

Chapter 2 Motion in One Dimension

Section 2.1 Describing Motion Section 2.2 Uniform Motion Section 2.3 Instantaneous Velocity Section 2.4 Acceleration Section 2.5 Motion with Constant Acceleration Section 2.6 Solving One-Dimensional Motion Section 2.7 Free Fall

Units

• Scientists use a system of units called *le Système International d'Unités*, commonly referred to as **SI Units**.

TABLE 1.1 Common SI units

Measurements and Significant Figures

• When we measure any quantity we can do so with only a certain *precision*.

• We state our knowledge of a measurement through the use of **significant figures**: digits that are reliably known.

Working with Numbers

In scientific notation, a number is expressed as a decimal number between 1 and 10 multiplied by a power of ten. In scientific notation, the diameter of the earth is 1.27×10^7 m.

A **prefix** can be used before a unit to indicate a multiple of 10 or $1/10$. Thus we can write the diameter of the earth as $12,700$ km, where the k in km denotes 1000.

Reading Question 1.2

The quantity 2.67×10^5 has how many significant figures?

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Chapter 2 Motion in One Dimension

Chapter Goal: To describe and analyze linear motion.

Motion Diagrams

• A good first step in analyzing motion is to draw a motion diagram, marking the position of an object in subsequent times.

• In this chapter, you'll learn to create motion diagrams for different types of motion along a line. Drawing pictures like this is a good staring point for solving problems.

Stop to Think

A bicycle is moving to the left with increasing speed. Which of the following motion diagrams illustrates this motion?

Section 2.1 Describing Motion

Representing Position

- We will use an *x***-axis** to analyze horizontal motion and motion on a ramp, with the positive end to the right.
- We will use a *y***-axis** to analyze vertical motion, with the positive end up.

Representing Position

The motion diagram of a student walking to school and a coordinate axis for making measurements

- Every dot in the motion diagram of the upper figure represents the student's position at a particular time.
- Lower figure shows the student's motion shows the student's position as a **graph** of *x* versus *t*.

From Position to Velocity

• On a position-versus-time graph, **a faster speed corresponds to a steeper slope**.

slope of graph =
$$
\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}
$$

• **The slope of an object's position-versus-time graph is the object's velocity at that point in the motion.**

From Position to Velocity

- We can deduce the **velocity-versus-time graph** from the positionversus-time graph.
- The velocity-versus-time graph is yet another way to represent an object's motion.

• Here is a motion diagram of a car moving along a straight road:

• Here is a motion diagram of a car moving along a straight road:

A graph of position versus time for a basketball player moving down the court appears as follows:

Which of the following velocity graphs matches the position graph?

Example 2.2 Analyzing a car's position graph

The figure gives the position-versus-time graph of a car.

- a. Draw the car's velocityversus-time graph.
- b. Describe the car's motion in words.

PREPARE The Figure is a graphical representation of the motion. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

Example 2.2 Analyzing a car's position graph (cont.)

SOLVE

a. From $t = 0$ s to $t = 2$ s ($\Delta t = 2$ s) the car's displacement is $\Delta x = -4$ m -0 m $= -4$ m. The velocity during this interval is

$$
v_x = \frac{\Delta x}{\Delta t} = \frac{-4 \text{ m}}{2 \text{ s}} = -2 \text{ m/s}
$$

The car's position does not change from $t = 2$ s to $t = 4$ s ($\Delta x = 0$ m), so $v_x = 0$ m/s. Finally, the displacement between $t = 4$ s and $t = 6$ s ($\Delta t = 2$ s) is $\Delta x = 10$ m. Thus the velocity during this interval is

$$
v_x = \frac{10 \text{ m}}{2 \text{ s}} = 5 \text{ m/s}
$$

These velocities are represented graphically.

Example 2.2 Analyzing a car's position graph (cont.)

SOLVE

b. The velocity-versus-time graph of Figure 2.12 shows the motion in a way that we can describe in a straightforward manner: The car backs up for 2 s at 2 m/s, sits at rest for 2 s, then drives forward at 5 m/s for 2 s.

ASSESS Notice that the velocity graph and the position graph look completely different. They should! The value of the velocity graph at any instant of time equals the *slope* of the position graph. Since the position graph is made up of segments of constant slope, the velocity graph should be made up of segments of constant *value*, as it is. This gives us confidence that the graph we have drawn is correct.

From Velocity to Position

- We can deduce the position -versus -time graph from the velocity -versus time graph.
- The sign of the velocity tells us whether the slope of the position graph is positive or negative.
- The magnitude of the velocity tells us how steep the slope is.

• A graph of velocity versus time for a hockey puck shot into a goal appears as follows:

• Which of the following position graphs matches the velocity graph?

Section 2.2 Uniform Motion

Uniform Motion

- Straight-line motion in which equal displacements occur during any successive equal -time intervals is called **uniform motion** or **constant velocity motion.**
- **An object's motion is uniform if and only if its position -versus -time graph is a straight line.**

Equations of Uniform Motion

• The velocity of an object in uniform motion tells us the amount by which its position changes during each second.

$$
v_x = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}
$$

$$
x_{\rm f} = x_{\rm i} + v_{\rm x} \, \Delta t
$$

Position equation for an object in uniform motion $(v_x$ is constant)

$$
\Delta x = v_x \, \Delta t
$$

• The displacement Δx is proportional to the time interval Δ*t*.

- Here is a position graph of an object:
- At $t = 1.5$ s, the object's velocity is

- A. 40 m/s
- B. 20 m/s
- C. 10 m/s
- D. -10 m/s
- E. None of the above

Reading Question 2.2

Which of the following is an example of uniform motion?

- A. A car going around a circular track at a constant speed.
- B. A person at rest starts running in a straight line in a fixed direction.
- C. A ball dropped from the top of a building.
- D. A hockey puck sliding in a straight line at a constant speed.

Section 2.3 Instantaneous Velocity

Instantaneous Velocity

- For one-dimensional motion, an object changing its velocity is either speeding up or slowing down.
- An object's velocity—a speed *and* a direction—at a specific *instant* of time *t* is called the object's **instantaneous velocity.**
- **From now on, the word "velocity" will always mean instantaneous velocity.**

Finding the Instantaneous Velocity

• Graphically, the slope of the curve at a point is the same as the slope of a straight line drawn *tangent* to the curve at that point. Calculating rise over run for the tangent line, we get

$$
v_x = (8.0 \text{ m})/(1.0 \text{ s}) = 8.0 \text{ m/s}
$$

• This is the same value we obtained from the closeup view. The slope of the tangent line is the instantaneous velocity at that instant of time.

Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is the

- A. Object's speed at that point.
- B. Object's average velocity at that point.
- C. Object's instantaneous velocity at that point.
- D. Object's acceleration at that point.
- E. Distance traveled by the object to that point.

From Velocity to Position, One More Time

• **The displacement Δ***x* **is equal to the area under the velocity graph during the time interval Δ***t***.**

• A car moves along a straight stretch of road. The following graph shows the car's position as a function of time:

- At what point (or points) do the following conditions apply?
	- The displacement is zero.
	- The speed is zero.
	- The speed is increasing.
	- The speed is decreasing.

Example 2.5 The displacement during a rapid start

The Figure shows the velocity-versus-time graph of a car pulling away from a stop. How far does the car move during the first 3.0 s?

PREPARE the Figure is a graphical representation of the motion. The question How far? indicates that we need to find a displacement Δ*x* rather than a position *x*. According to Equation 2.7, the car's displacement v_r (m/s) The velocity is steadily $\Delta x = x_f - x_i$ between $t = 0$ s 16 increasing. and $t = 3$ s is the area under

the curve from $t = 0$ s to

 $t = 3$ s.

Example 2.5 The displacement during a rapid start (cont.)

SOLVE The curve in this case is an angled line, so the area is that of a triangle:

 Δx = area of triangle between $t = 0$ s and $t = 3$ s

$$
=\frac{1}{2}
$$
 × base × height $=\frac{1}{2}$ × 3 s × 12 m/s = 18 m

The car moves 18 m during the first 3 seconds as its velocity changes from 0 to 12 m/s. v_x (m/s)

Section 2.4 Acceleration

Acceleration

- We define a new motion concept to describe an object whose velocity is changing.
	- The ratio of $\Delta v_x / \Delta t$ is the *rate of change of velocity*.
	- The ratio of $\Delta v_x / \Delta t$ is the *slope of a velocity-versus-time graph.*

$$
a_x = \frac{\Delta v_x}{\Delta t}
$$

Definition of acceleration as the rate of change of velocity

Representing Acceleration

• We can find an acceleration graph from a velocity graph.

The Sign of the Acceleration

An object can move right or left (or up or down) while either speeding up or slowing down. Whether or not an object that is slowing down has a negative acceleration depends on the direction of motion.

> The object is moving to the right $(v_r > 0)$ and speeding up.

The object is moving to the left $(v_r < 0)$ and slowing down.

The Sign of the Acceleration (cont.)

An object can move right or left (or up or down) while either speeding up or slowing down. Whether or not an object that is slowing down has a negative acceleration depends on the direction of motion.

> The object is moving to the right $(v_r > 0)$ and slowing down.

The object is moving to the left $(v_r < 0)$ and speeding up.

- The motion diagram shows a particle that is slowing down. The sign of the position *x* and the sign of the velocity v_x are:
	- A. Position is positive, velocity is positive.
	- B. Position is positive, velocity is negative.
	- C. Position is negative, velocity is positive.
	- D. Position is negative, velocity is negative.

• A particle has velocity \vec{v}_1 as it moves from point 1 to point 2. The acceleration is shown. What is its velocity vector \vec{v}_2 as it moves away from point 2?

• Here is a motion diagram of a car speeding up on a straight road:

- The sign of the acceleration a_x is
	- A. Positive.
	- B. Negative.
	- C. Zero.

- The motion diagram shows a particle that is slowing down. The sign of the acceleration a_x is:
	- A. Acceleration is positive.
	- B. Acceleration is negative.

Reading Question 2.4

If an object is speeding up,

- A. Its acceleration is positive.
- B. Its acceleration is negative.
- C. Its acceleration can be positive or negative depending on the direction of motion.

Example Problem

A ball moving to the right traverses the ramp shown below. Sketch a graph of the velocity versus time, and, directly below it, using the same scale for the time axis, sketch a graph of the acceleration versus time.

• Which velocity-versus-time graph goes with this acceleration graph?

Section 2.5 Motion with Constant Acceleration

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Constant Acceleration Equations

• We can use the acceleration to find $(v_x)_f$ at a later time t_f .

$$
a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t}
$$

$$
(v_x)_f = (v_x)_i + a_x \Delta t
$$

Velocity equation for an object with constant acceleration

• We have expressed this equation for motion along the *x*-axis, but it is a general result that will apply to any axis.

Constant Acceleration Equations

- The velocity-versus-time graph for constant-acceleration motion is a straight line with value (v_x) at time t_i and slope a_x .
- The displacement Δ*x* during a time interval Δ*t* is the area under the velocity-versus-The displacement Δx is the area under this curve: the sum time graph shown in the of the areas of a triangle ... shaded area of the figure. **(b)** Velocity ... and a rectangle.

Constant Acceleration Equations

For **motion with constant acceleration**:

• Velocity changes steadily:

• The position changes as the square of the time interval:

Final and initial position (m)

\n
$$
\mathbf{x}_{\mathbf{f}} = \mathbf{x}_{\mathbf{i}}^{\mathbf{i}} + (\mathbf{y}_{\mathbf{x}})_{\mathbf{i}} \Delta t + \frac{1}{2} a_{\mathbf{x}} (\mathbf{\Delta} t)^{2}
$$
\nInitial velocity (m/s)

\n
$$
\mathbf{f}(\mathbf{x}_{\mathbf{f}}) = \mathbf{f}(\mathbf{x}_{\mathbf{f}})_{\mathbf{f}(\mathbf{x}_{\mathbf{f}})} \Delta t + \frac{1}{2} a_{\mathbf{x}} (\mathbf{\Delta} t)^{2}
$$
\nReceleration (m/s)

• We can also express the change in velocity in terms of **distance, not time**:

Final and initial velocity (m/s)

\n
$$
\begin{aligned}\n\vec{v} &= (\vec{v}_x)^2 + 2a_x \Delta x \quad (3) \\
\text{Acceleration} &= (\vec{v}_x)^2 + 2a_x \Delta x \quad (4) \\
\text{Acceleration} &= (\vec{v}_x)^2 \quad \text{Change in} \\
(m/s^2) &= \text{position (m)}\n\end{aligned}
$$

Text: p. 43

Motion with Constant Acceleration

• We can use the slope of the graph in the velocity graph to determine the acceleration of the rocket.

$$
a_y = \frac{\Delta v_y}{\Delta t} = \frac{27 \text{ m/s}}{1.5 \text{ s}} = 18 \text{ m/s}^2
$$

Example 2.8 Coming to a stop

As you drive in your car at 15 m/s, you see a child's ball roll into the street ahead of you. You hit the brakes and stop as quickly as you can. In this case, you come to rest in 1.5 s.

How far does your car travel as you brake to a stop?

PREPARE The problem statement gives us a description of motion in words. To help us visualize the situation, the figure

illustrates the key features of the motion with a motion diagram and a velocity graph. The graph is based on the car slowing from 15 m/s to 0 m/s in 1.5 s.

Example 2.8 Coming to a stop (cont.)

SOLVE We've assumed that your car is moving to the right, so its initial velocity is $(v_x)_i = +15$ m/s. After you come to rest, your final velocity is $(v_x)_f = 0$ m/s. We use the definition of acceleration:

$$
a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t} = \frac{0 \text{ m/s } - 15 \text{ m/s}}{1.5 \text{ s}} = -10 \text{ m/s}^2
$$

An acceleration of -10 m/s^2 (really -10 m/s per second) means the car slows by 10 m/s every second.

Now that we know the acceleration, we can compute the distance that the car moves as it comes to rest using the second constant acceleration equation:

$$
x_{\rm f} - x_{\rm i} = (v_x)_{\rm i} \Delta t + \frac{1}{2} a_x (\Delta t)^2
$$

= (15 \text{ m/s})(1.5 \text{ s}) + \frac{1}{2} (-10 \text{ m/s}^2)(1.5 \text{ s})^2 = 11 \text{ m}

Section 2.7 Free Fall

Free Fall

- If an object moves under the influence of gravity only, and no other forces, we call the resulting motion **free fall**.
- **Any two objects in free fall, regardless of their mass, have the same acceleration.**
- On the earth, air resistance is a factor. For now we will restrict our attention to situations in which air resistance can be ignored.

Free Fall

• The figure shows the motion diagram for an object that was released from rest and falls freely. The diagram and the graph would be the same for all falling objects.

Free Fall

- **The free-fall acceleration always points down**, no matter what direction an object is moving.
- Any object moving under the influence of gravity only, and no other force, is in free fall.

 $\vec{a}_{\text{free fall}} = (9.80 \text{ m/s}^2, \text{ vertically downward})$

Standard value for the acceleration of an object in free fall

- A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration *a^y* is
	- A. Positive.
	- B. Negative.
	- C. Zero.

• An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. At which point of the trajectory is the arrow's acceleration the greatest? The least? Ignore air resistance; the only force acting is gravity.

• An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. At which point of the trajectory is the arrow's acceleration the greatest? The least? Ignore air resistance; the only force acting is gravity.

Same at all points

• An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. Which graph best represents the vertical velocity of the arrow as a function of time? Ignore air resistance; the only force acting is gravity.

Example 2.14 Analyzing a rock's fall

A heavy rock is dropped from rest at the top of a cliff and falls 100 m before hitting the ground. How long does the rock take to fall to the ground, and what is its velocity when it hits?

PREPARE The figure shows a visual overview with all necessary data. We have placed the origin at the ground, which makes

Example 2.14 Analyzing a rock's fall (cont.)

SOLVE Free fall is motion with the specific constant acceleration $a_y = -g$.

The relation between time and distance gives:

Using the relation between time and distance with $(v_y)_i = 0$ m/s and $t_i = 0$ s, we find

$$
y_{\rm f} = y_{\rm i} + (v_{\rm y})_{\rm i} \Delta t + \frac{1}{2} a_{\rm y} (\Delta t)^2 = y_{\rm i} - \frac{1}{2} g (\Delta t)^2 = y_{\rm i} - \frac{1}{2} g t_{\rm f}^2
$$

We can now solve for t_f :

$$
t_{\rm f} = \sqrt{\frac{2(y_{\rm i} - y_{\rm f})}{g}} = \sqrt{\frac{2(100 \text{ m} - 0 \text{ m})}{9.80 \text{ m/s}^2}} = 4.52 \text{ s}
$$

Now that we know the fall time, we can use the first kinematic equation to find $(v_y)_f$:

$$
(v_y)_f = (v_y)_i - g \Delta t = -gt_f = -(9.80 \text{ m/s}^2)(4.52 \text{ s})
$$

= -44.3 m/s

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Summary: Important Concepts

Velocity is the rate of change of position:

$$
v_x = \frac{\Delta x}{\Delta t}
$$

Acceleration is the rate of change of velocity:

$$
a_x = \frac{\Delta v_x}{\Delta t}
$$

The units of acceleration are $m/s²$.

An object is speeding up if v_r and a_r have the same sign, slowing down if they have opposite signs.

Summary: Important Concepts

A position-versus-time graph plots position on the vertical axis against time on the horizontal axis.

Summary: Applications

Uniform motion

An object in uniform motion has a constant velocity. Its velocity graph is a horizontal line; its position graph is linear.

Kinematic equation for uniform motion:

$$
x_{\rm f} = x_{\rm i} + v_{\rm x} \, \Delta t
$$

Uniform motion is a special case of constantacceleration motion, with $a_x = 0$.

Summary: Applications

Motion with constant acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.

Kinematic equations for motion with constant acceleration:

$$
\begin{aligned} \n\psi_x\text{r} &= (\nu_x\text{r}_1 + a_x \, \Delta t \\ \nx_f &= x_i + (\nu_x\text{r}_1 \, \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\ \n(\nu_x\text{r}_f^2 &= (\nu_x\text{r}_1\text{r}_1 + 2a_x \, \Delta x \, \Delta x) \n\end{aligned}
$$

Summary: Applications

Free fall

Free fall is a special case of constantacceleration motion. The acceleration has magnitude $g = 9.80$ m/s² and is always directed vertically downward whether an object is moving up or down.

