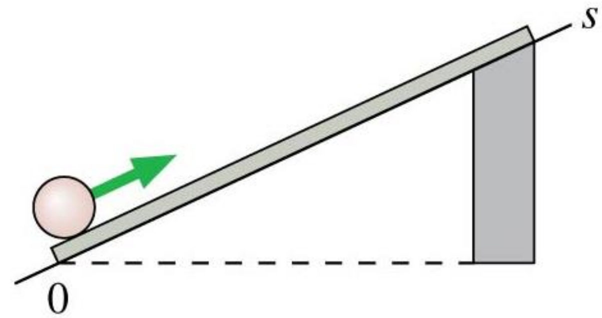
- We choose the coordinate system to have the *x*-axis along the ramp and the *y*-axis perpendicular. All motion will be along the *x*-axis.
- The acceleration parallel to the ramp is a component of the free-fall acceleration the object would have if the ramp vanished:

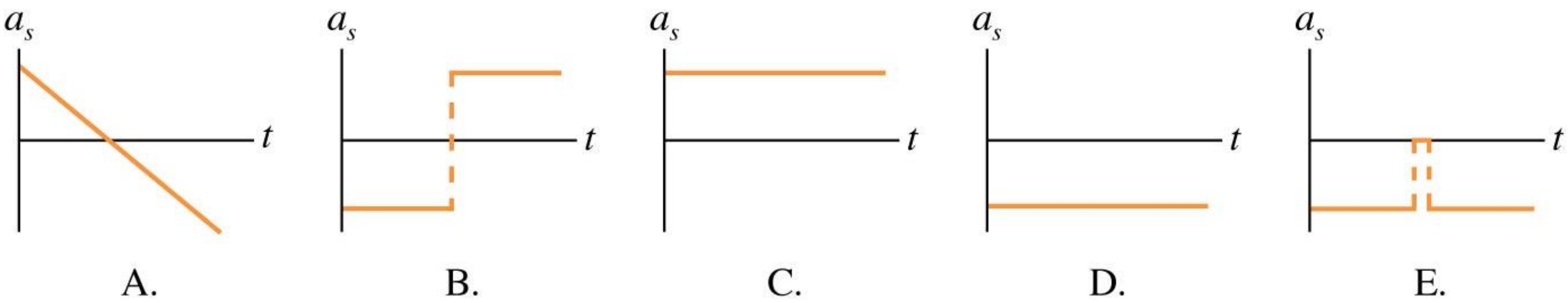
$$a_x = \pm g \sin\theta$$



QuickCheck 3.14

- A ball rolls up the ramp, then back down. Which is the correct acceleration graph?





Reading Question 3.3

The acceleration of a cart rolling down a ramp depends on

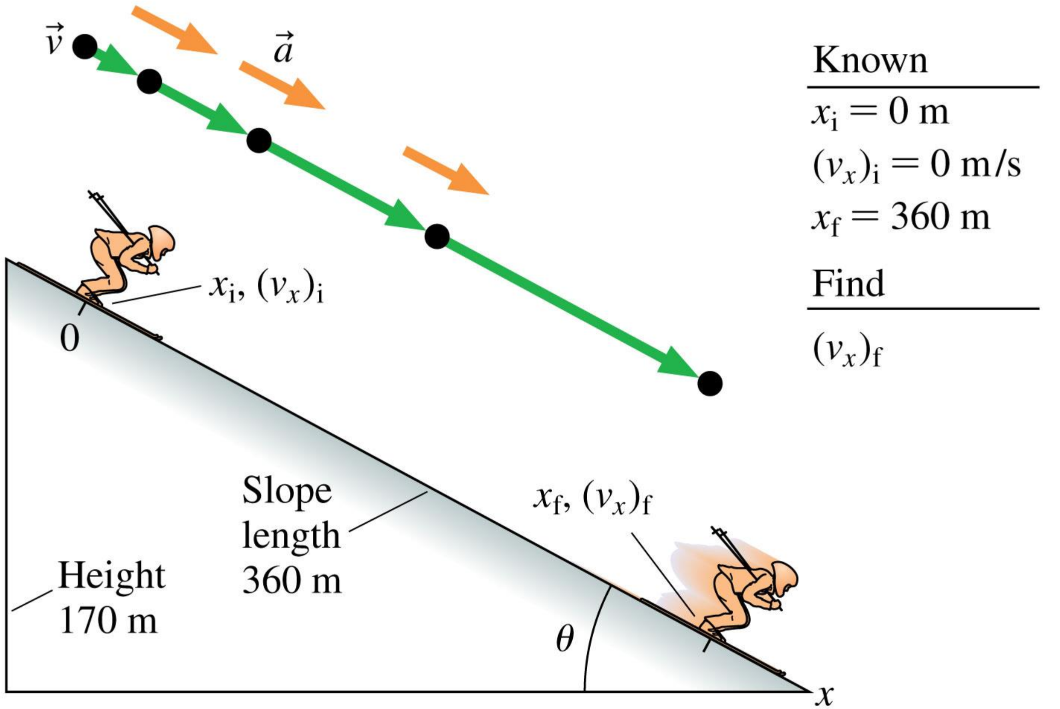
A. The angle of the ramp.
B. The length of the ramp.
C. Both the angle of the ramp and the length of the ramp.
D. Neither the angle of the ramp or the length of the ramp.

Example 3.6 Maximum possible speed for a skier

PREPARE The skier started from rest.

What is the fastest speed at the end of this run?

We put the *x*-axis along the slope.



Example 3.6 Maximum possible speed for a skier (cont.)

SOLVE The fastest possible run would be one without any friction or air resistance.

The acceleration is in the positive *x*-direction, so we use the positive sign.

The precedent figure shows that the 360-m-long slope is the hypotenuse of a triangle of height 170 m, so we use trigonometry to find

$$\sin\theta = \frac{170\text{ m}}{360\text{ m}}$$

which gives $\theta = \sin^{-1}(170/360) = 28°$. And then:

$$a_x = +g\sin\theta = (9.8\text{ m/s}^2)(\sin 28°) = 4.6\text{ m/s}^2$$

Example 3.6 Maximum possible speed for a skier (cont.)

For linear motion with constant acceleration, we can use the third of the kinematic equations:

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x.$$

The initial velocity $(v_x)_i$ is zero; thus

This is the distance along the slope, the length of the run.

$$(v_x)_f = \sqrt{2a_x\Delta x} = \sqrt{2(4.6\text{ m/s}^2)(360\text{ m})} = 58\text{ m/s}$$

This is the fastest that any skier could hope to be moving at the end of the run. Any friction or air resistance would decrease this speed.