Chapter 2 – Kinematics of Particles – p1

- Kinematics of a Particle
- Rectilinear Motion
- Particle Motion Cases
- Kinematic Equations
- Relative Rectilinear Motion

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What is Dynamics?

Scientific Definition

Dynamics is a scientific discipline that deals with **systems** undergoing changes in **state**.

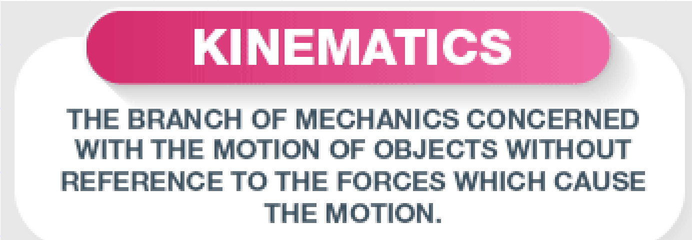
Engineering Definition

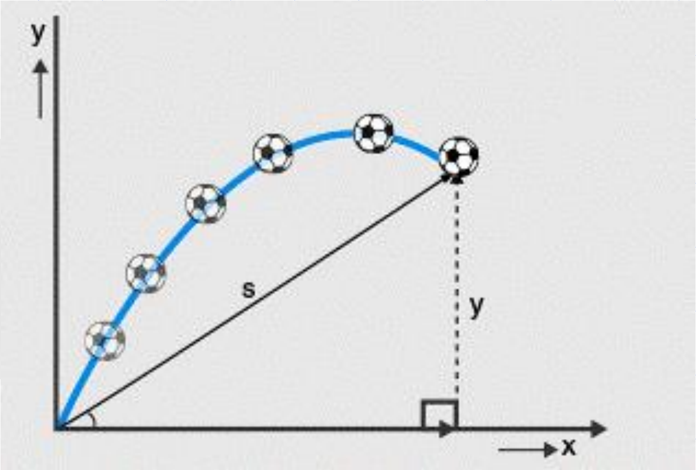
Dynamics is a branch of Mechanics that deals with the relation between **forces** and the **motion** of *bodies*.

Dynamics is divided into two parts:

<u>Kinematics</u> – the study of motion without reference to any forces

- Study of the geometry of motion
- A purely mathematical construct





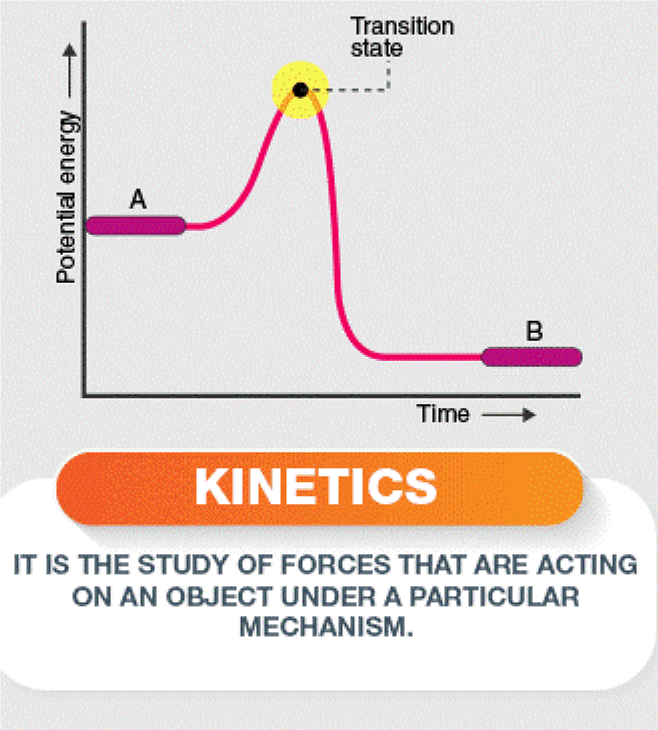
Kinetics – the study of motion that results from forces action on bodies

- Used to predict motion caused by given forces

  \- OR -

  to determine the forces required to produce a given motion

- Based on physical law



Kinematics of a Particle

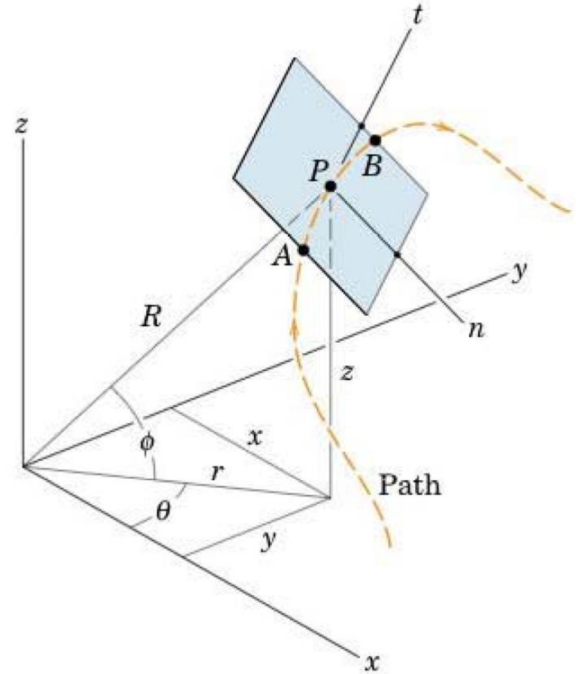
Although each of these planes is rather large, from a distance their motion can be modeled as if each plane were a particle

Kinematics of Particles

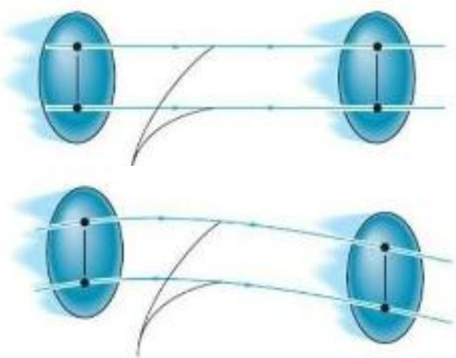
Relates motion and time w/o ref. to forces that cause the motion

Here, "motion" is quantified by

- Position $\bar{r}$
- Velocity $\bar{v}$
- Acceleration $\bar{a}$



Motion Types:

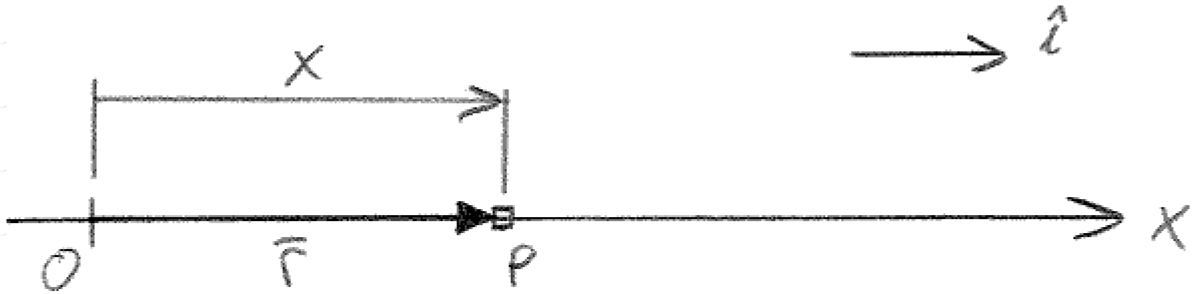


Path of curvilinear translation

1. Rectilinear
2. Curvilinear
3. Relative
4. Dependent

Rectilinear Motion of Particles

Particle is constrained to move in a straight line.



$$\boxed{\bar{r} = \overrightarrow{OP} = x\,\hat{i}}$$

Position Vector

Comments:

A Scalar

- Magnitude of x specifies <u>distance</u> of P away from O

- sign of x indicates the <u>direction</u> of P relative to O

Scalar notation suffices.
(Rectilinear Motion only!)

Position x

x = algebraic dist. from O to P

(m, ft, mi, etc.)

Velocity v

We define

$$\boxed{v = \frac{dx}{dt}}$$

Instantaneous velocity

Acceleration a

Ave. Acc. $= \dfrac{\Delta v}{\Delta t}$ $\qquad \left(\dfrac{m/s}{s} = m/s^2,\ ft/s^2,\ \text{etc.} \right)$

In the limit as $\Delta t \to 0$

$$\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2}}$$

Instantaneous Acceleration

Shorthand: $\qquad (\dot{\ }) = \dfrac{d(\ )}{dt}$

$$\Rightarrow \qquad v = \dot{x}$$

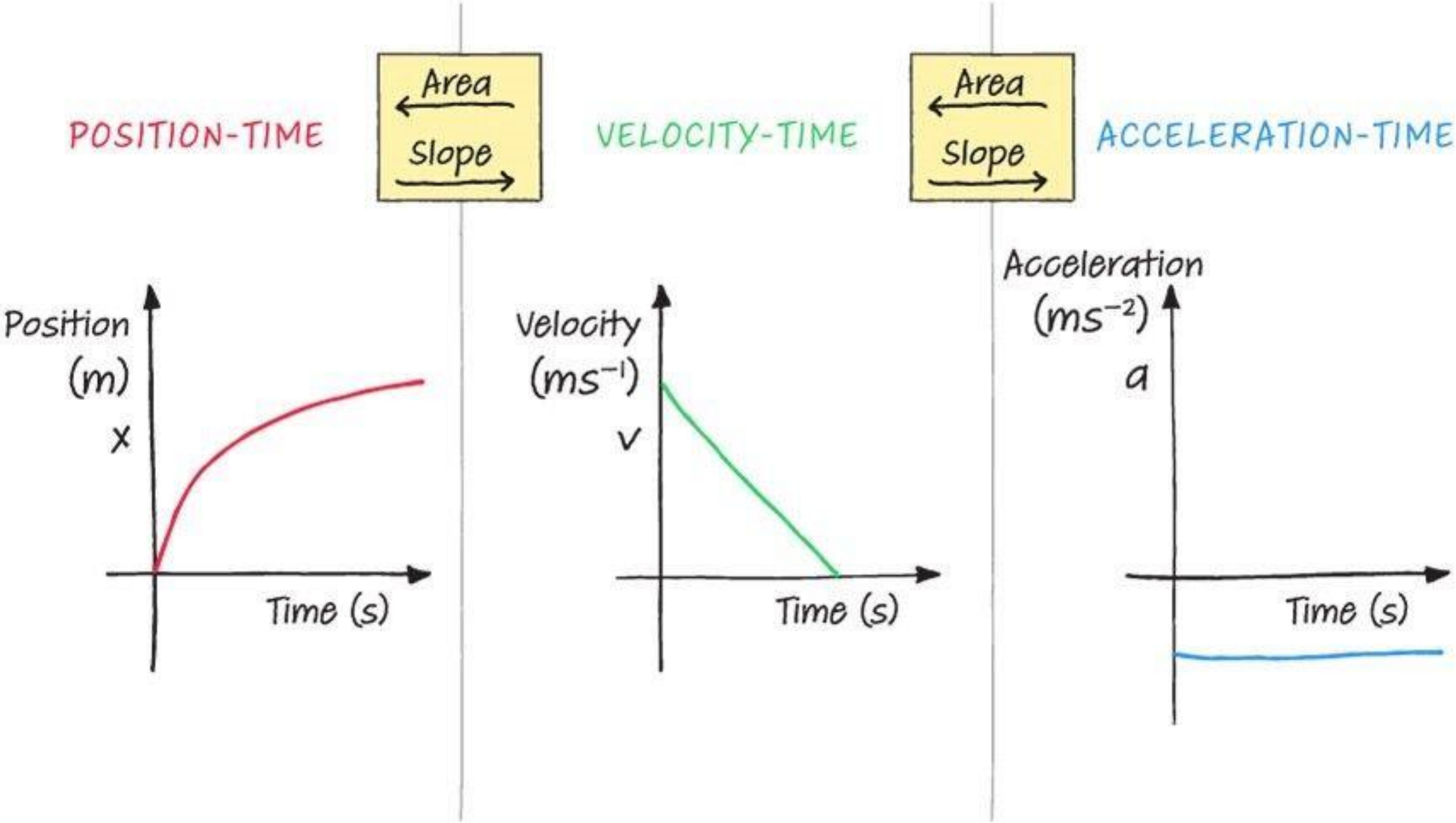
$$a = \dot{v} = \ddot{x}$$

IMPORTANT POINTS

- Dynamics is concerned with bodies that have accelerated motion
- Kinematics is a study of the geometry of the motion
- Kinetics is a study of the forces that cause the motion
- Rectilinear kinematics refers to straight-line motion

- Speed refers to the magnitude of velocity
- Average speed is the total distance traveled divided by the total time; this is different from the average velocity which is the displacement divided by the time

- The acceleration, $a = dv/dt$, is negative when the particle is slowing down or decelerating.
- A particle can have an acceleration and yet have zero velocity


POSITION-TIME
Area
Slope
VELOCITY-TIME
Area
Slope
ACCELERATION-TIME
Position (m)
x
Time (s)
Velocity (ms⁻¹)
v
Time (s)
Acceleration (ms⁻²)
a
Time (s)

Example 1

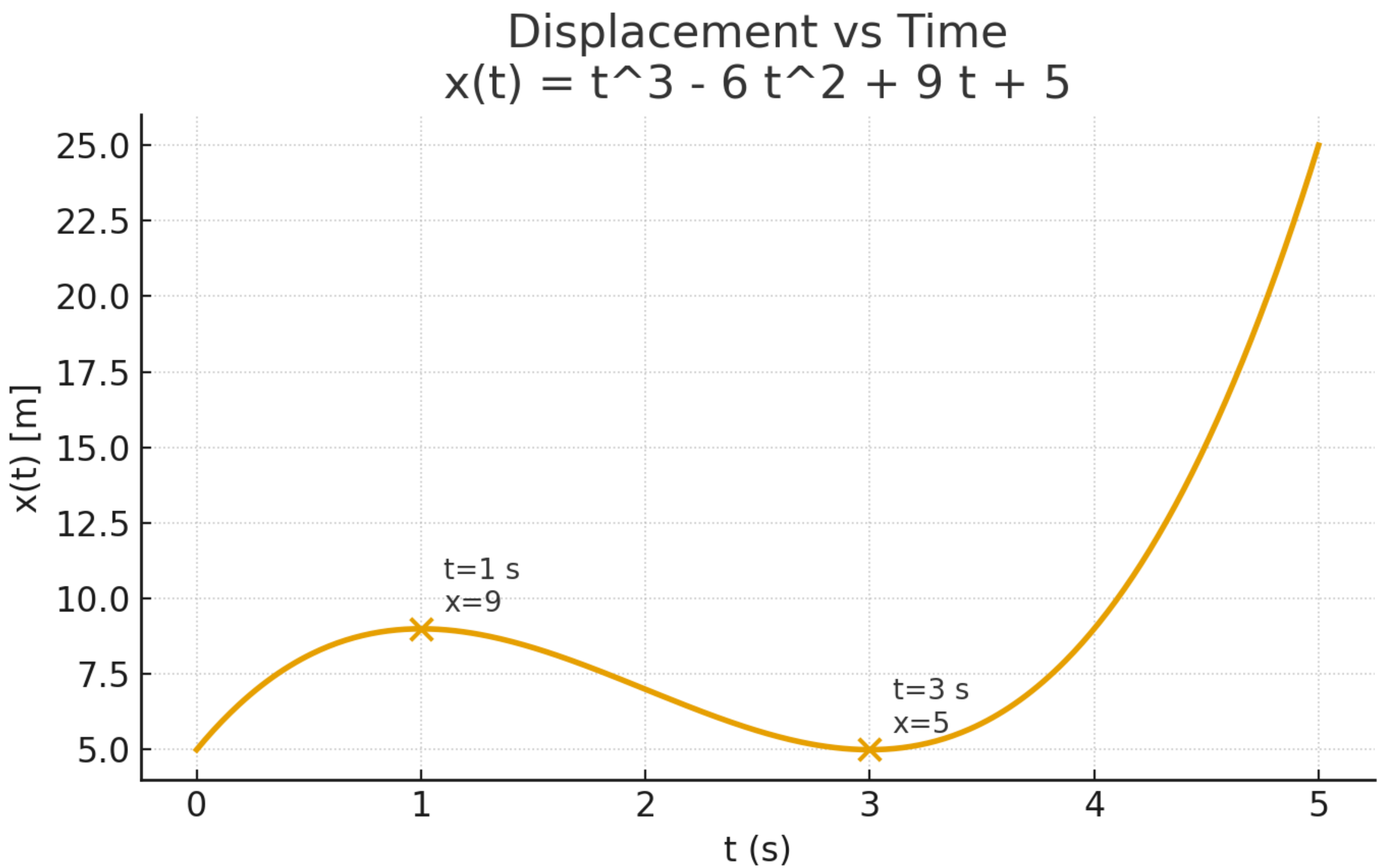
- A particle moves along a line with position:

  $$x(t) = t^3 - 6t^2 + 9t + 5 \quad (m, t \text{ in } s)$$

- Tasks:

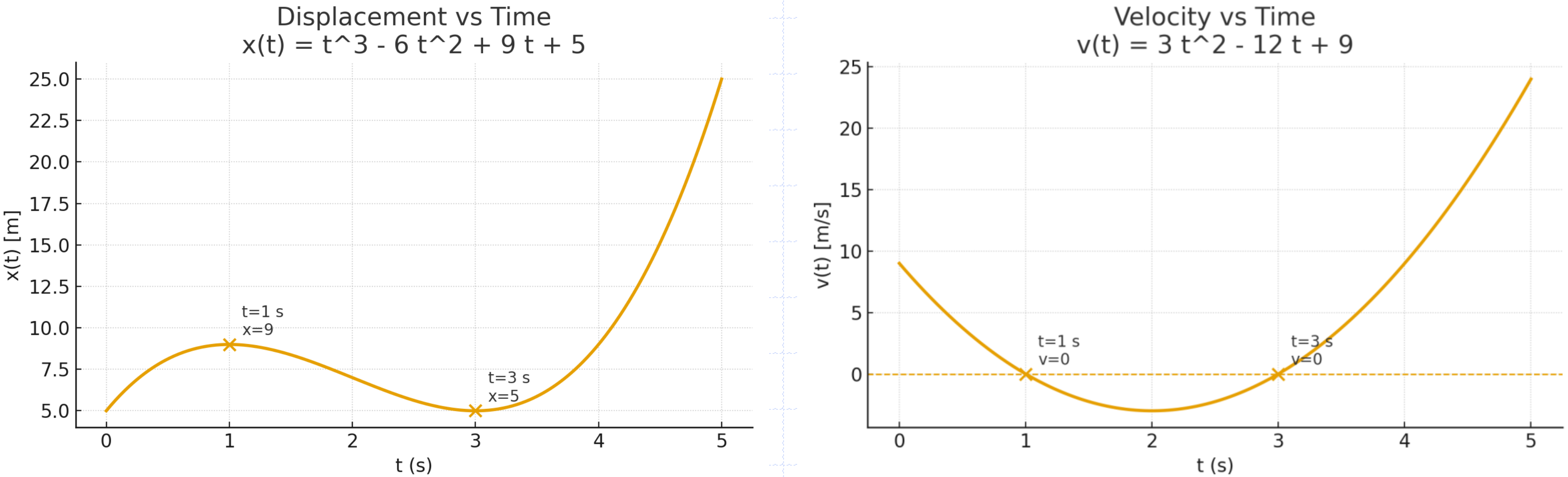
a) Find the total distance traveled from $t=0$ to $t=5$ s.

b) Determine acceleration at the critical times when velocity is zero.


Displacement vs Time
x(t) = t^3 - 6 t^2 + 9 t + 5
x(t) [m]
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
t=1 s
x=9
t=3 s
x=5
0
1
2
3
4
5
t (s)

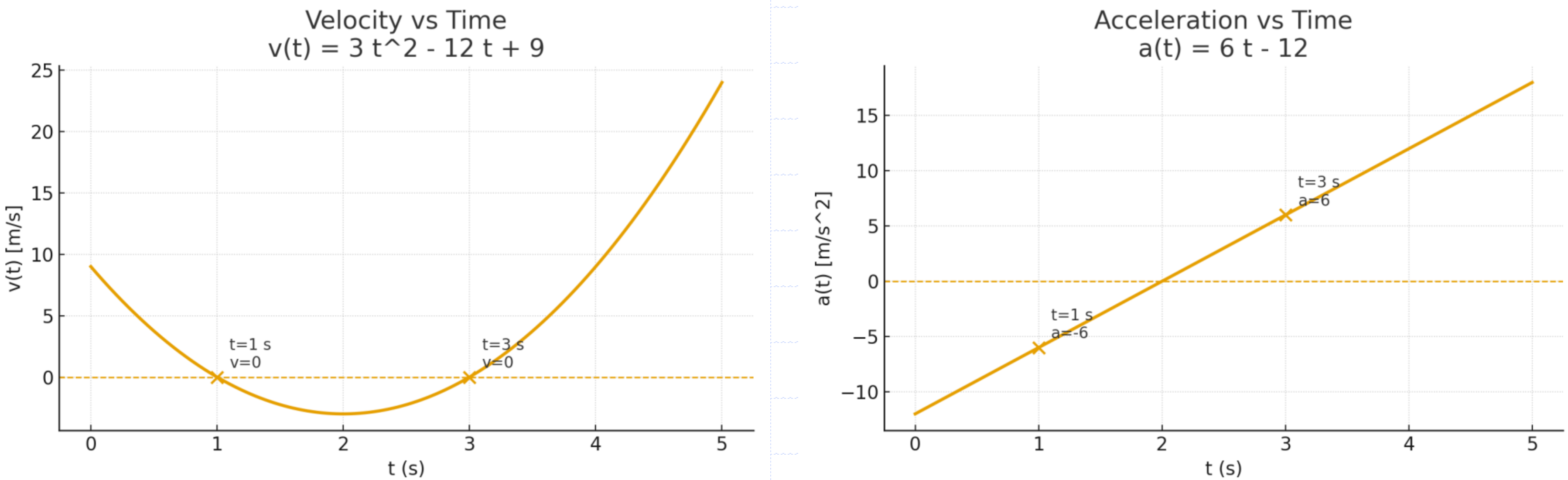
Step 1. Velocity

- $v(t) = dx/dt = 3t^2 - 12t + 9 = 3(t-1)(t-3)$
- Turning points where $v(t)=0$:
  $t = 1$ s, $t = 3$ s



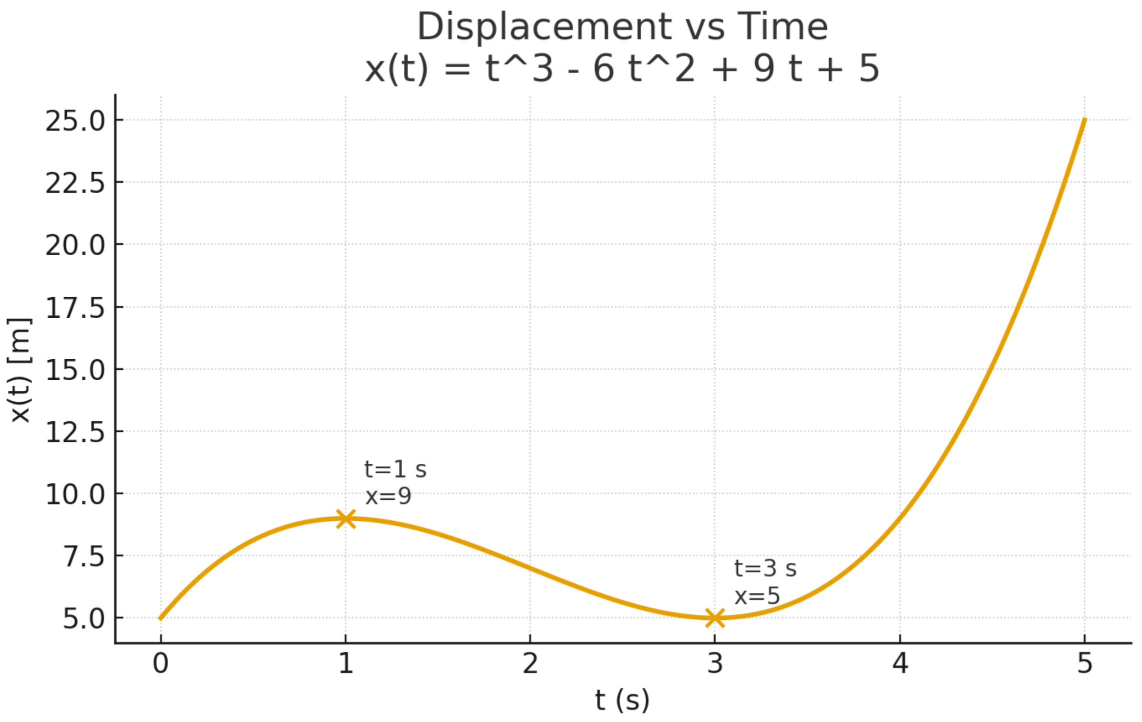
Step 2. Acceleration

- $a(t) = dv/dt = 6t - 12$
- $a(1) = -6$ m/s² (negative, local maximum)
- $a(3) = +6$ m/s² (positive, local minimum)



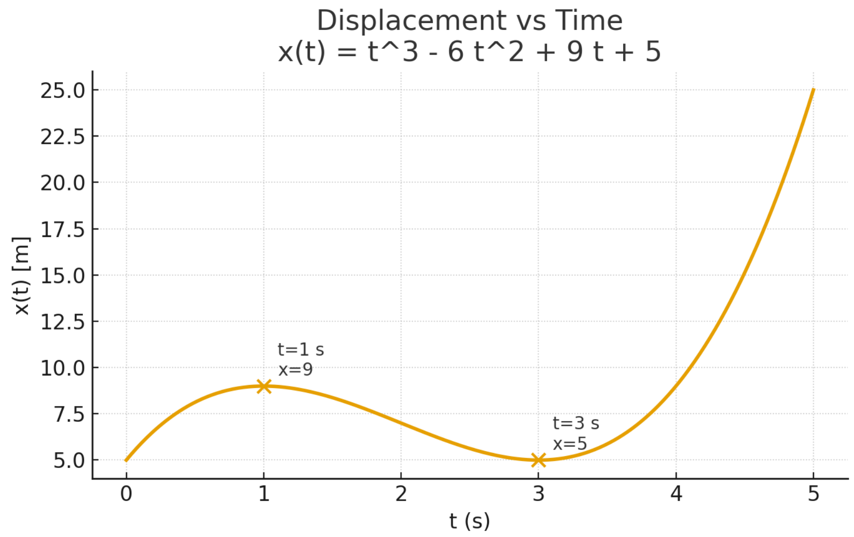
Step 3. Positions

- Evaluate position at key times:
- $x(0) = 5$ m
- $x(1) = 9$ m
- $x(3) = 5$ m
- $x(5) = 25$ m



Step 4. Distances

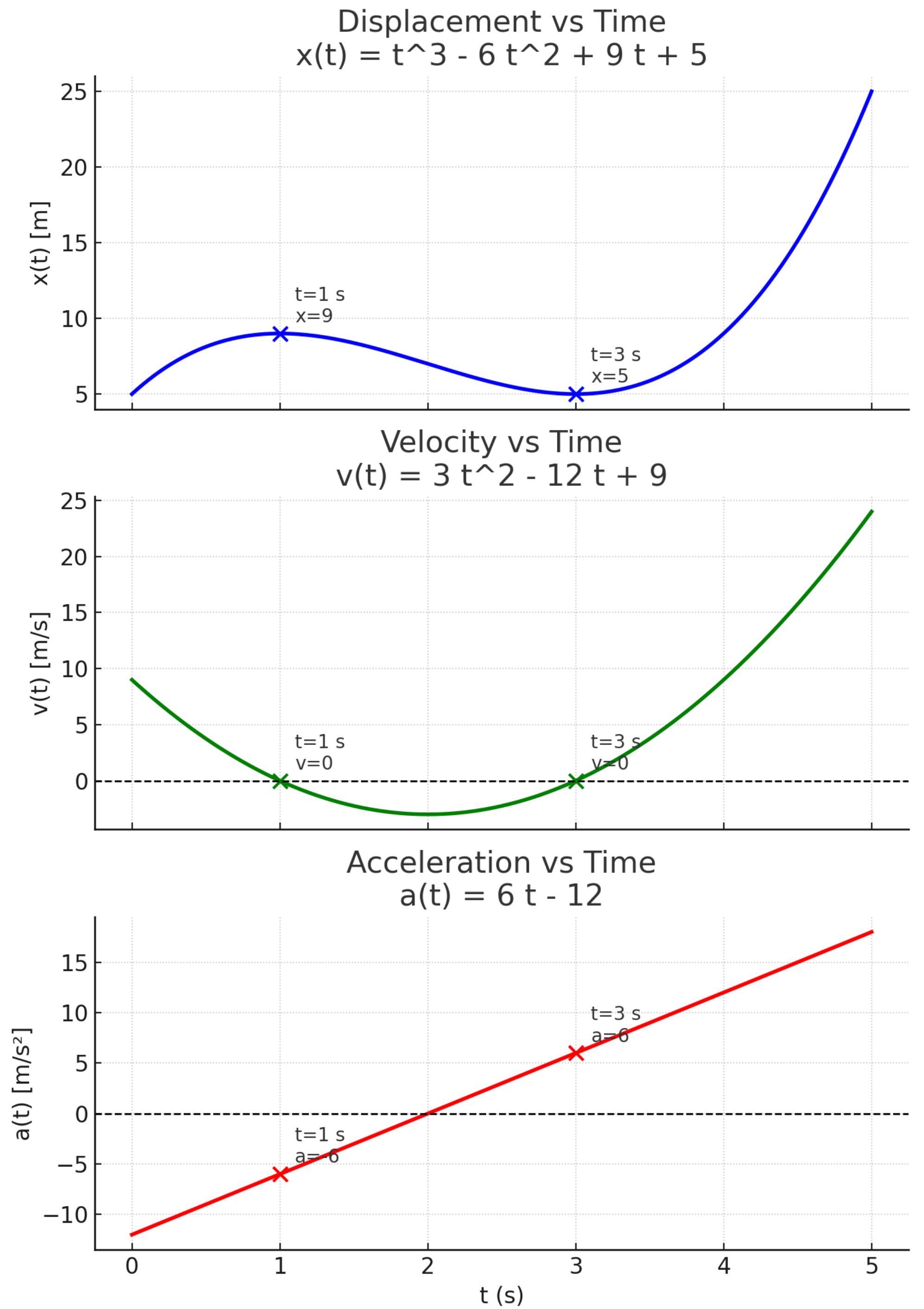
- Distance traveled in each interval:
- $0 \rightarrow 1$ s: $|9 - 5| = 4$ m
- $1 \rightarrow 3$ s: $|5 - 9| = 4$ m
- $3 \rightarrow 5$ s: $|25 - 5| = 20$ m
- **Total Distance = 28 m (Answer)**



Notes

- Displacement vs Distance:
displacement = 20 m,
distance = 28 m
- At t=1 s and t=3 s: velocity = 0,
acceleration changes sign

Displacement, Velocity, and Acceleration



Determination of Particle Motions

- Typically acc. Known (measured, via $\bar{F} = m\bar{a}$)
- Vel. & pos. found via integration

Common Cases:

① $a = a(t)$

② $a = a(x)$

③ $a = a(v)$

Integrate Directly
(Show by Example)

Special Cases:

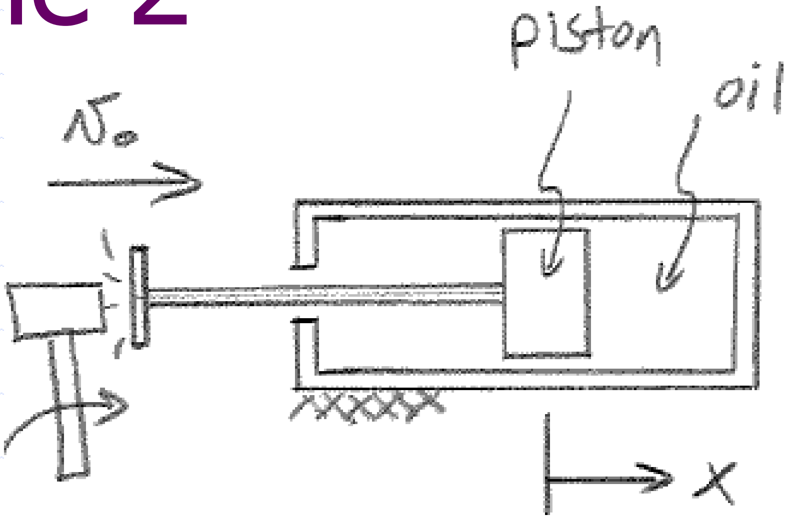
④ $a = 0$

⑤ $a = \text{const}$

lead to

Kinematic Equations

Example 2



@ $t = 0$, $x = 0$, $v = v_0 > 0$

$a(v) = -Kv$ (Case ③)

$K > 0$ is constant

Express (A) $v = v(t)$

(B) $x = x(t)$

(C) $v = v(x)$

PART A: Find $v = v(t)$

$$a(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a(v)} = \frac{dv}{-Kv}$$

$$\Rightarrow \int_0^t dt = \frac{-1}{K} \int_{v_0}^{v} \frac{dv}{v}$$

$$\Rightarrow \quad t - 0 = \frac{-1}{K}\left(\ln|v| - \ln|v_0|\right)$$

$$= \frac{-1}{K}\ln\left(\frac{v}{v_0}\right)$$

$$\Rightarrow \quad \ln\left(\frac{v}{v_0}\right) = -Kt$$

$$\Rightarrow \quad \frac{v}{v_0} = e^{-Kt}$$

$$\therefore \quad v(t) = v_0 e^{-Kt}$$

PART B: Find $X = X(t)$

$$v(t) = \frac{dX}{dt}$$

Known from (A)

$$\Rightarrow \quad dX = v(t)\,dt = v_0 e^{-Kt}\,dt$$

$$\Rightarrow \quad \int_{X_0}^{X} dX = v_0 \int_0^t e^{-Kt}\,dt$$

$$\Rightarrow \quad x - \cancelto{0}{x_0} = v_0 \cdot \frac{-1}{k} e^{-kt} \Big|_0^t$$

$$= \frac{-v_0}{k}\left(e^{-kt} - 1\right)$$

$$\therefore \quad x(t) = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

PART C: Find $v = v(x)$

$$a(v) = \frac{dv}{dt} = \frac{dv}{dx}\frac{\cancel{dx}}{\cancel{dt}} = \frac{dv}{dx} v$$

Given

$$\Rightarrow \quad dx = \frac{v\,dv}{a(v)} = \frac{v\,dv}{-Kv} = \frac{-1}{K}\,dv$$

$$\Rightarrow \quad \int_{x_0}^{x} dx = \frac{-1}{k} \int_{v_0}^{v} dv$$

$$\Rightarrow \quad x - \cancelto{0}{x_0} = \frac{-1}{k} (v - v_0)$$

$$\Rightarrow \quad v - v_0 = -kx$$

$$\therefore \quad v(x) = v_0 - kx$$

Uniform Rectilinear Motion

Case ④:

$$a = \frac{dv}{dt} = 0 \quad \Rightarrow \quad v = \text{const.}$$

Then $v = \dfrac{dx}{dt} \Rightarrow dx = v\,dt$

$$\Rightarrow x - x_0 = v(t - 0)$$

$$\boxed{x = x_0 + vt}$$

pos. - time

Uniformly Accelerated Rectilinear Motion

Case 5: $\quad a = \text{const.}$

Then

$$a = \frac{dv}{dt} \quad \Rightarrow \quad dv = a\,dt$$

$$\Rightarrow \quad v - v_0 = a(t-0)$$

$$\boxed{v = v_0 + at}$$

vel. - time

Now consider

$$v = \frac{dx}{dt} \Rightarrow dx = \underbrace{v(t)}_{v_0 + at}\, dt$$

$$\Rightarrow dx = (v_0 + at)\, dt$$

$$\Rightarrow x - x_0 = v_0 t + a\frac{t^2}{2}$$

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2} a t^2}$$ pos - time

Another relationship - invoke chain rule

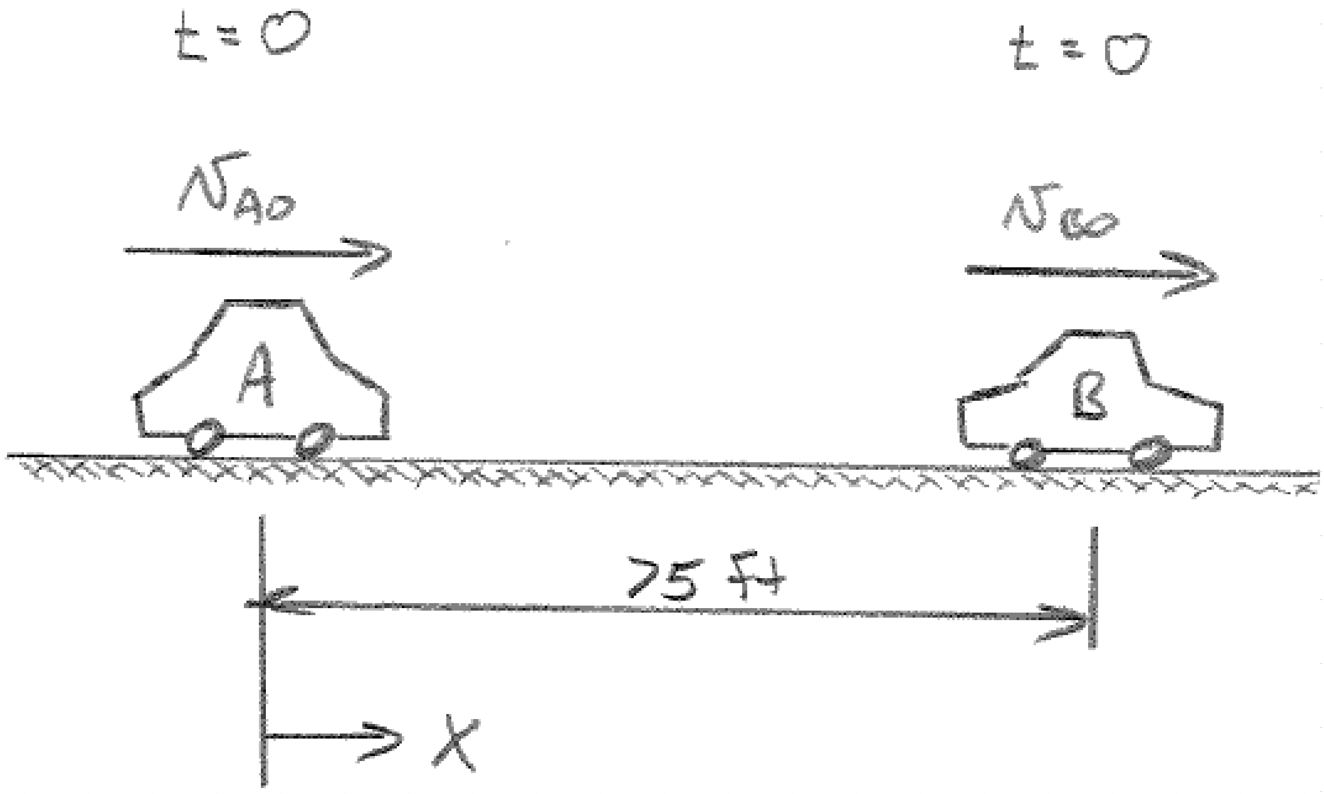
$$a = \frac{dv}{dt} = v\frac{dv}{dx} \Rightarrow v\,dv = a\,dx$$

$$\Rightarrow \frac{v^2}{2} - \frac{v_0^2}{2} = a(x - x_0)$$

$$\boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

vel. - pos.

Example 3



Data:

$$v_A(t=0) = v_{A0} = 24 \text{ mph}, \qquad a_A = 1.8 \text{ ft/s}^2$$

$$v_B(t=0) = v_{B0} = 36 \text{ mph}, \qquad a_B = -1.2 \text{ ft/s}^2$$

Find: (A) When & where A overtakes B

(B) corresponding speed of each car

PART A: Use Kinematic eqns for $a = \text{const.}$

$$v = v_0 + at \quad \text{——— (1)}$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 \quad \text{— (2)} \leftarrow \text{pos. - time}$$

$$v^2 - v_0^2 = 2a(x - x_0) \quad \text{— (3)}$$

Car A: $x_A(t) = x_{A0} + v_{A0} t + \frac{1}{2} a_A t^2$

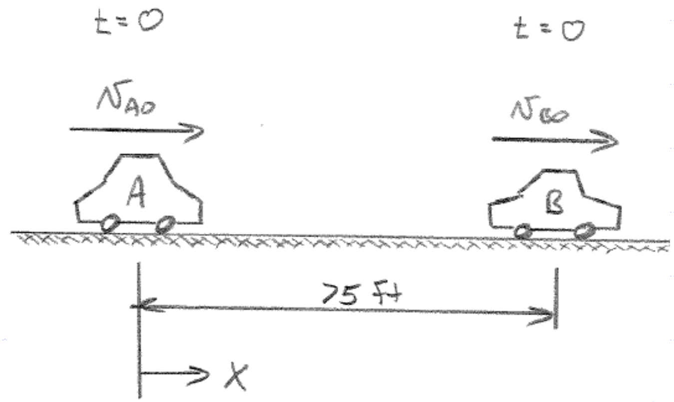
Car B: $x_B(t) = x_{B0} + v_{B0} t + \frac{1}{2} a_B t^2$ — (4)

Here, $x_{A0} = 0$, $x_{B0} = 75$ ft

$$v_{A0} = \left(24\,\frac{\text{mi}}{\text{hr}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right)\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right) = 35.2\ \text{ft/s}$$

$$v_{B0} = 36\ \text{mph} = 52.8\ \text{ft/s}$$

a_A, a_B given



A overtakes B when $x_A(t) = x_B(t)$

$$\Rightarrow\quad x_{A0} + v_{A0}t + \tfrac{1}{2}a_A t^2 = x_{B0} + v_{B0}t + \tfrac{1}{2}a_B t^2$$

$$\Rightarrow\quad (a_A - a_B)t^2 + 2(v_{A0} - v_{B0})t + 2(x_{A0} - x_{B0}) = 0$$

$$\Rightarrow\quad (3\ \text{ft/s}^2)\,t^2 + (-35.2\ \text{ft/s})\,t + (-150\ \text{ft}) = 0$$

of form $at^2 + bt + c = 0$

so $$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad t_1 = 15.05\ s$$

$$t_2 = -3.32\ s \leftarrow \underline{\text{nonsense}}$$

$$\therefore \quad t = t_1 = 15.1\ s$$

For positions, plug $t = t_1$ into ④.

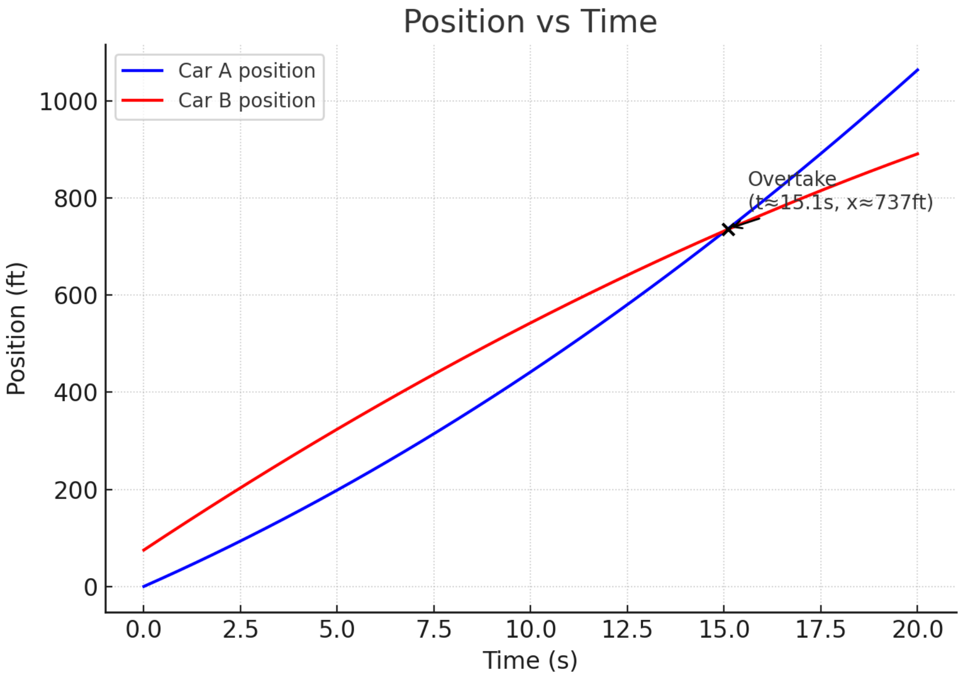
$$X_p = 737\ \text{ft}$$

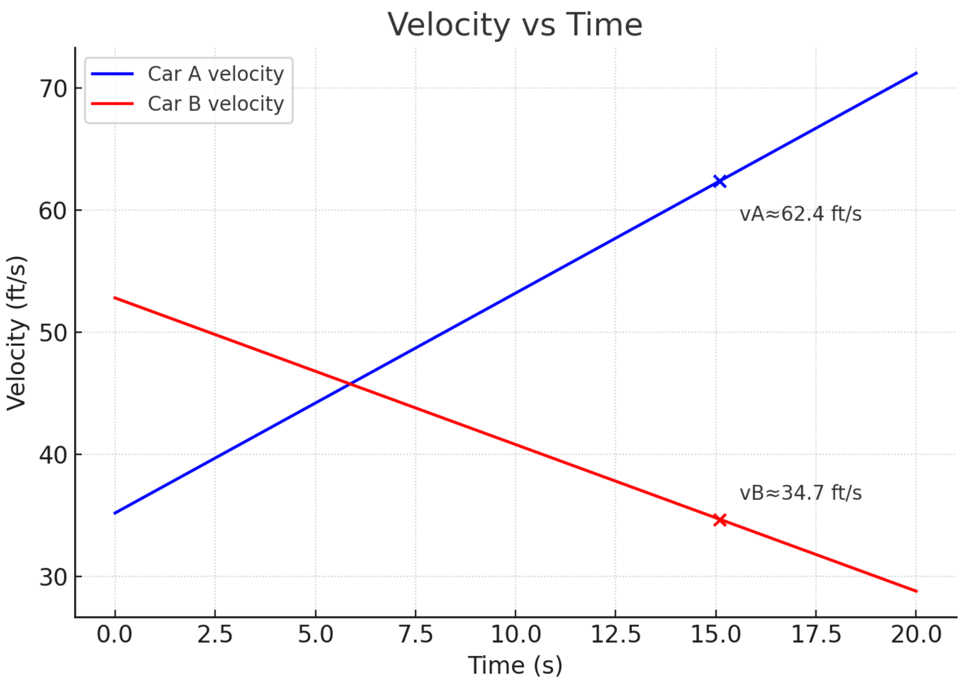
PART B: Find $v_A(t_1)$, $v_B(t_1)$

Easy! Use ①.

$$v_A(t_1) = v_{A0} + a_A t_1 = 62.3 \text{ ft/s} = 42.5 \text{ mph}$$

$$v_B(t_1) = v_{B0} + a_B t_1 = 34.7 \text{ ft/s} = 23.7 \text{ mph}$$


Position vs Time
Car A position
Car B position
Overtake
(t≈15.1s, x≈737ft)
Position (ft)
Time (s)
1000
800
600
400
200
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

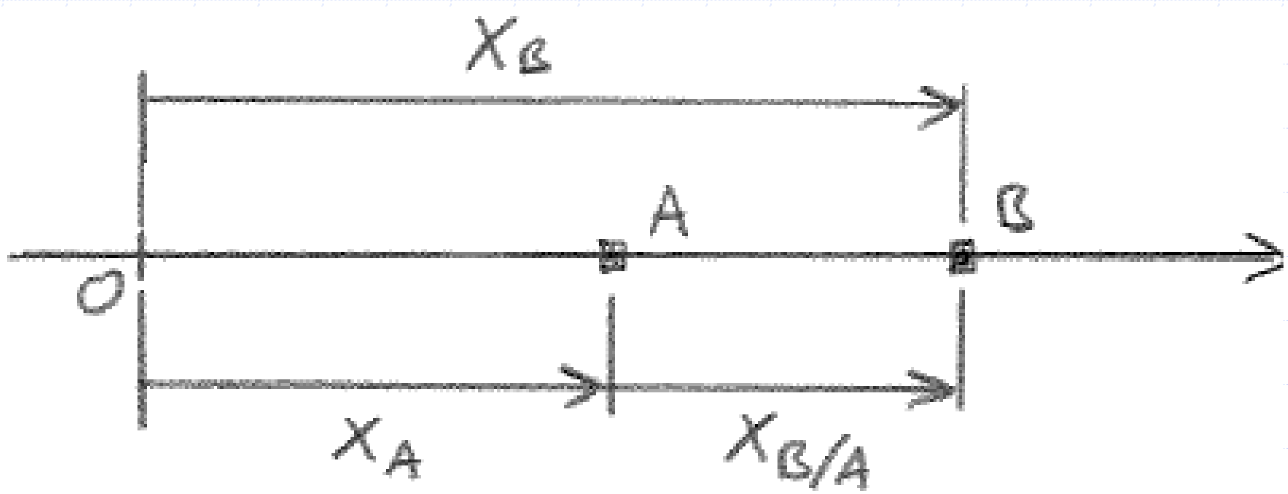

Velocity vs Time
Car A velocity
Car B velocity
vA≈62.4 ft/s
vB≈34.7 ft/s
Velocity (ft/s)
Time (s)
70
60
50
40
30
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Relative Rectilinear Motion

→ Relative motions are often more important than absolute motions.

We define $x_{B/A} = x_B - x_A$

B/A reads "B with respect to A"



$\frac{d}{dt}$ to obtain

$$v_{B/A} = v_B - v_A$$

$$a_{B/A} = a_B - a_A$$

Rectilinear Motion

Velocity and Acceleration

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

Cases 1,2,3: $a = a(t),\ a = a(x),\ a = a(v)$

Integrate directly.

Rectilinear Motion

Velocity and Acceleration

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

Cases 4,5: $a = 0\ ,\ a = \text{constant}$

$$x = x_o + vt$$

$$v = v_o + at$$

$$x - x_o = v_o t + \tfrac{1}{2} a t^2$$

$$v^2 - v_0^2 = 2a(x - x_o)$$