Chapter 2 – Part 2
Moment – Couple – Resultant

STATICS, AGE-1330

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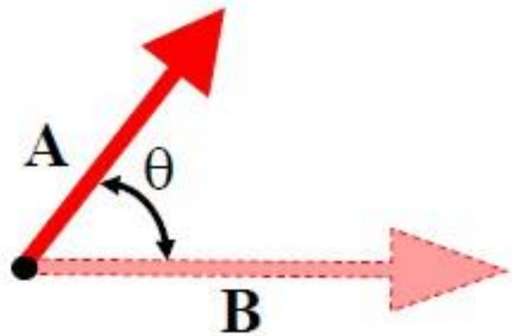
1. Dot and Vector Product

Product of 2 Vectors: Dot Product

- **Dot Product** *(Scalar product)*
  - $\mathbf{A}.\mathbf{B} = AB\cos\theta$



  - **Applications**
    - Determination of the angle between two vectors

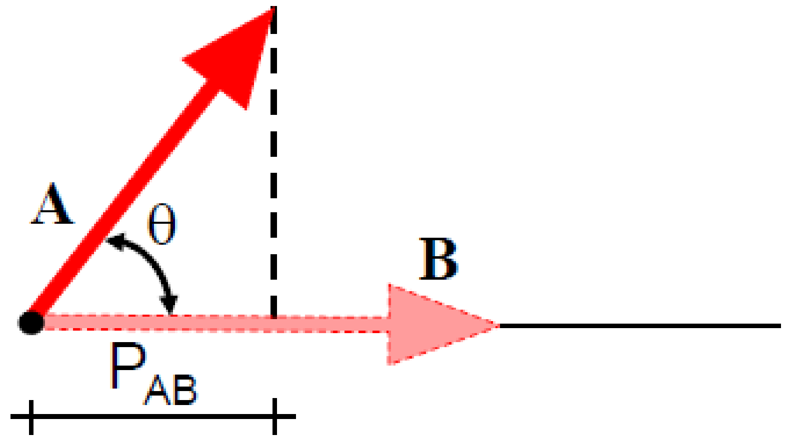
$$\mathbf{A}.\mathbf{B} = (A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k}).(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k}) = A_xB_x+A_yB_y+A_zB_z$$

$$\mathbf{A}.\mathbf{B} = AB\cos\theta$$

Obtain θ

Product of 2 Vectors: Dot Product

- **Applications**
  - Determination of the projection of a vector on a given axis



$\mathbf{A}.\mathbf{B} = A_x B_x + A_y B_y + A_z B_z$

$P_{AB} = A\cos\theta = (\mathbf{A}.\mathbf{B})/B$

Cross Product

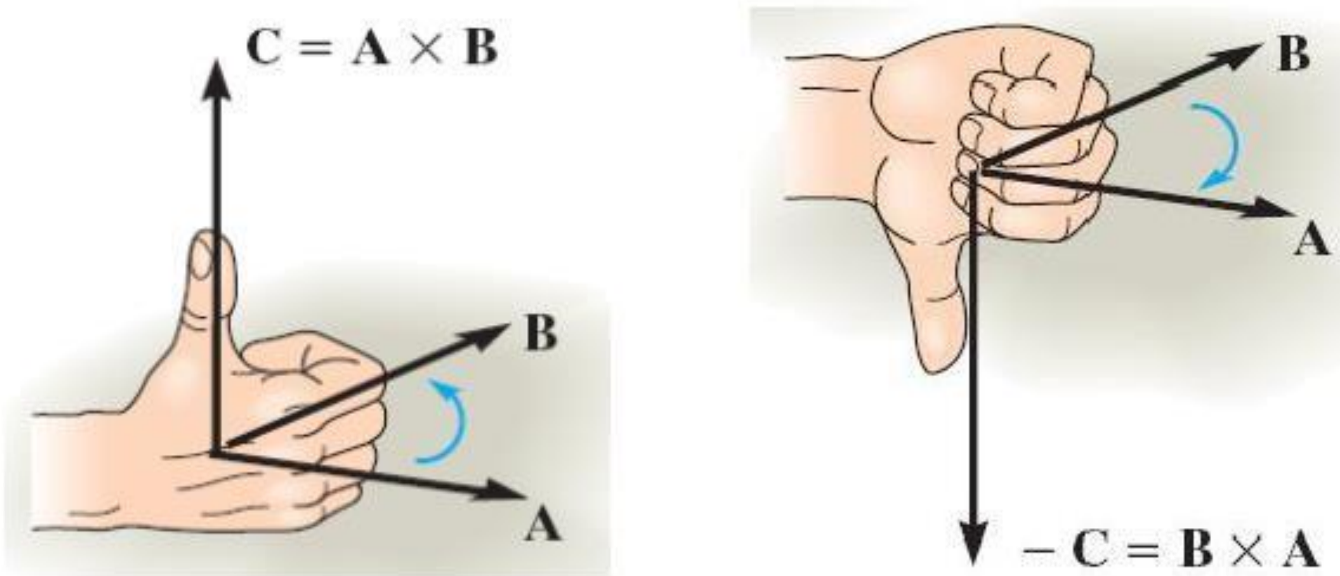
Another method of vector multiplication

$$C = A \times B$$

Read as **C** equals **A** cross **B**

Product of 2 Vectors: Cross Product

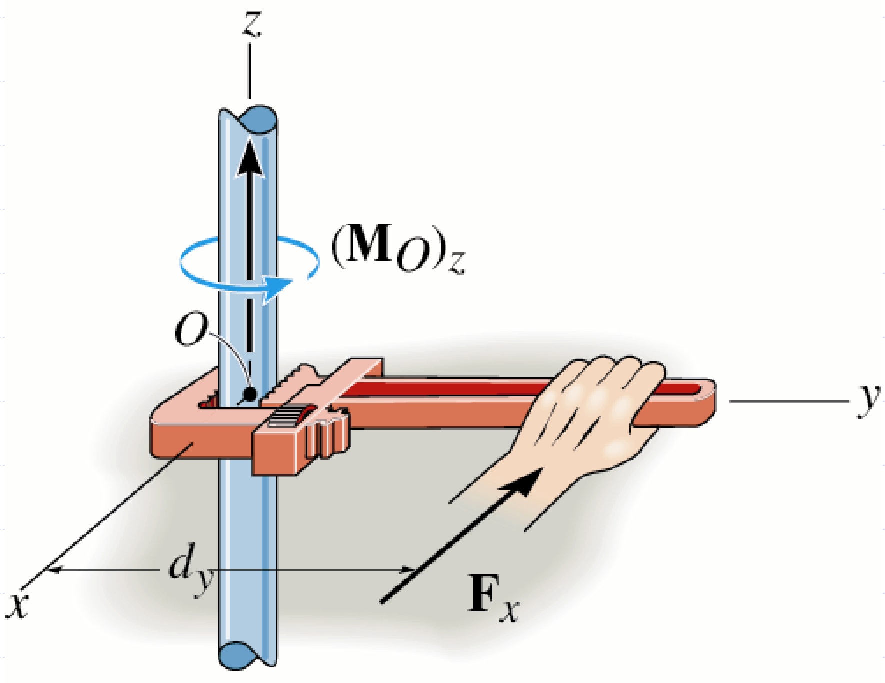
- **Cross Product** *(Vector Product)*
  - $\mathbf{C} = \mathbf{A} \times \mathbf{B}$
    - $C = AB \sin\theta$



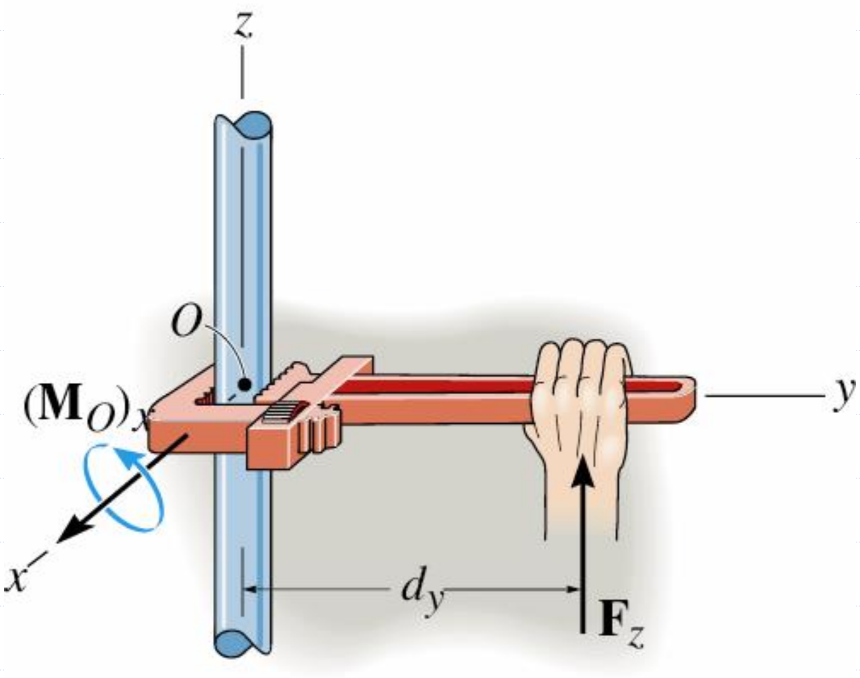
2. Moment of a Force

Moment of a Force

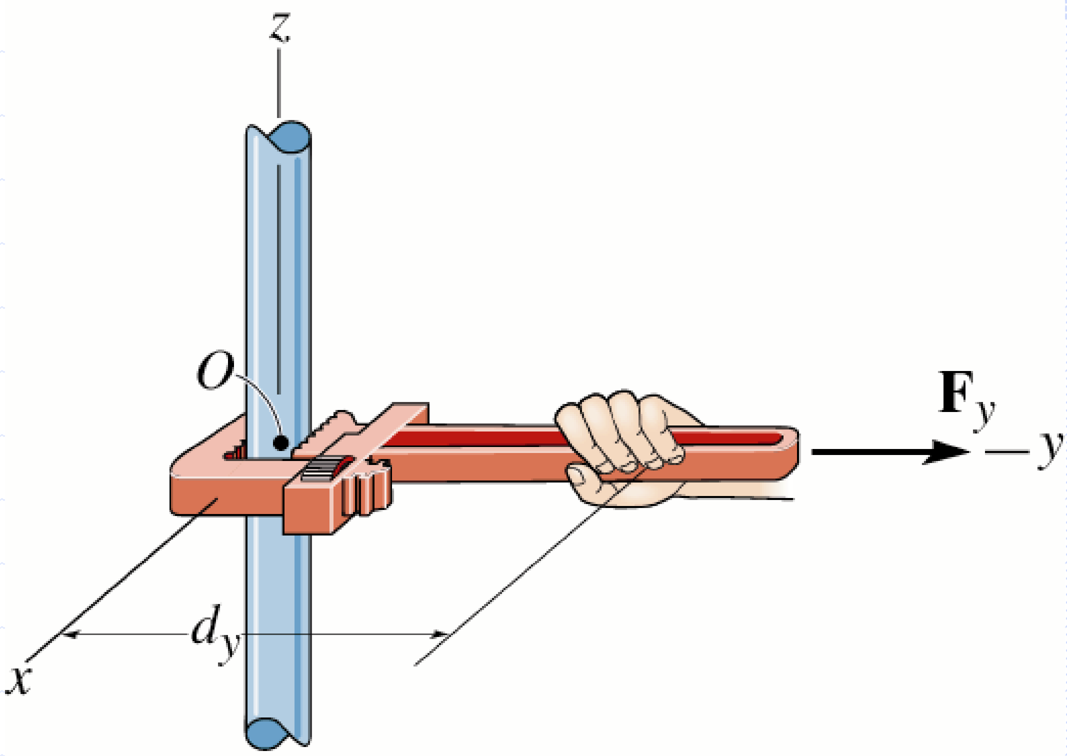
The moment of a force about a point or an axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis



F_x - horizontal force
d_y - distance from point O to force
$\mathbf{M_o}$ - moment of force about point O
$(\mathbf{M_o})_z$ - moment of force about axis z



F_z - horizontal force
d_y - distance from point O to force
$\mathbf{M_o}$ - moment of force about point O
$(\mathbf{M_o})_x$ - moment of force about axis z

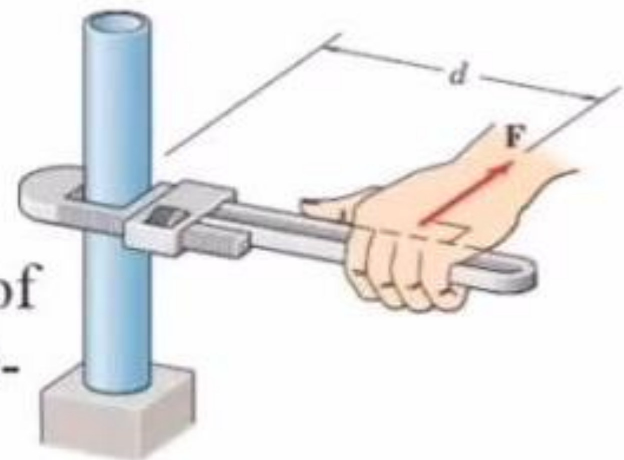


No Moment

2- Moment

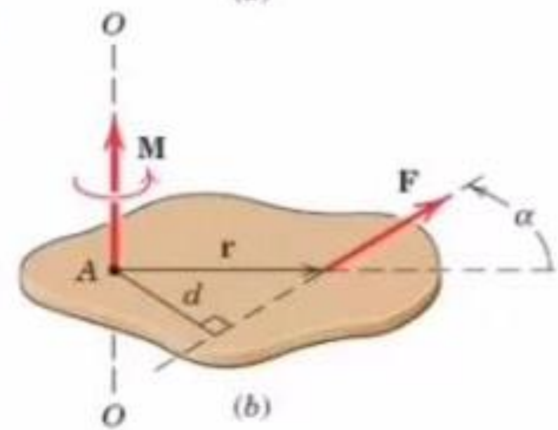
A force can rotate a body about an axis

The <u>magnitude</u> of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to ***the magnitude of the force and to the moment arm d.***

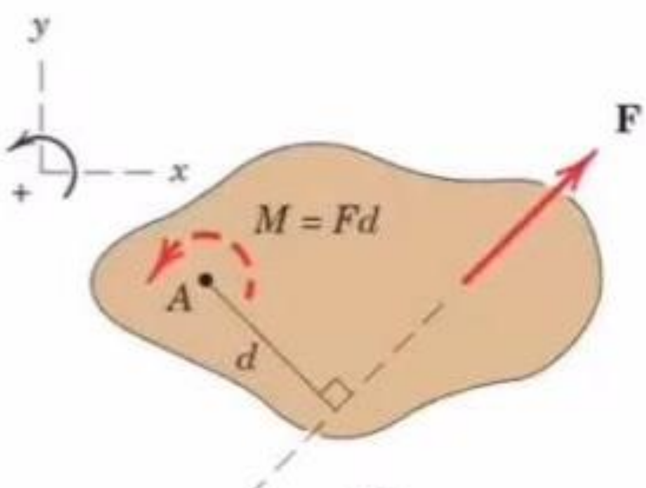


(a)

$$M = Fd \quad \text{(N.m)}$$

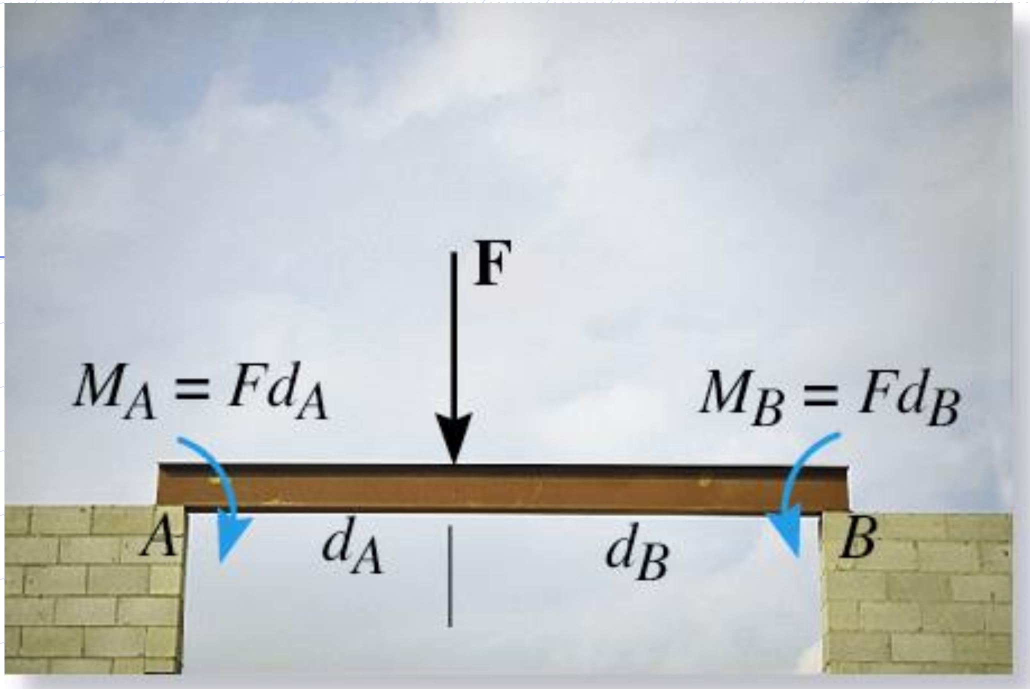


(b)



(d)

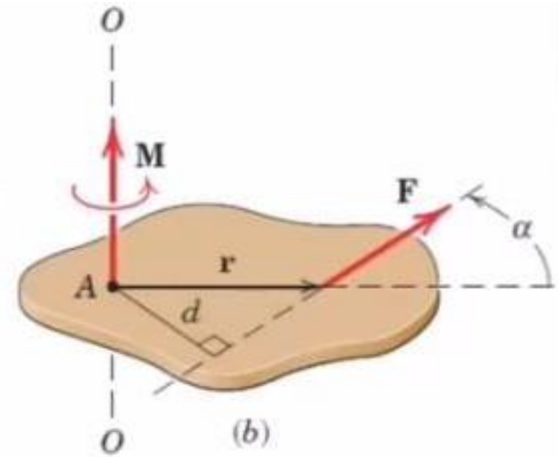
NOTE: The moment is a vector **M** perpendicular to the plane of the body



Do not actually need rotation to have a moment. Moment is the tendency to cause rotation

The Cross Product

A vector approach for moment calculations. The moment of **F** about point A of may be represented by the cross-product expression.



$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

where **r** is a **position vector** which runs from the moment reference point *A* to any point on the line of action of **F**.

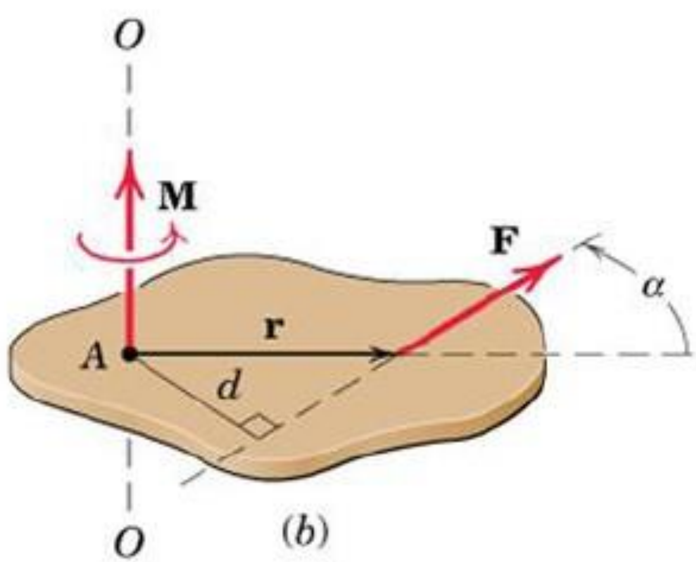
The magnitude

$$M = Fr \sin \alpha = Fd$$

NOTE: $\mathbf{M} = \mathbf{r} \times \mathbf{F}$ $\quad$ $\mathbf{F} \times \mathbf{r} = -\mathbf{M}$

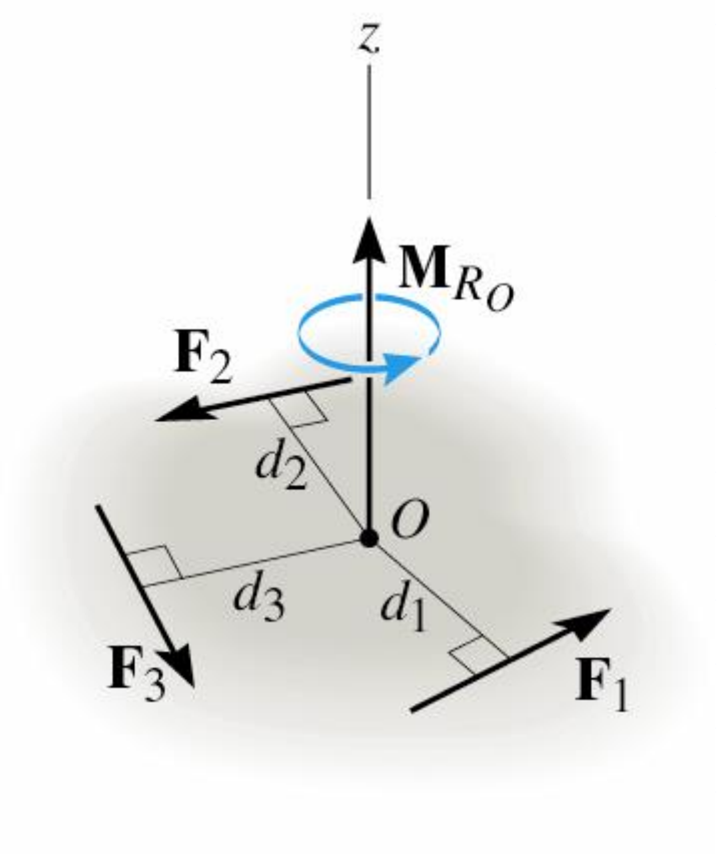
Moment of a Force

- Characteristic
  - Moment arm ($d = r\,sin\alpha$) does not depend on the particular point on the line of action of **F** to which the vector **r** is directed
    - **Sliding vector**
      - Line of action same as the moment axis

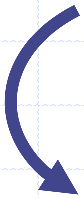


(b)

Resultant Moment of a System of Coplanar Forces



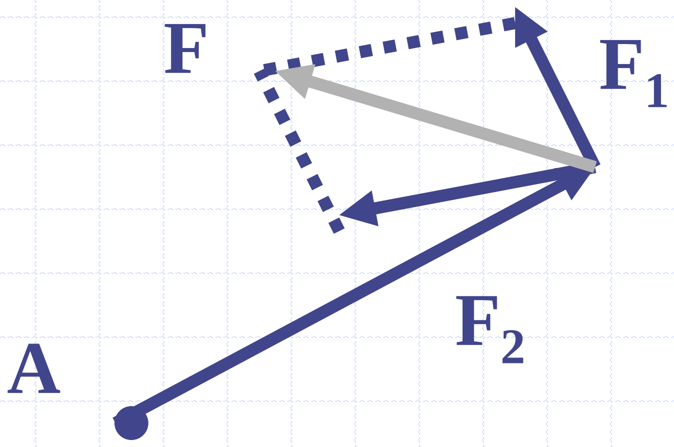
$$+M_{R_O} = \sum Fd$$



Counterclockwise is positive by scalar sign convention

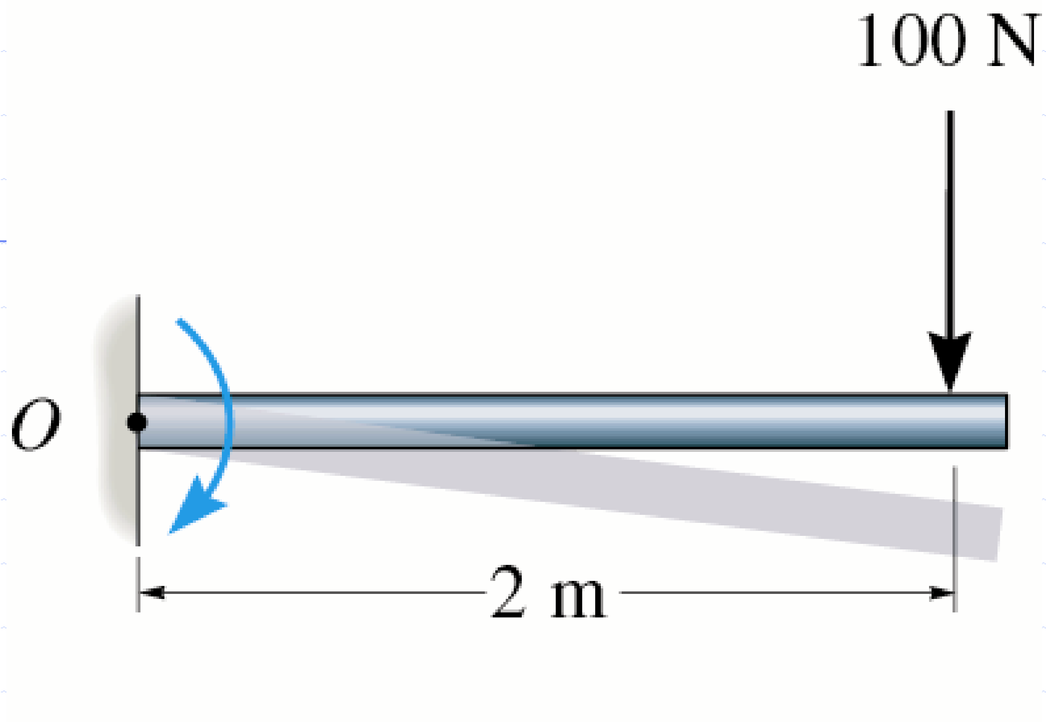
Principle of Moments

The moment of a force about a point is equal to the sum of the moments of the force's components about the point.

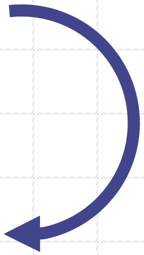


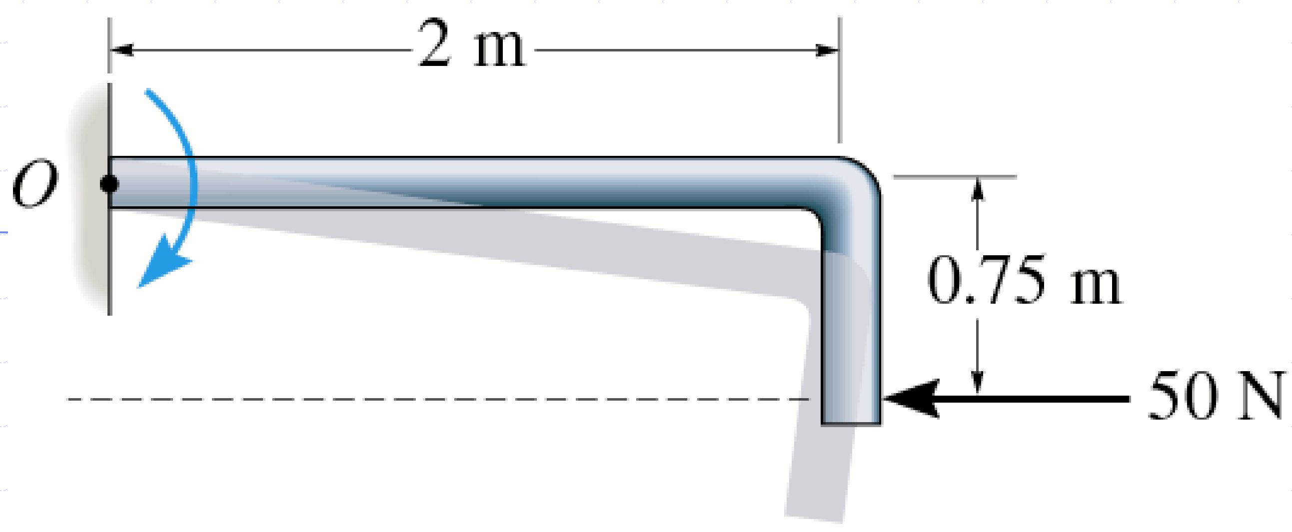
Example 1

For each case, find the moment of the force about the point O

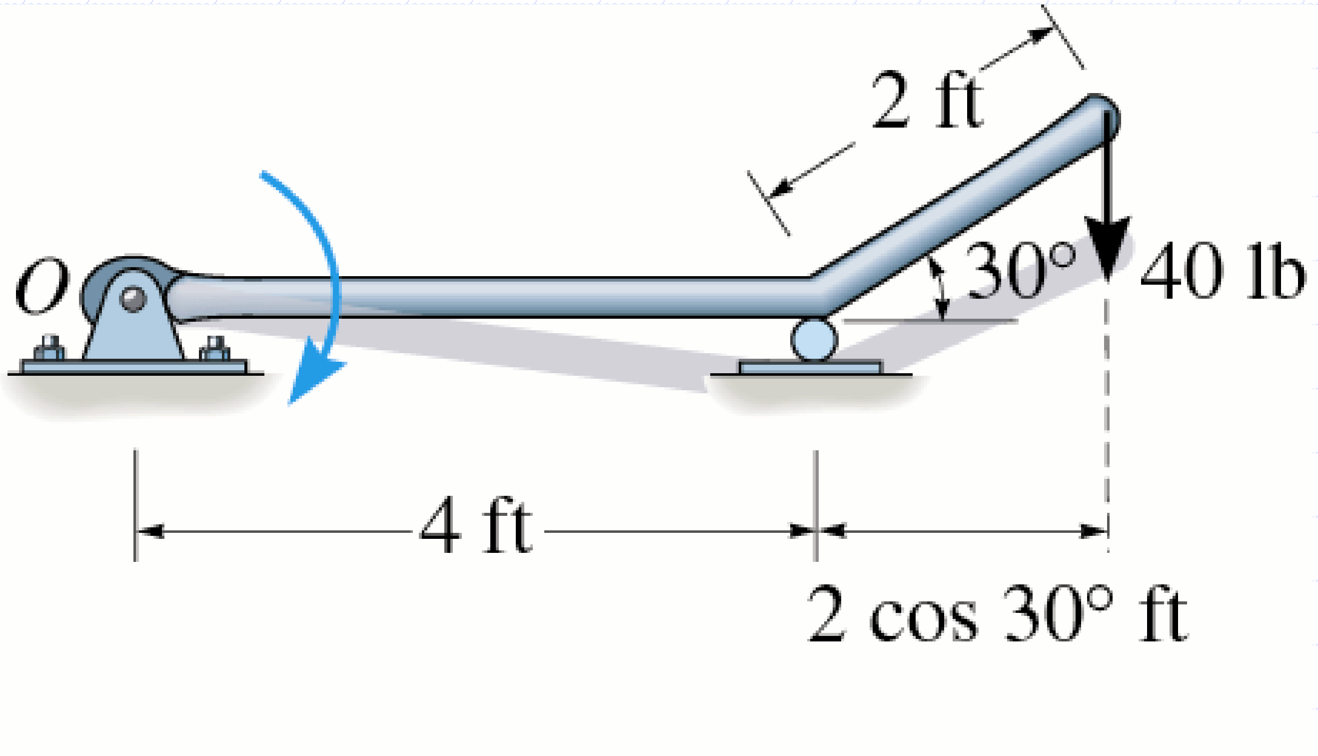


$$M_O = (100\,N)(2\,m) = 200\,N \cdot m$$

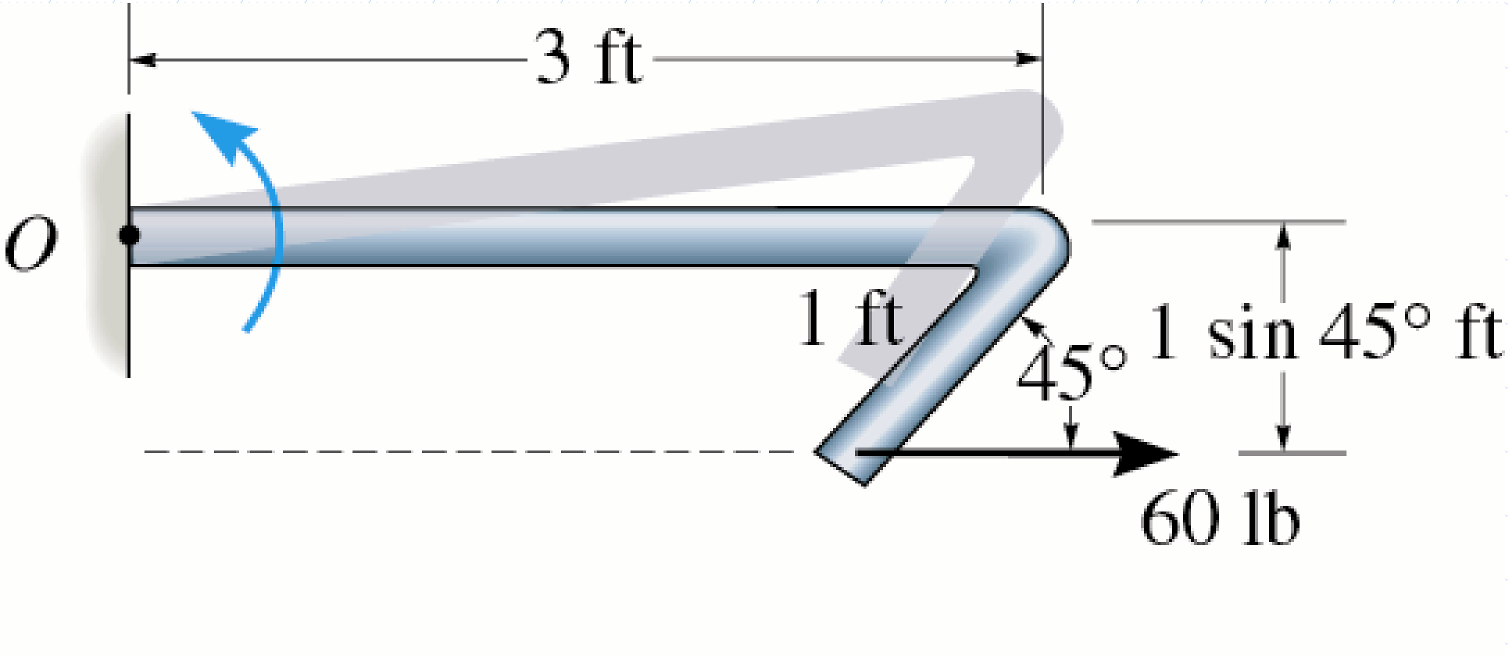




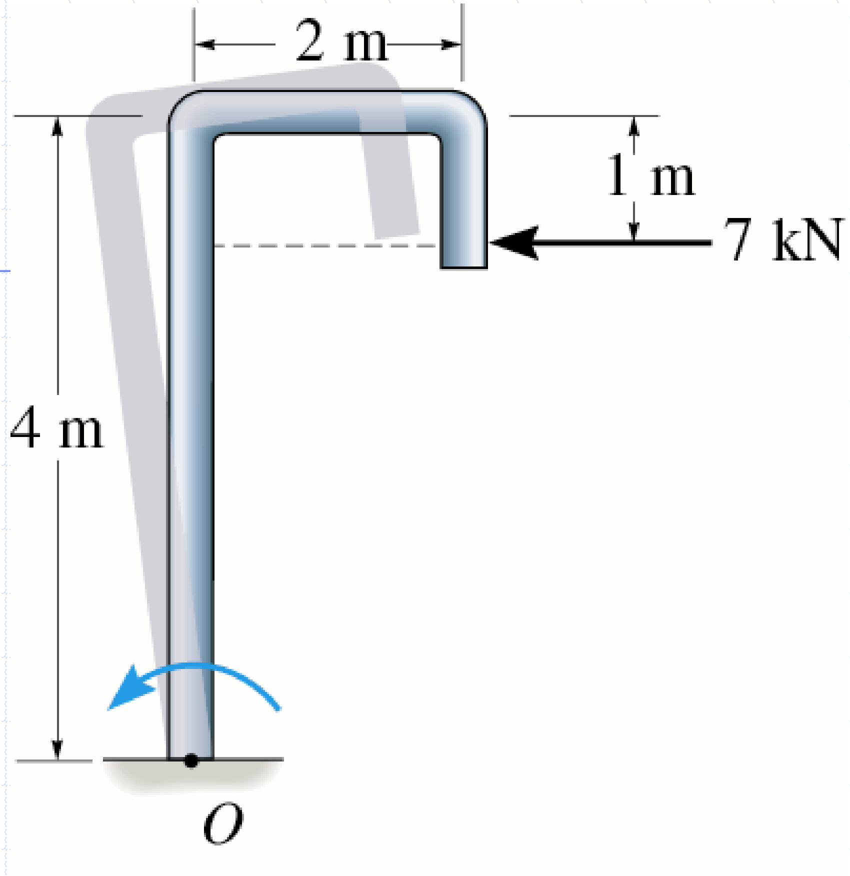
$$M_O = (50N)(0.75m) = 37.5\ N \cdot m$$



$$M_O = (40\,\text{lb})\left(4 + 2\cos 30^{\circ}\ \text{ft}\right) = 229\,\text{lb}\cdot\text{ft}$$



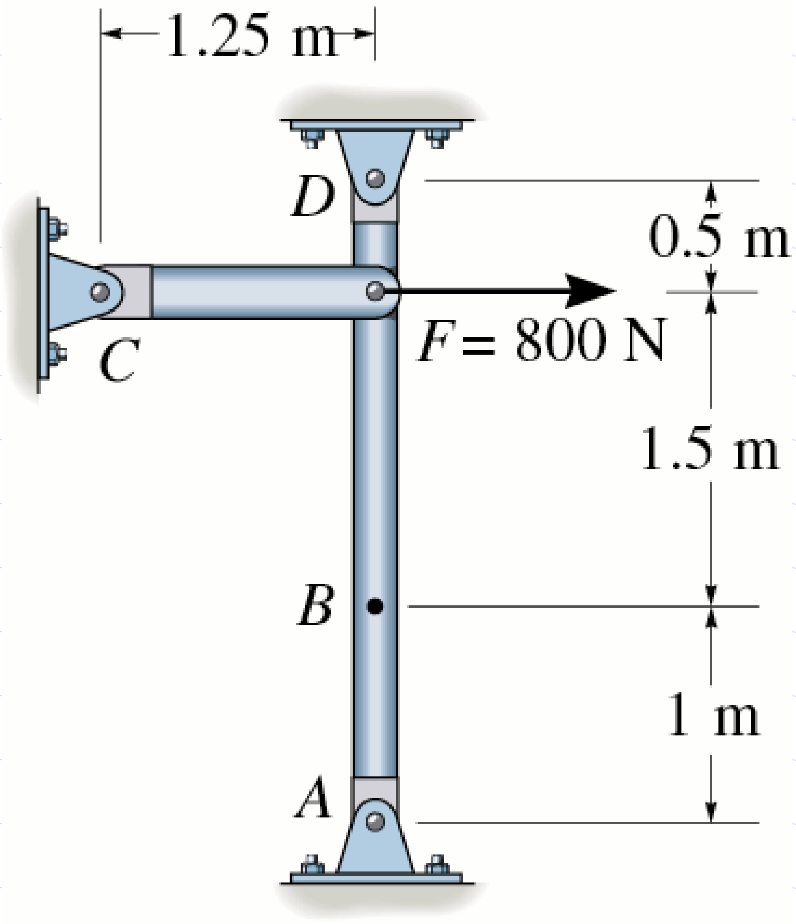
$$M_O = (60\,\mathrm{lb})\left(1 \sin 45^{o}\,\mathrm{ft}\right) = 42.4\,\mathrm{lb \cdot ft}$$



$$M_O = (7\,\text{kN})(4-1\,\text{m}) = 21.0\,\text{kN}\cdot\text{m}$$

Example 2

Determine the moment of the 800 N force about points A, B, C, and D



$$M_A = 800\text{ N }(2.5\text{ m}) = 2000\text{ N}\cdot\text{m}$$

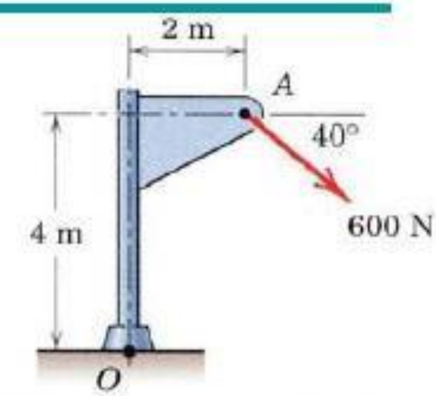
$$M_B = 800\text{ N }(1.5\text{ m}) = 1200\text{ N}\cdot\text{m}$$

$$M_C = 800\text{ N }(0\text{ m}) = 0\text{ N}\cdot\text{m}$$

$$M_D = 800\text{ N }(0.5\text{ m}) = 400\text{ N}\cdot\text{m}$$

Moment: Example

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.

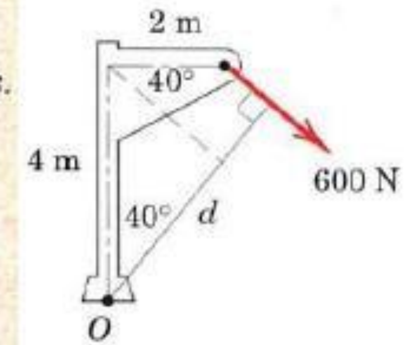


Solution. (*I*) The moment arm to the 600-N force is

$$d = 4 \cos 40^\circ + 2 \sin 40^\circ = 4.35 \text{ m}$$

By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

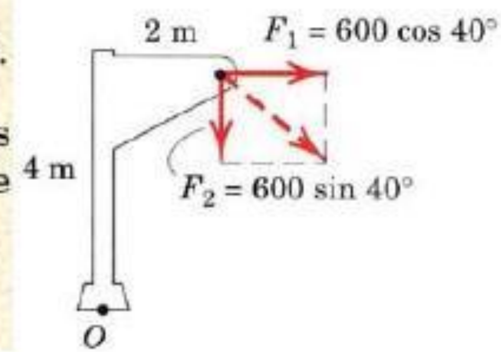


(*II*) Replace the force by its rectangular components at A

$$F_1 = 600 \cos 40^\circ = 460 \text{ N}, \qquad F_2 = 600 \sin 40^\circ = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

$$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$



(*III*) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$$d_1 = 4 + 2 \tan 40^\circ = 5.68 \text{ m}$$

Moment: Example

and the moment is

$$M_O = 460(5.68) = 2610\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

(IV) Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$$d_2 = 2 + 4 \cot 40° = 6.77\ \text{m}$$

and the moment is

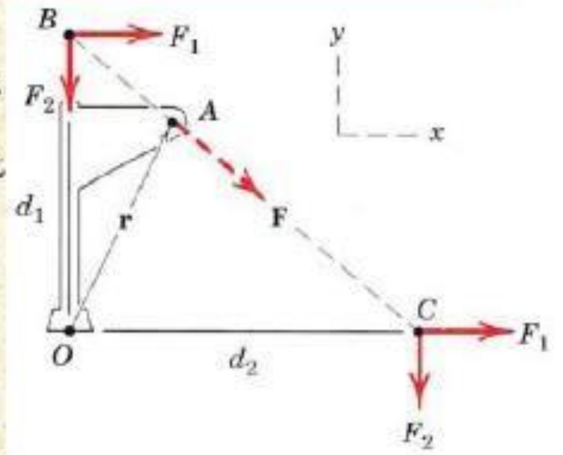
$$M_O = 386(6.77) = 2610\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

(V) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°)$$
$$= -2610\mathbf{k}\ \text{N}\cdot\text{m}$$

The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

$$M_O = 2610\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$



3. Couple

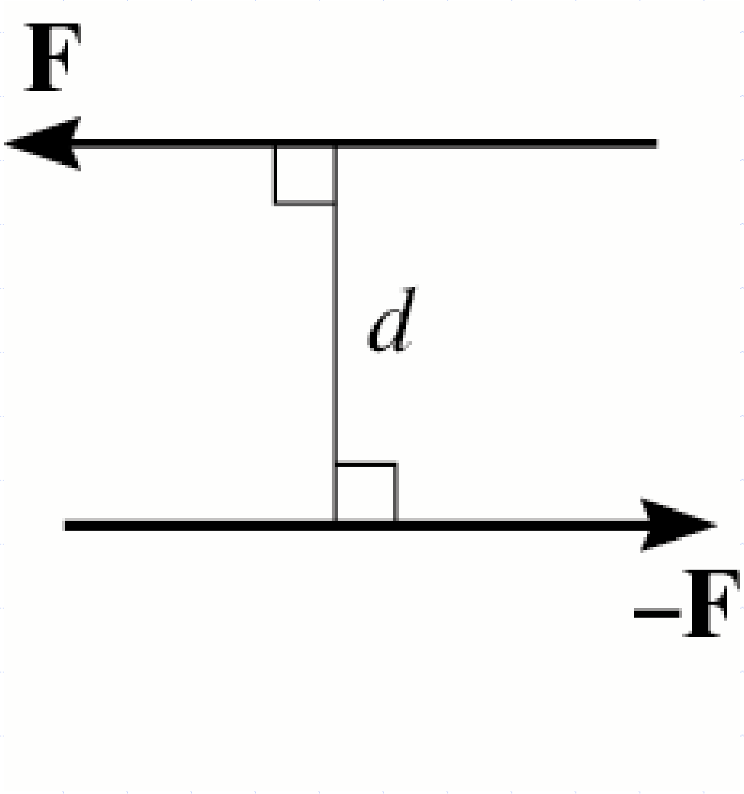
Moment of a Couple

A Couple consists of two parallel forces, equal magnitude, opposite directions, and separated a distant “d” apart.

A Couple Moment about any point O equals the sum of the moments of both forces.

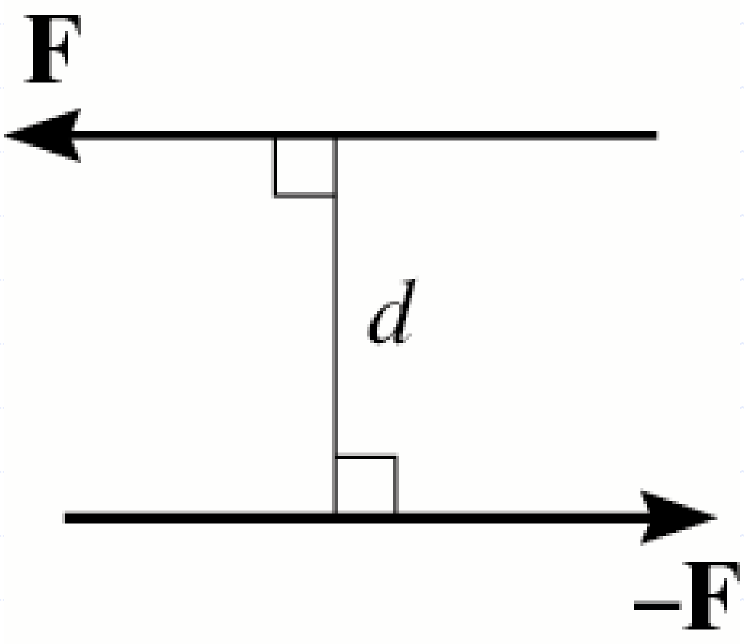
Moment of a Couple

A couple is two parallel forces having the same magnitude and opposite directions separated by a distance d.



Moment of a Couple

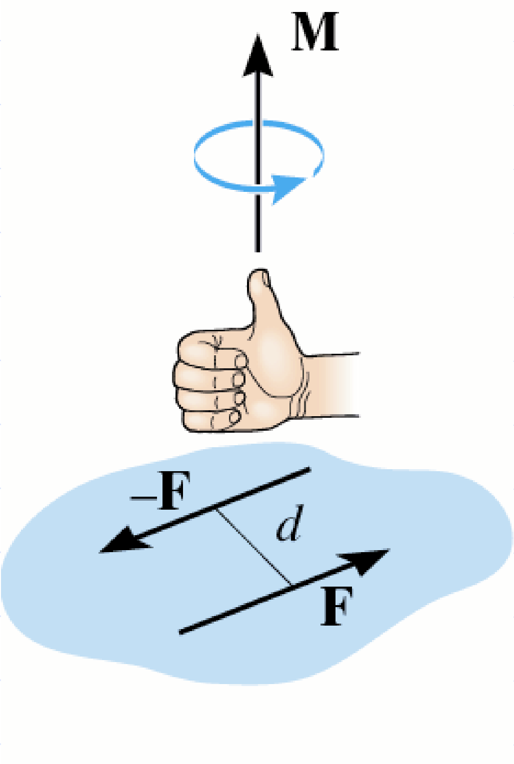
Resultant Force is zero. Effect of couple is a moment



Moment of Couple

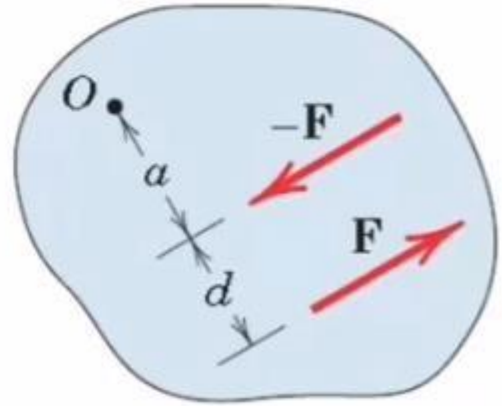
Scalar formulation:

Magnitude of couple moment is M = Fd. Direction is perpendicular to plane of forces. RHR applies



3- Couple

Couple is the moment produced by two equal, opposite, and non-collinear forces.



This couple has a magnitude *M* *through any point O*:

$$M = F(a + d) - Fa$$

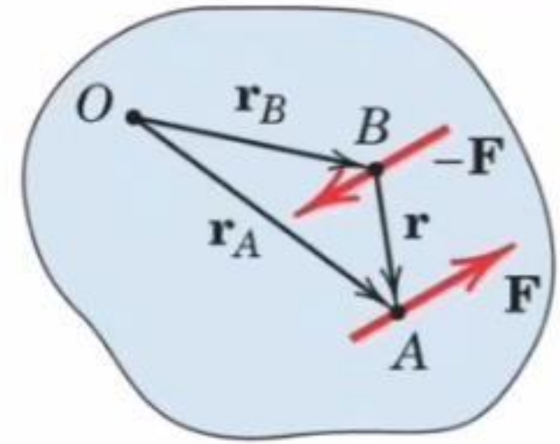
OR

$$M = Fd$$

NOTE: the moment of a couple has the **same value** for all moment centers.

Vector Algebra Method

With the cross-product, the combined moment about point ***O*** of the forces:



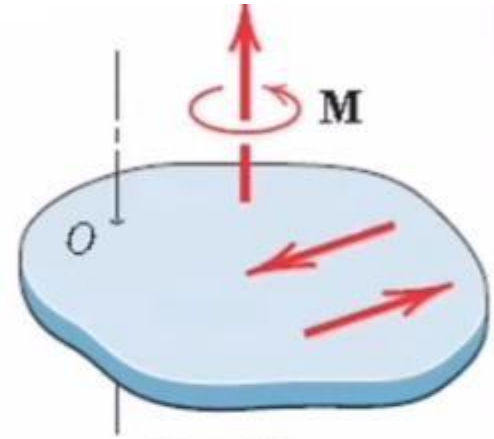
$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

Because $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, we can express **M** as:

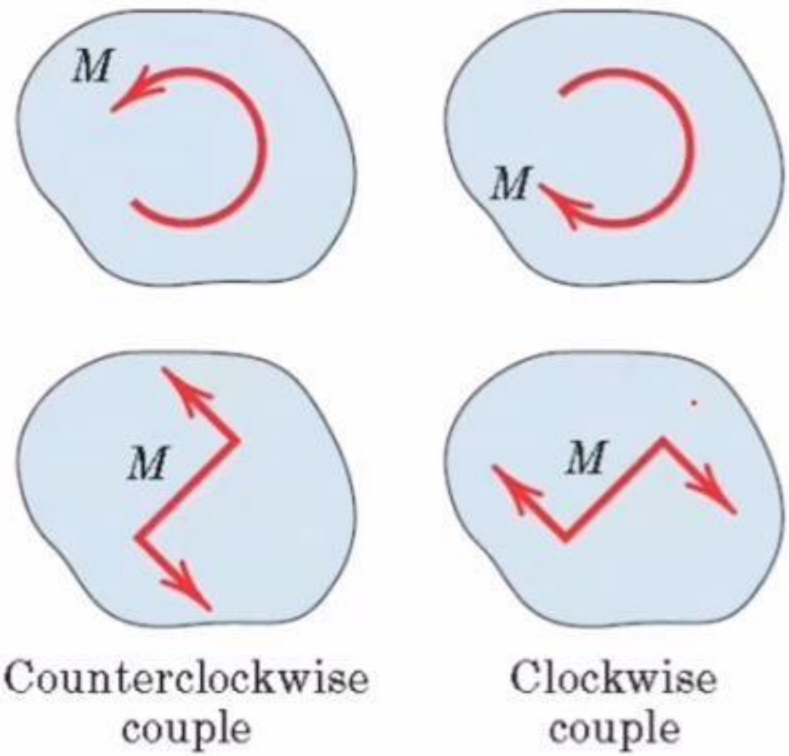
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

NOTE: the moment expression contains no reference to the moment center O and, therefore, is the same for all moment centers.

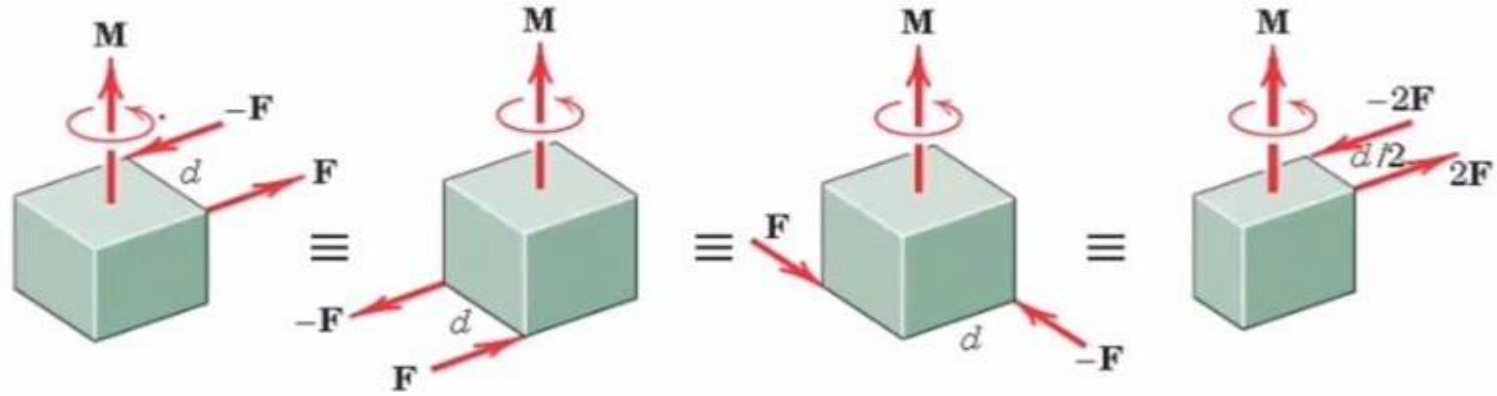
Thus, we may represent **M** by **a free vector**, where the direction of **M** is normal to the plane of the couple and the sense of **M** is established by the right-hand rule.



We may represent the direction of moment or couple vector as : clockwise or counterclockwise



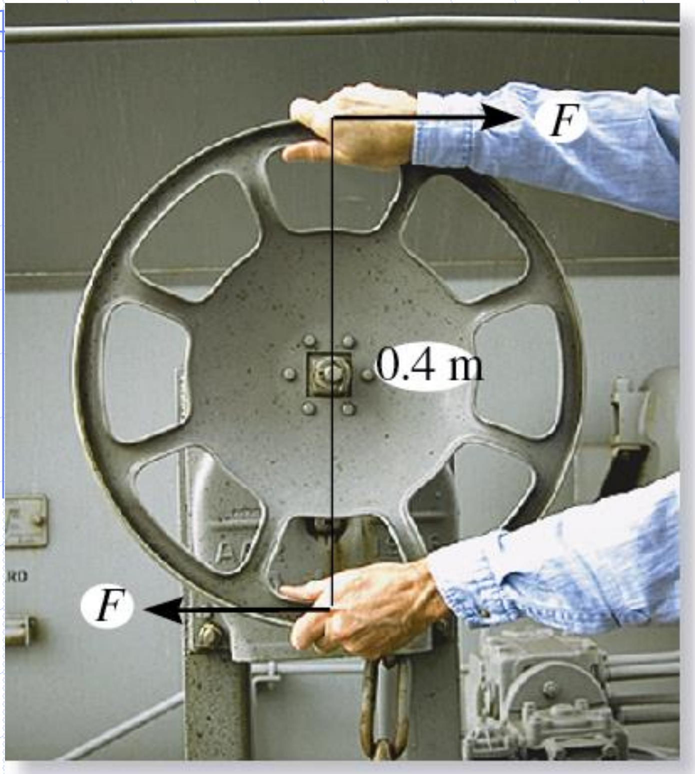
Equivalent Couples

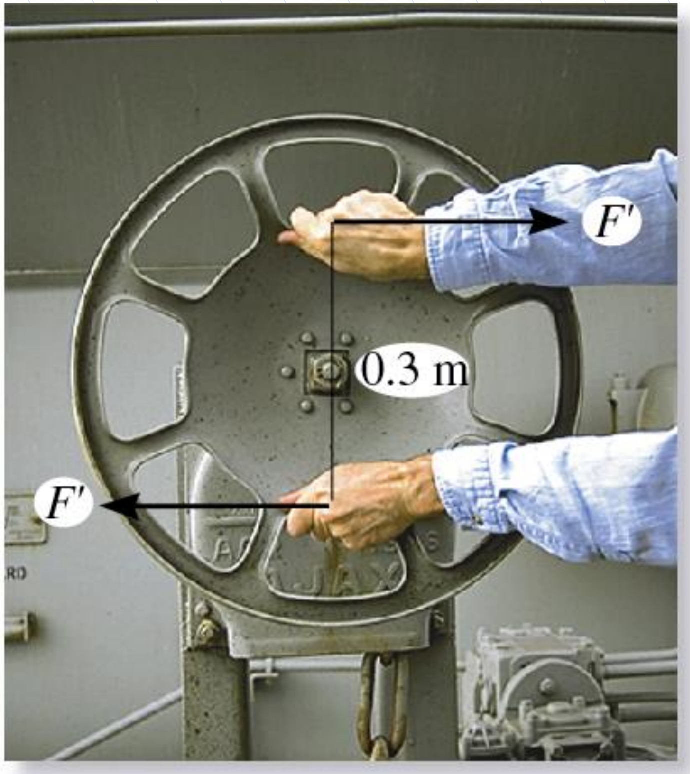


In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

$$M = Fd$$

Moment directions may be accounted for by using a stated sign convention, such as a **plus sign (+) for counterclockwise** moments and a **minus sign (-) for clockwise moments**, or vice versa.


F
0.4 m
F


F'
0.3 m
F'

Couple: Example

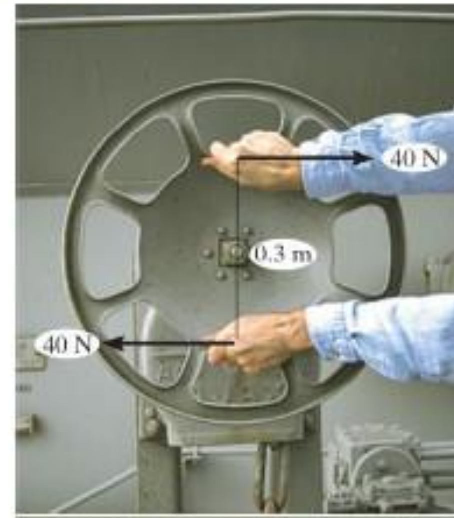
Moment reqd to turn the shaft connected at center of the wheel = 12 Nm

- First case: Couple Moment produced by 40 N forces = 12 Nm
- Second case: Couple Moment produced by 30 N forces = 12 Nm

  If only One hand is used → F = 60N

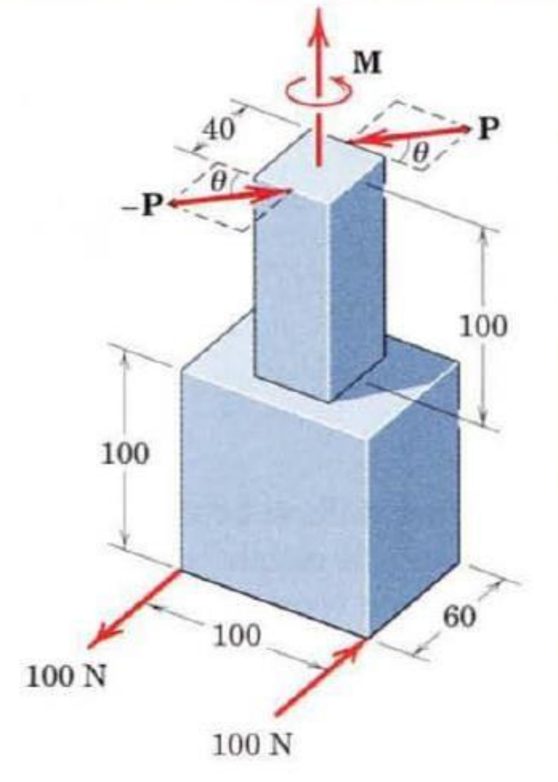
Same couple moment will be produced even if the shaft is not connected at the center of the wheel

→ Couple Moment is a Free Vector



Sample Problem 2/6

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and −**P**, each of which has a magnitude of 400 N. Determine the proper angle θ.



Dimensions in millimeters

Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $$M = 100(0.1) = 10\ \text{N}\cdot\text{m}$$

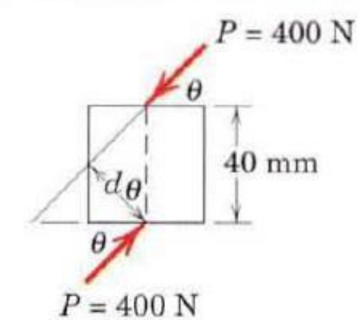
The forces **P** and −**P** produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

Equating the two expressions gives

$$10 = 400(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ \qquad \textit{Ans.}$$



Helpful Hint

① Since the two equal couples are parallel free vectors, the only dimensions which are relevant are those which give the perpendicular distances between the forces of the couples.

4. Force System Resultants

4- Resultants

The resultant of a system of forces: is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body: is the condition in which the resultant of all forces acting on the body is ***zero.***

Equivalent Systems: Resultants

Equilibrium

Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.

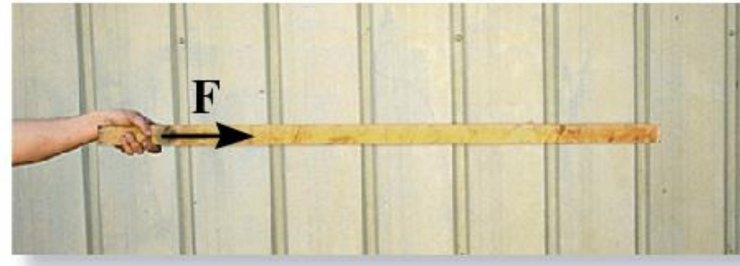
→Condition studied in Statics

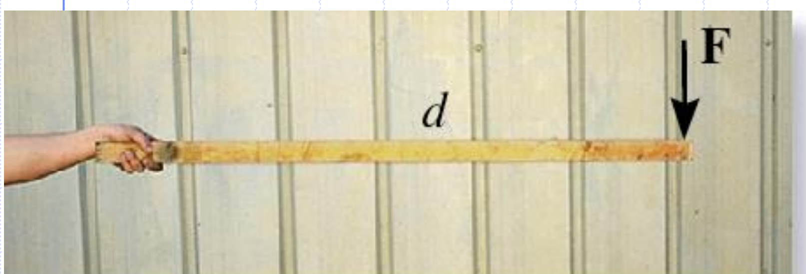
When the resultant of all forces on a body is not zero, acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body.

→ Condition studied in Dynamics

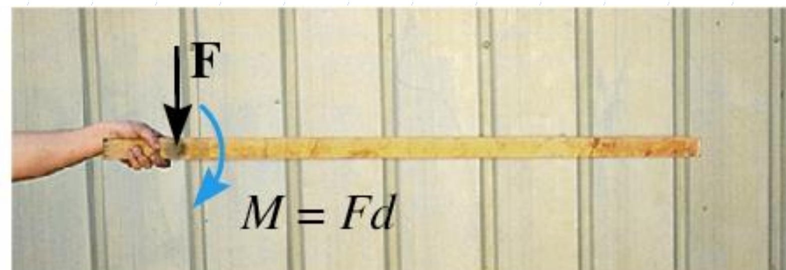

F

=

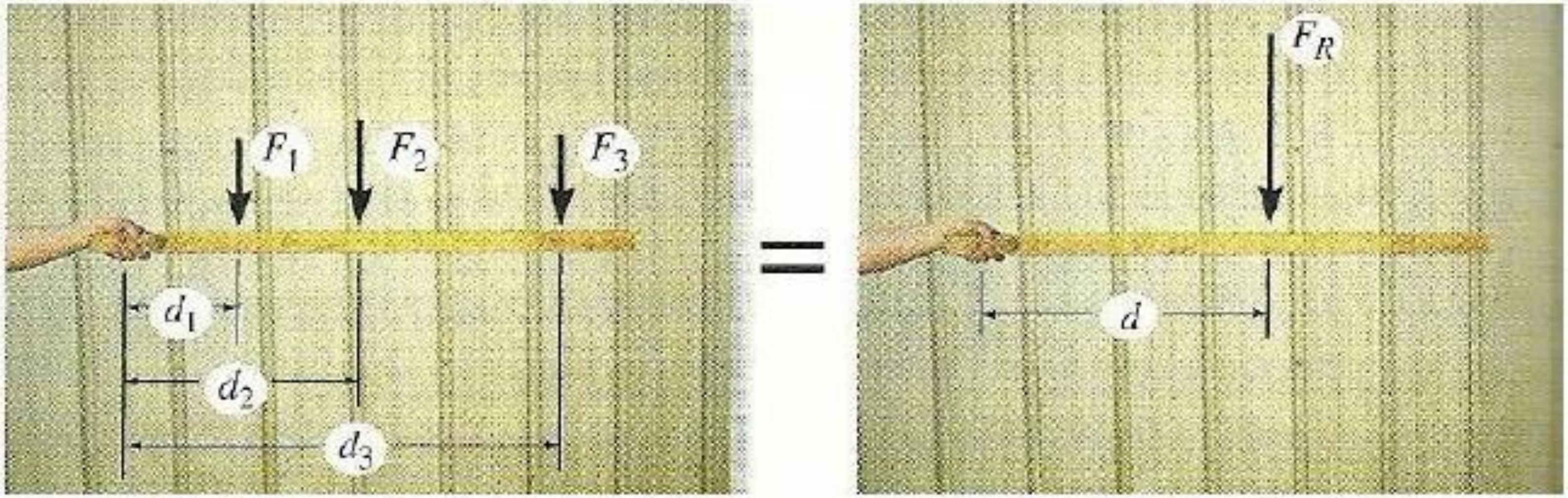

F


F
d

=


F
M = Fd

Equivalent Systems: Resultants



$F_R = F_1 + F_2 + F_3$

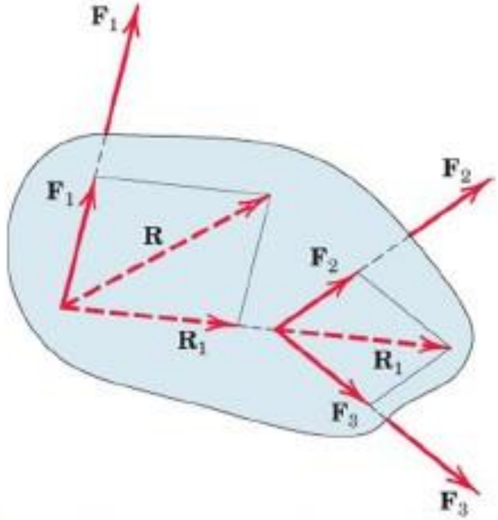
How to find d?

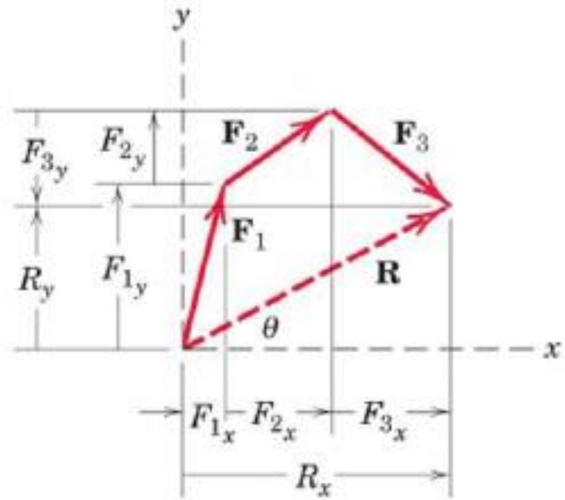
Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

$F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3$

Equivalent Systems: Resultants

Vector Approach: Principle of Transmissibility can be used





Magnitude and direction of the resultant force R is obtained by forming the force polygon where the forces are added head to tail in any sequence

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

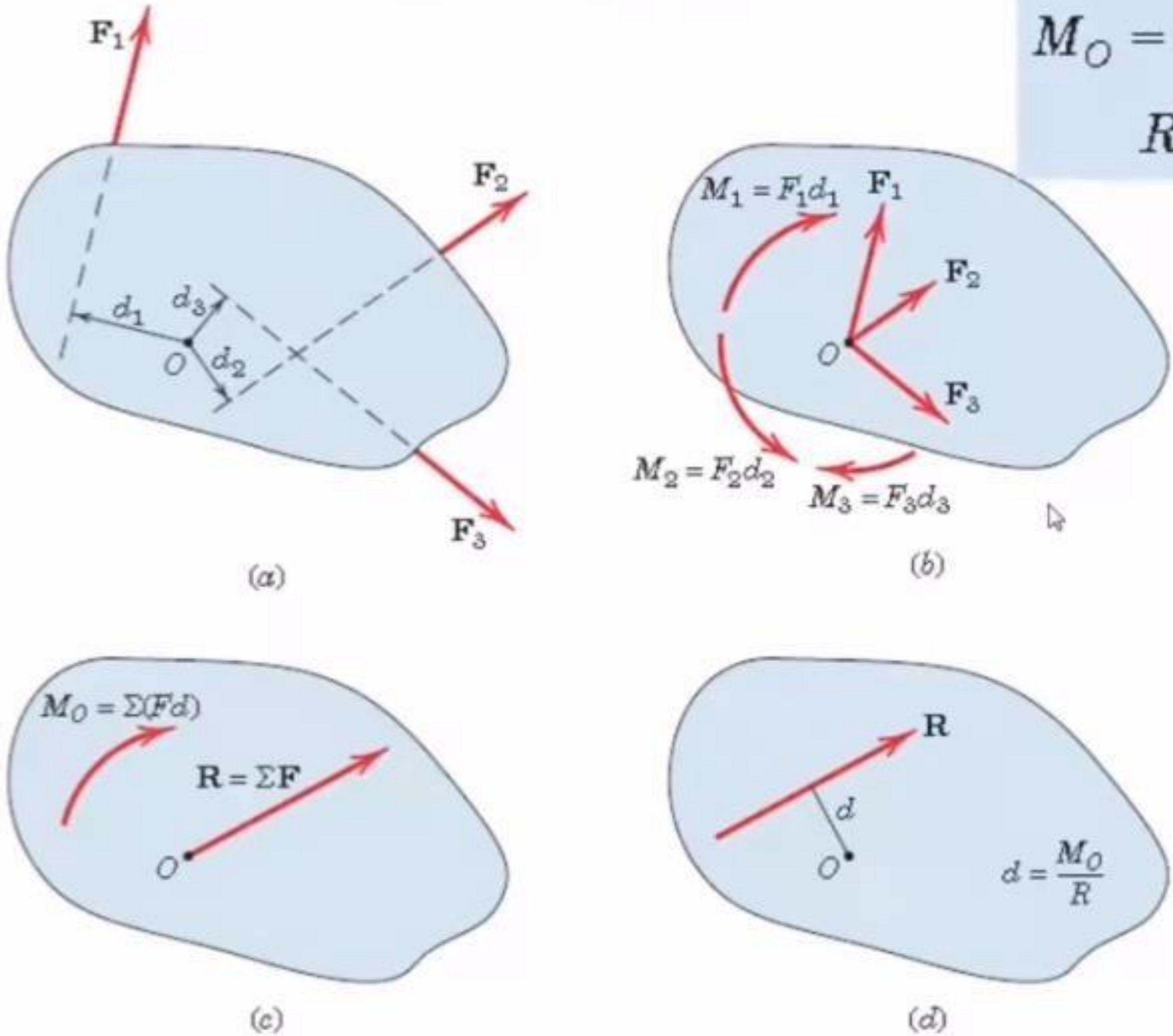
$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Algebraic Method

$$R = \Sigma F$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

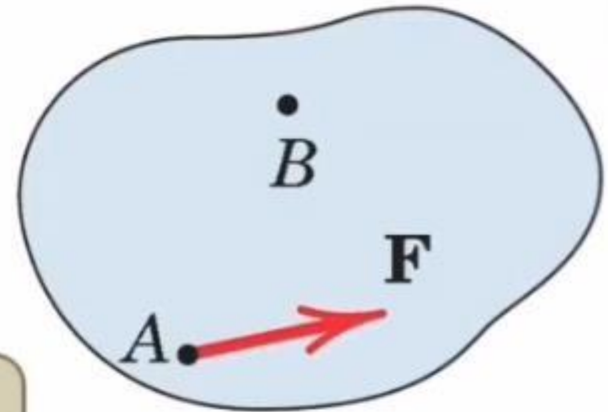


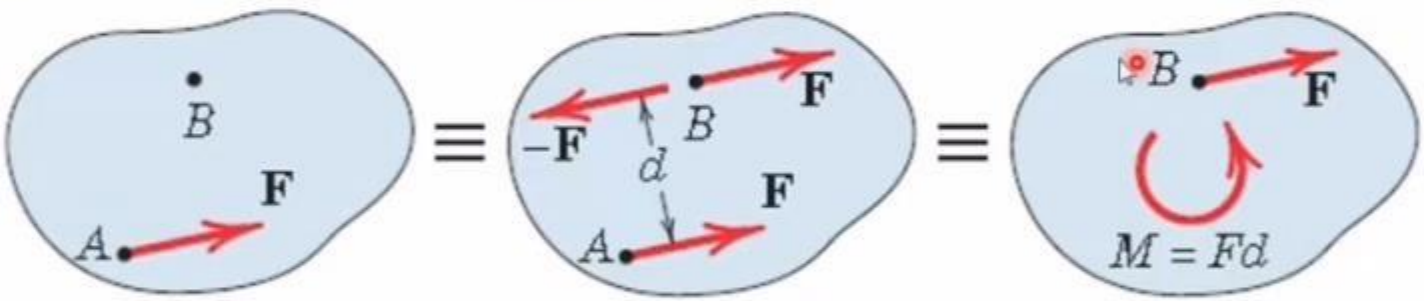
Force–Couple Systems

A force acting on a body

+ any fixed axis which does not intersect the line of the force.

= The same force at the fixed point **+** couple moment ($M=Fd$)



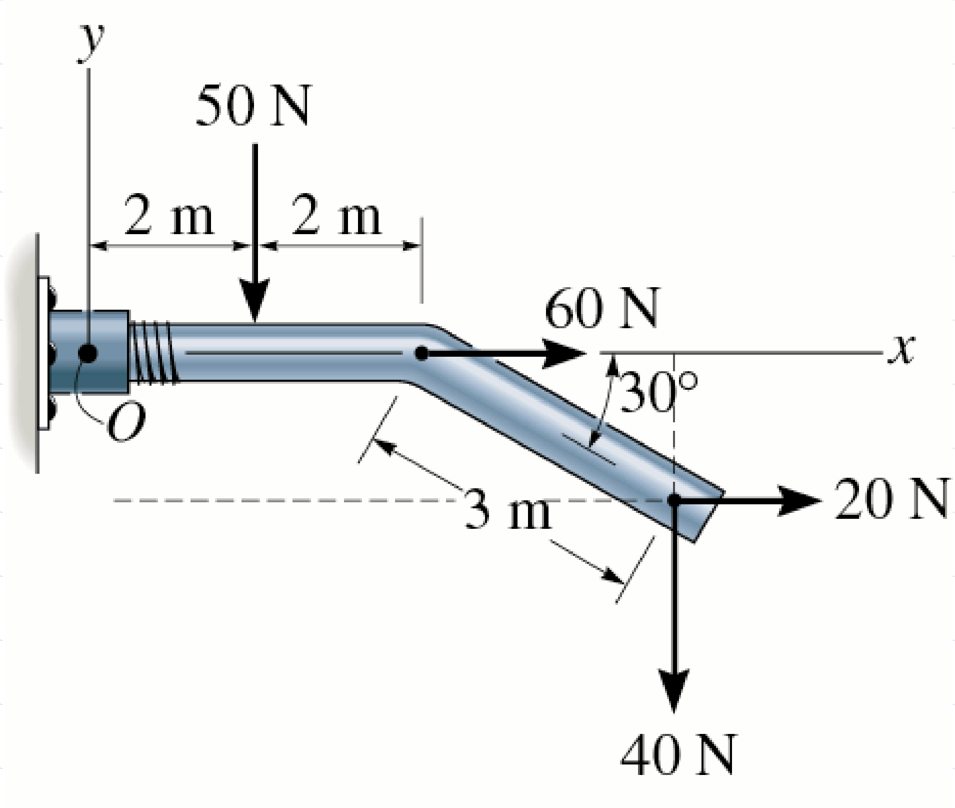


The combination of the force and couple in the right-hand is referred to as *a force–couple system.*

➢ *We may reverse this process.*

Example 3

Determine the resultant moment of the four forces about the base point O.



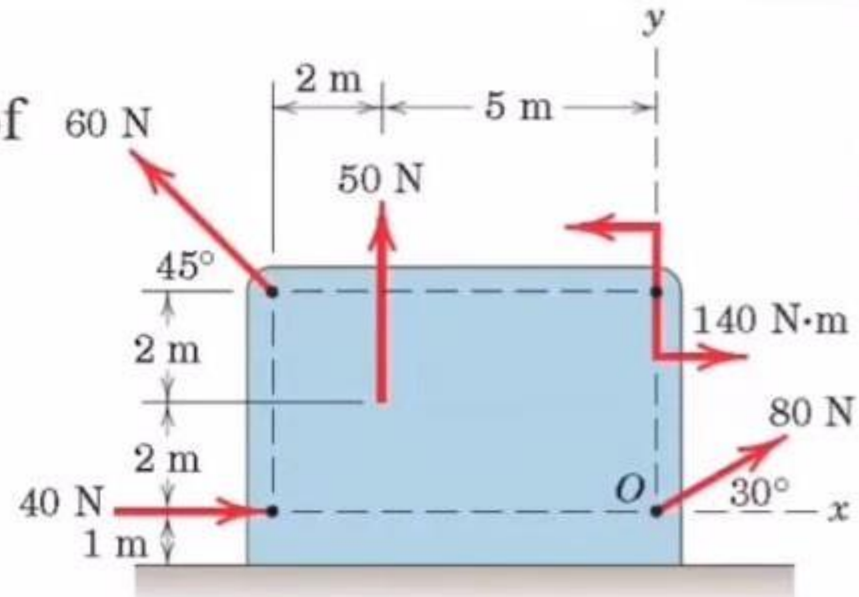
$$(+ccw) \quad M_{R_O} = \sum Fd$$

$$M_{R_O} = -50N(2m) + 60N(0)$$
$$+20N(3\sin 30^o\ m) - 40N(4 + 3\cos 30^o\ m)$$

$$M_{R_O} = -334\ N \cdot m = 334\ N \cdot m\ (cw)$$

SAMPLE PROBLEM

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution.

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80\cos 30° - 60\cos 45° = 66.9\text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80\sin 30° + 60\cos 45° = 132.4\text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3\text{ N}$

$\left[\theta = \tan^{-1}\frac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1}\frac{132.4}{66.9} = 63.2°$

$$[M_O = \Sigma(Fd)]$$

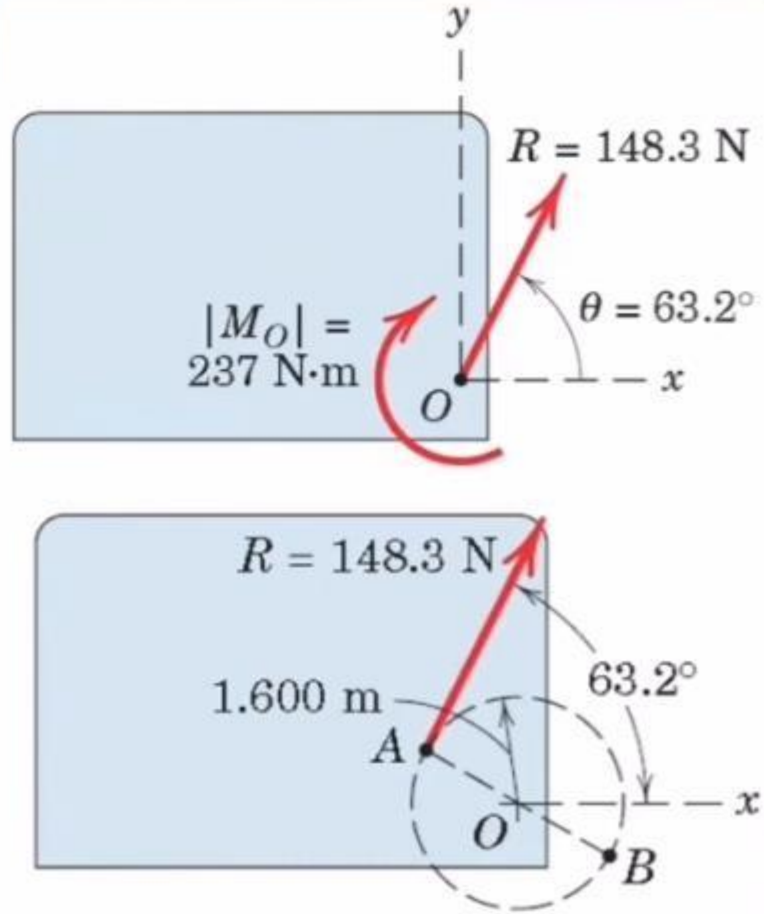
$$M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$$

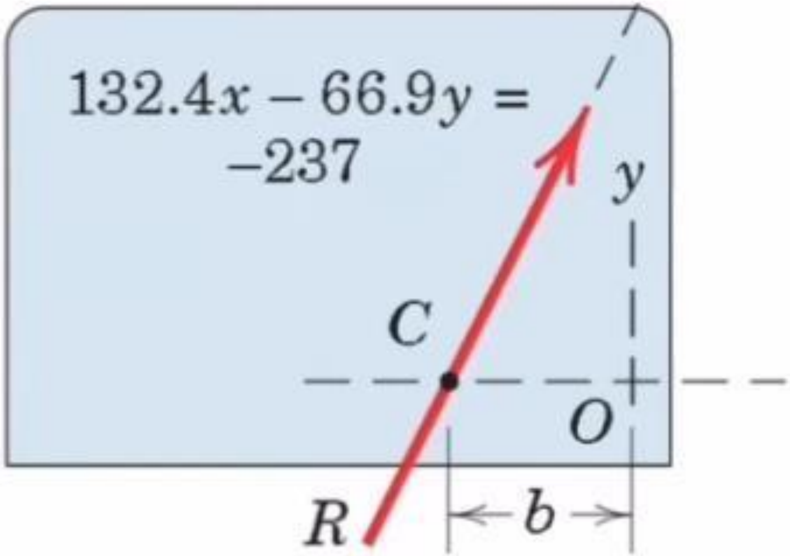
$$= -237 \text{ N} \cdot \text{m}$$

$$[Rd = |M_O|]$$

$$148.3d = 237$$

$$d = 1.600 \text{ m}$$



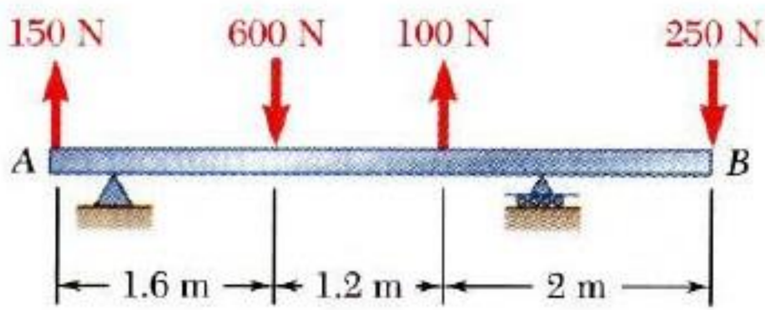


The resultant **R** may also be located

$$R_y b = |M_O|$$

$$b = \frac{237}{132.4} = 1.792 \text{ m}$$

Example on Equivalent System



For the beam, reduce the system of forces shown to

(a) an **equivalent force-couple** system at ***A***,

(b) an **equivalent force couple** system at ***B***, and

(c) a **single force** or **resultant**

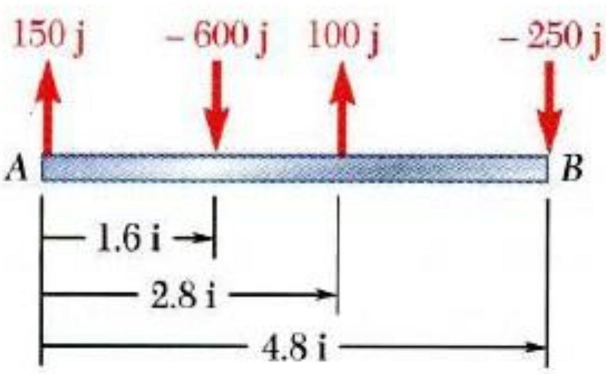
<u>Solution:</u>

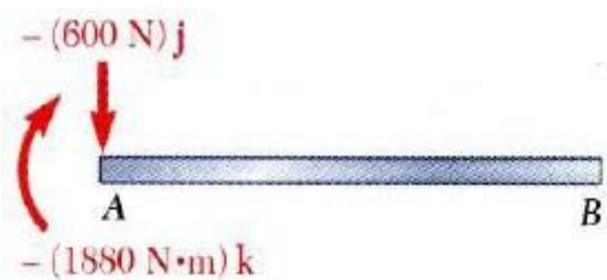
a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Example on Equivalent System





<u>SOLUTION:</u>

a) Compute the resultant force and the resultant couple at *A*.

$$\vec{R} = \sum \vec{F}$$

$$= (150\ \mathrm{N})\vec{j} - (600\ \mathrm{N})\vec{j} + (100\ \mathrm{N})\vec{j} - (250\ \mathrm{N})\vec{j}$$

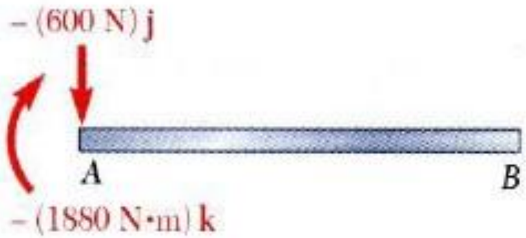
$$\boxed{\vec{R} = -(600\ \mathrm{N})\vec{j}}$$

$$\vec{M}_A^R = \sum\left(\vec{r} \times \vec{F}\right)$$

$$= \left(1.6\vec{i}\right)\times\left(-600\vec{j}\right) + \left(2.8\vec{i}\right)\times\left(100\vec{j}\right) + \left(4.8\vec{i}\right)\times\left(-250\vec{j}\right)$$

$$\boxed{\vec{M}_A^R = -(1880\ \mathrm{N}\cdot\mathrm{m})\vec{k}}$$

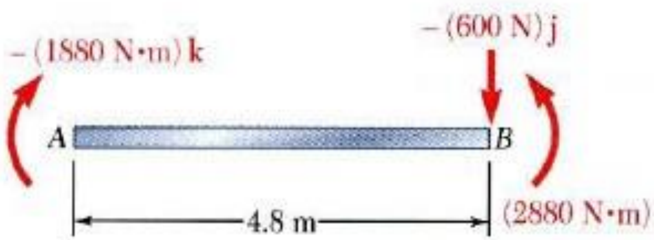
Example on Equivalent System



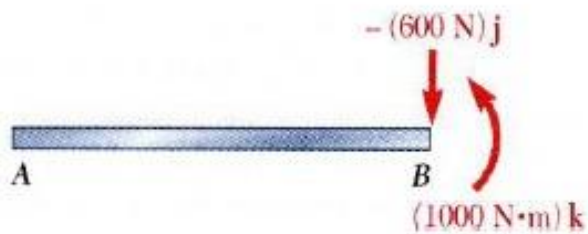
b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

The force is unchanged by the movement of the force-couple system from *A* to *B*.

$$\boxed{\vec{R} = -(600\,\mathrm{N})\vec{j}}$$



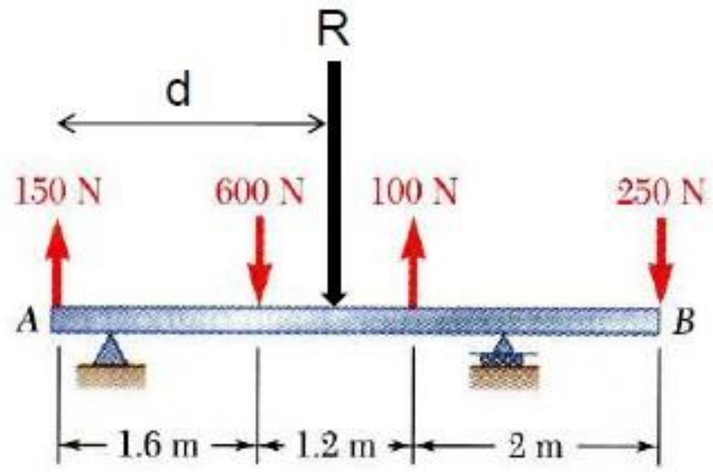
The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.



$$\begin{aligned}\vec{M}_B^R &= \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R} \\ &= -(1880\,\mathrm{N\cdot m})\vec{k} + (-4.8\,\mathrm{m})\vec{i} \times (-600\,\mathrm{N})\vec{j} \\ &= -(1880\,\mathrm{N\cdot m})\vec{k} + (2880\,\mathrm{N\cdot m})\vec{k}\end{aligned}$$

$$\boxed{\vec{M}_B^R = +(1000\,\mathrm{N\cdot m})\vec{k}}$$

Example on Equivalent System



c)

$$F_R = F_1 + F_2 + F_3 + F_4$$

$$R = 150 - 600 + 100 - 250 = -600\ N$$

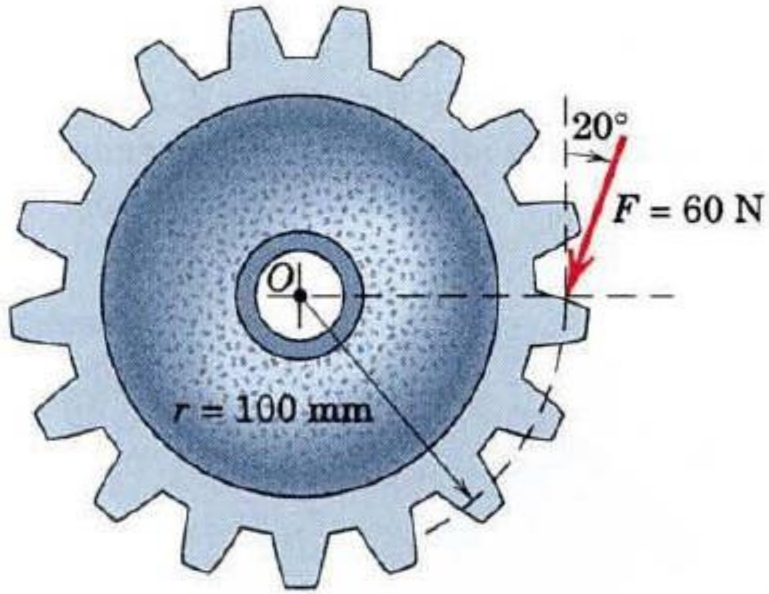
$$F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$$

$$d = 3.13\ m$$

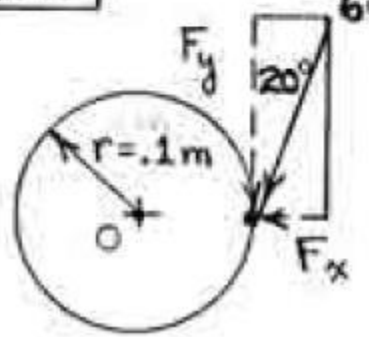
5. Additional Exercises

2/31 A force **F** of magnitude 60 N is applied to the gear. Determine the moment of **F** about point O.

Ans. $M_O = 5.64\ \text{N}\cdot\text{m CW}$



2/31

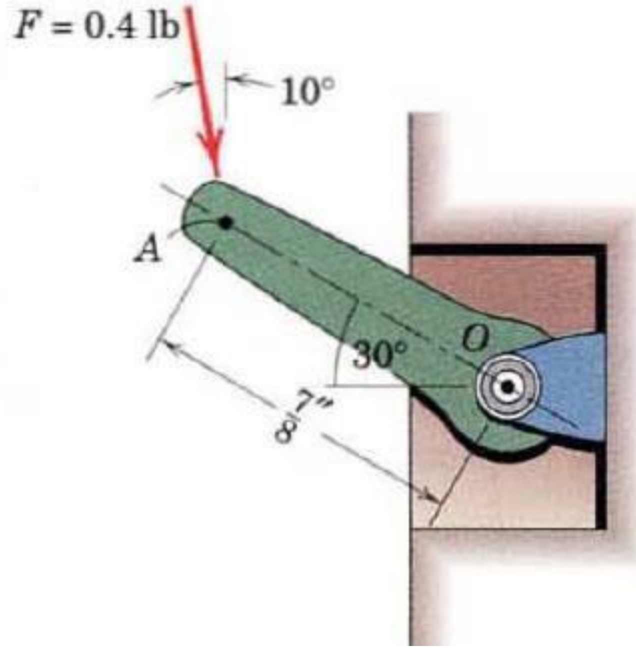


$+\circlearrowright M_o = r\,F_y$

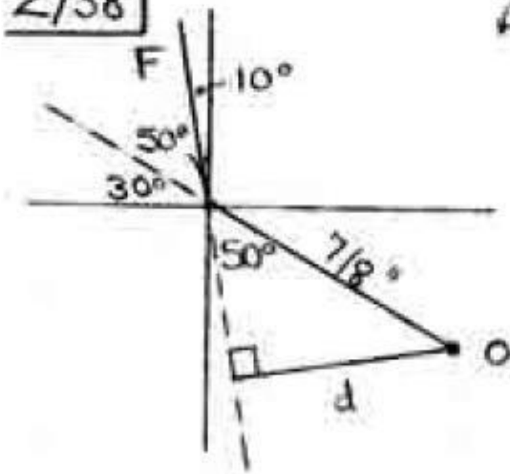
$= (0.1)(60 \cos 20°)$

$= \underline{5.64\ \text{N}\cdot\text{m}}$

2/38 Compute the moment of the 0.4-lb force about the pivot O of the wall-switch toggle.



2/38



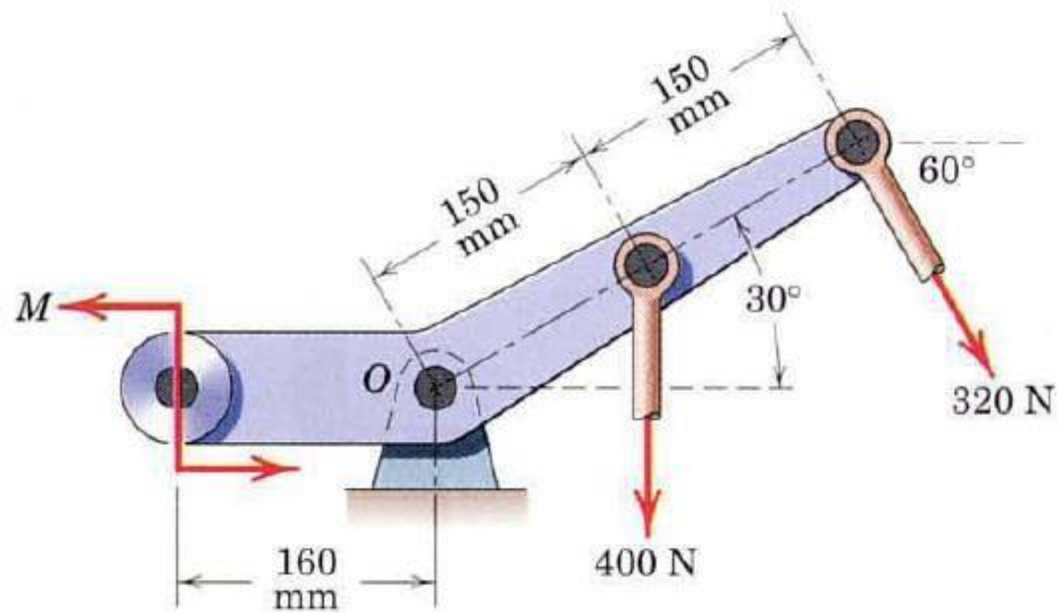
$$\curvearrowleft + M_O = Fd$$

$$= 0.4\left(\frac{7}{8}\sin 50^\circ\right)$$

$$= \underline{0.268 \text{ lb-in.}}$$

2/77 If the resultant of the two forces and couple M passes through point O, determine M.

Ans. $M = 148.0\ \text{N}\cdot\text{m CCW}$

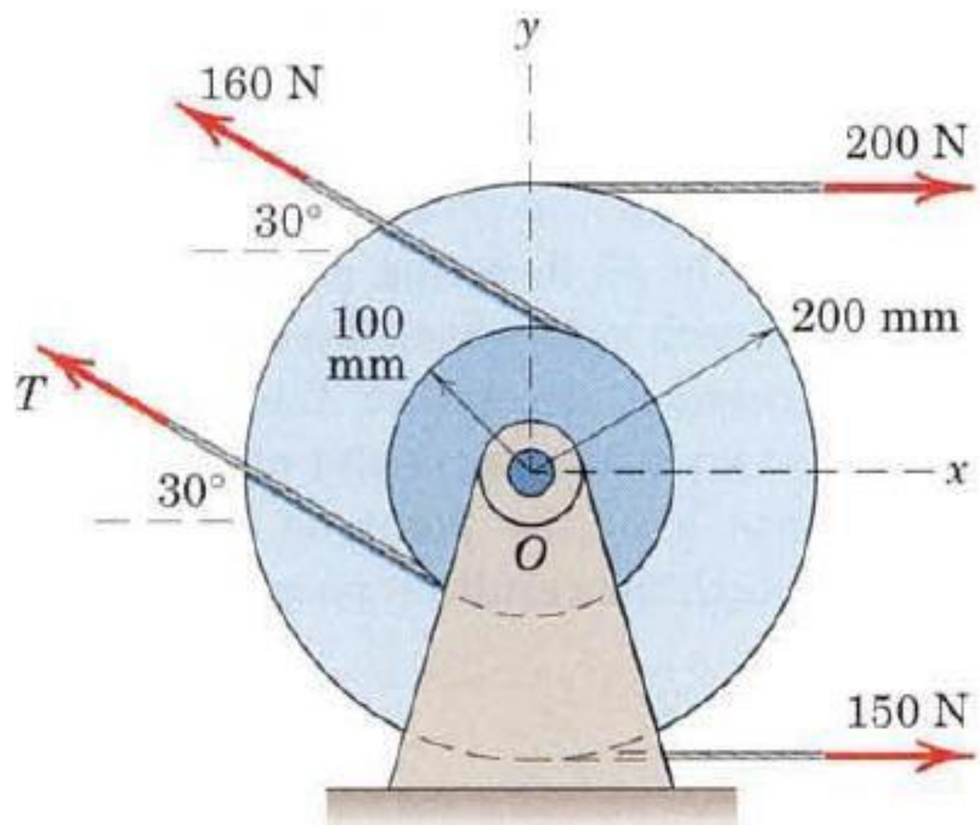


2/77 $M_o = 0$, so

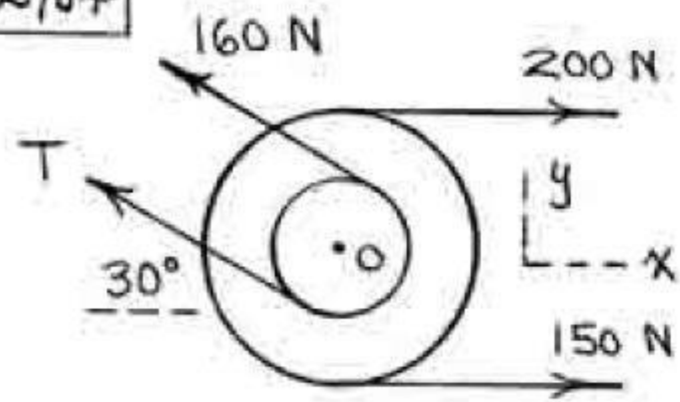
$$\circlearrowleft M - 400(0.150 \cos 30°) - 320(0.300) = 0$$

$$M = 148.0\ \text{N}\cdot\text{m}$$

2/84 Two integral pulleys are subjected to the belt tensions shown. If the resultant **R** of these forces passes through the center O, determine T and the magnitude of **R** and the counterclockwise angle θ it makes with the x-axis.



2/84



$\left\{ \begin{array}{l} r_1 = 0.1 \text{ m} \\ r_2 = 0.2 \text{ m} \end{array} \right.$

$+\circlearrowleft \Sigma M_O = 0: \; 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$

$\underline{T = 60 \text{ N}}$

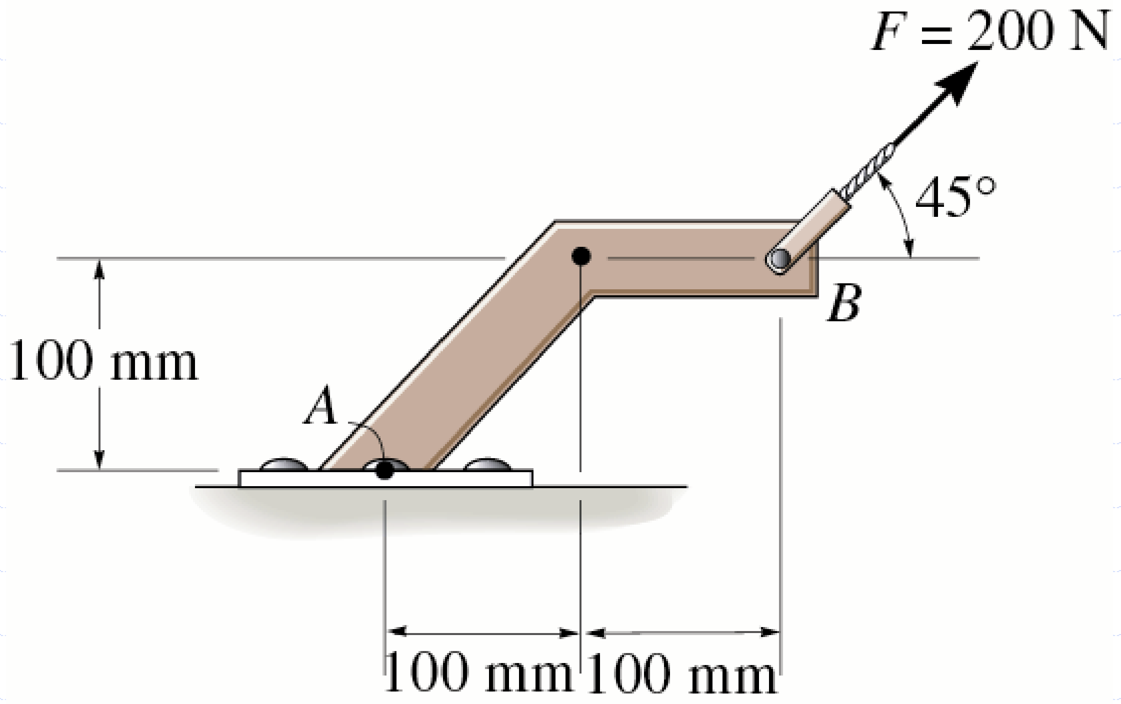
$R_x = \Sigma F_x = 200 + 150 - (160 + 60)\cos 30° = 159.5 \text{ N}$

$R_y = \Sigma F_y = (160 + 30)\sin 30° = 110 \text{ N}$

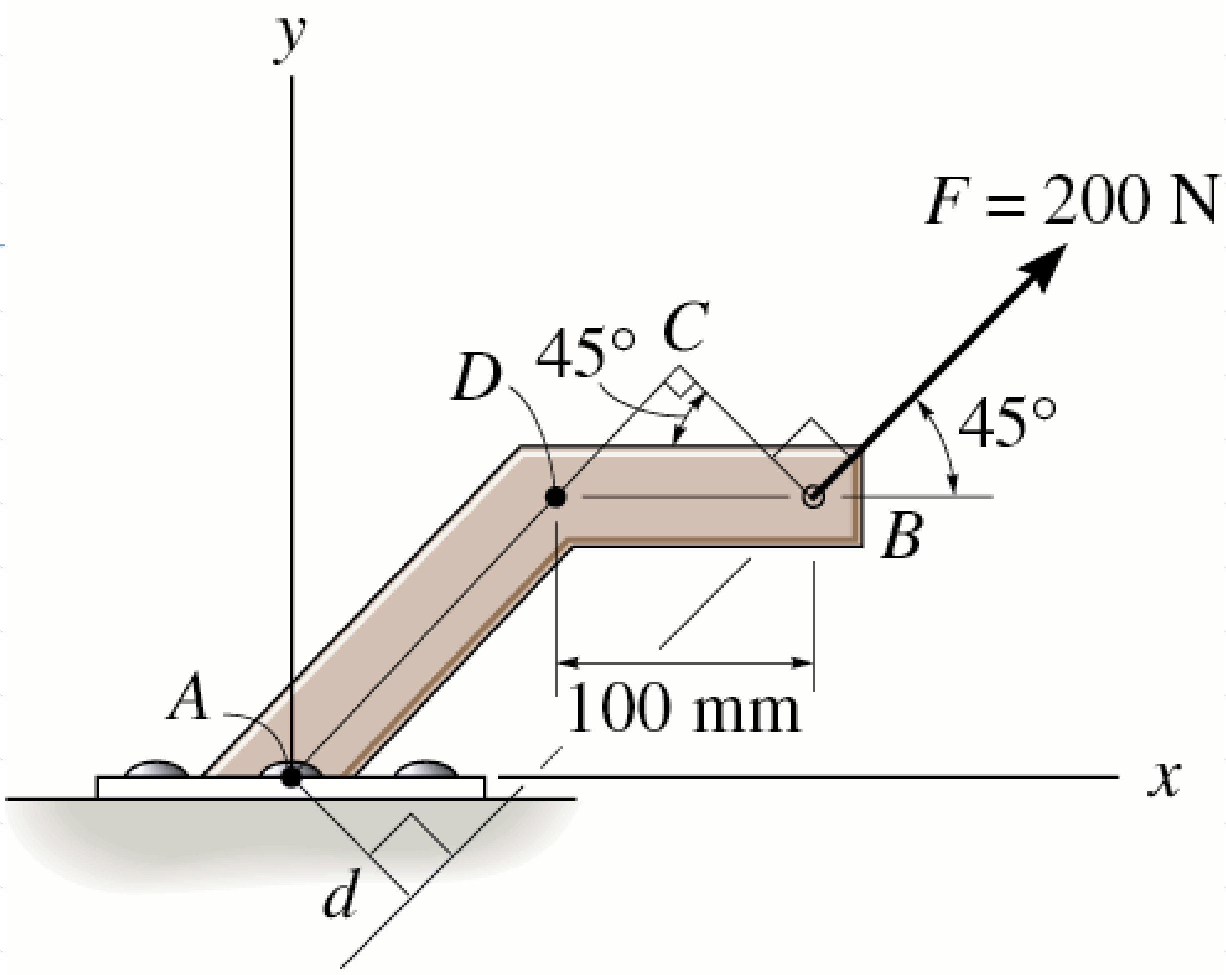
$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \text{ N}}$

$\theta = \tan^{-1}(R_y / R_x) = \underline{34.6°}$

Example 1



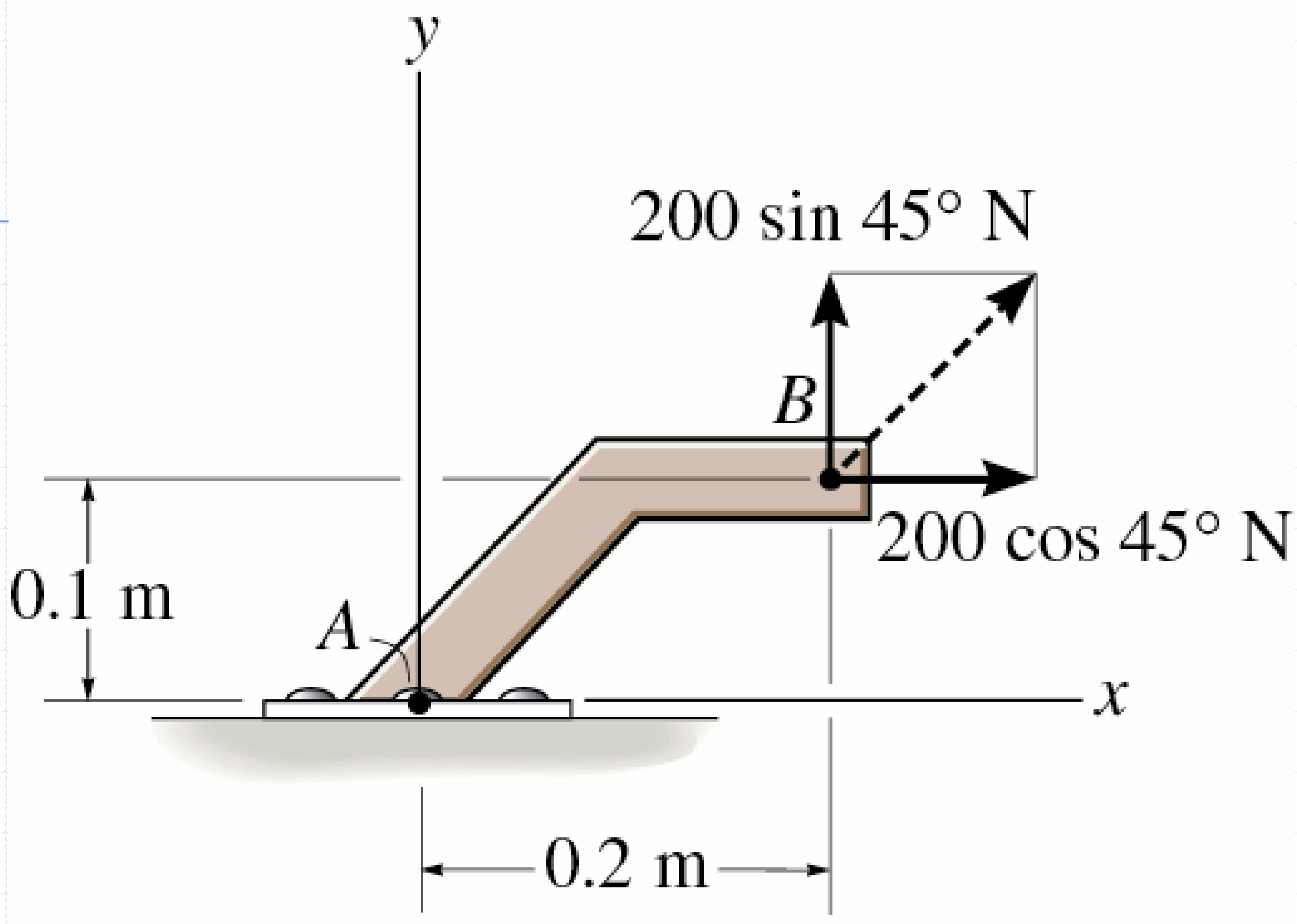
Determine the moment of the force about A.


y
F = 200 N
45°
C
D
45°
B
100 mm
A
x
d

$$CB = d = 100\cos 45^{o} = 70.71\text{mm} = 0.07071\text{m}$$

$$M_A = Fd = (200\text{N})(0.07071\text{m}) = 14.1\,\text{N}\cdot\text{m}$$

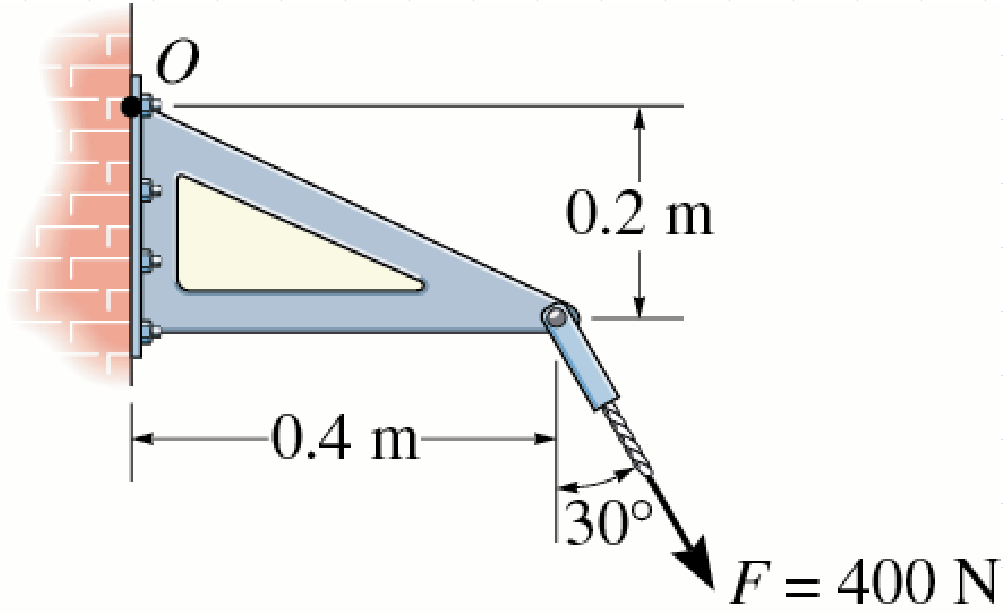
$$\vec{M}_A = (14.1\hat{k})\,\text{N}\cdot\text{m}$$


y
200 sin 45° N
B
200 cos 45° N
0.1 m
A
x
0.2 m

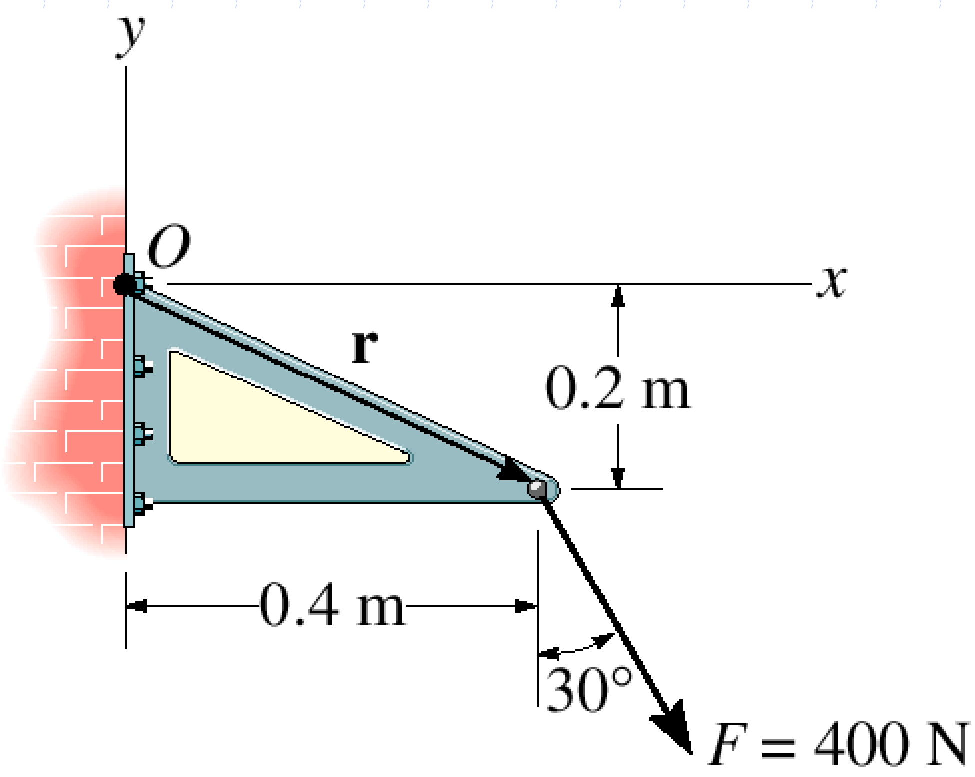
$$
\begin{aligned}
M_A &= \sum Fd \\
&= \left(200 \sin 45^{o}\,\mathrm{N}\right)\left(0.20\,\mathrm{m}\right) - \left(200 \cos 45^{o}\,\mathrm{N}\right)\left(0.10\,\mathrm{m}\right) \\
&= 14.1\,\mathrm{N}\cdot\mathrm{m}
\end{aligned}
$$

$$
\vec{M}_A = \left(14.1\hat{k}\right)\mathrm{N}\cdot\mathrm{m}
$$

Example 2



Determine the moment of the force about 0.


y
O
x
r
0.2 m
0.4 m
30°
F = 400 N

$$(+\text{ccw})$$

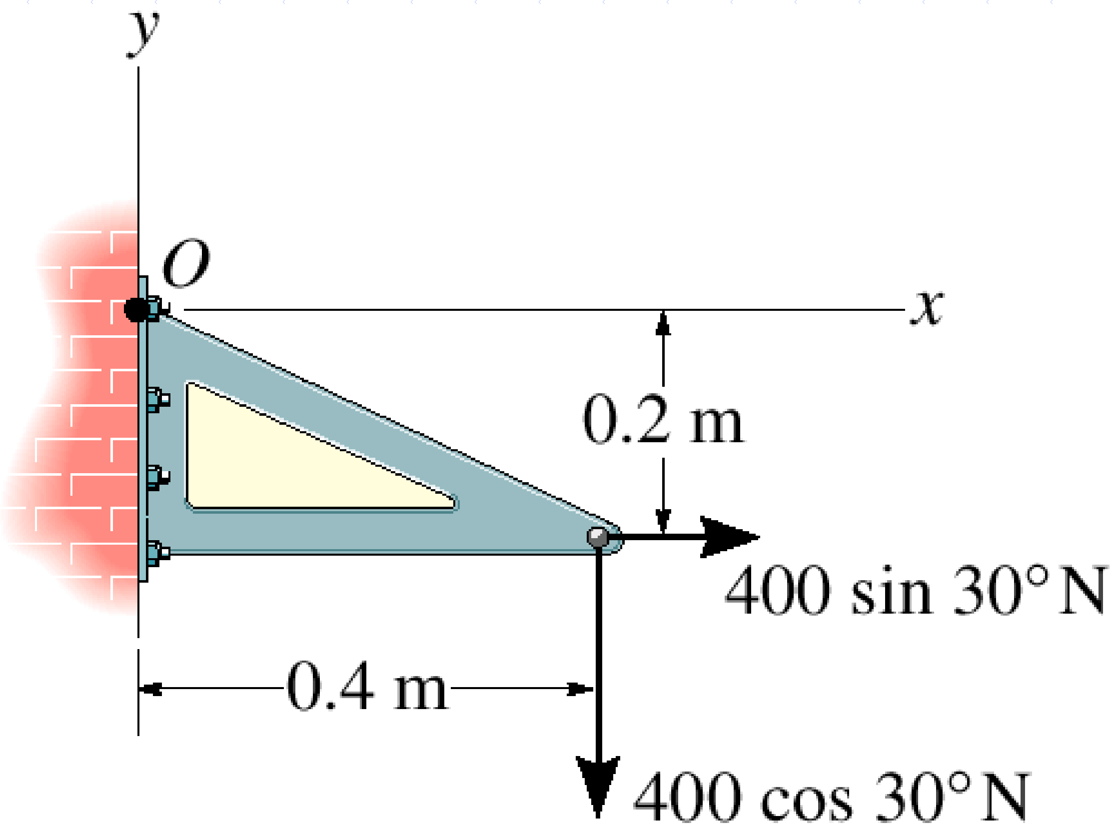
$$M_O = \left(400\sin 30^\circ\,\text{N}\right)(0.2\,\text{m})$$

$$-\left(400\cos 30^\circ\,\text{N}\right)(0.4\,\text{m})$$

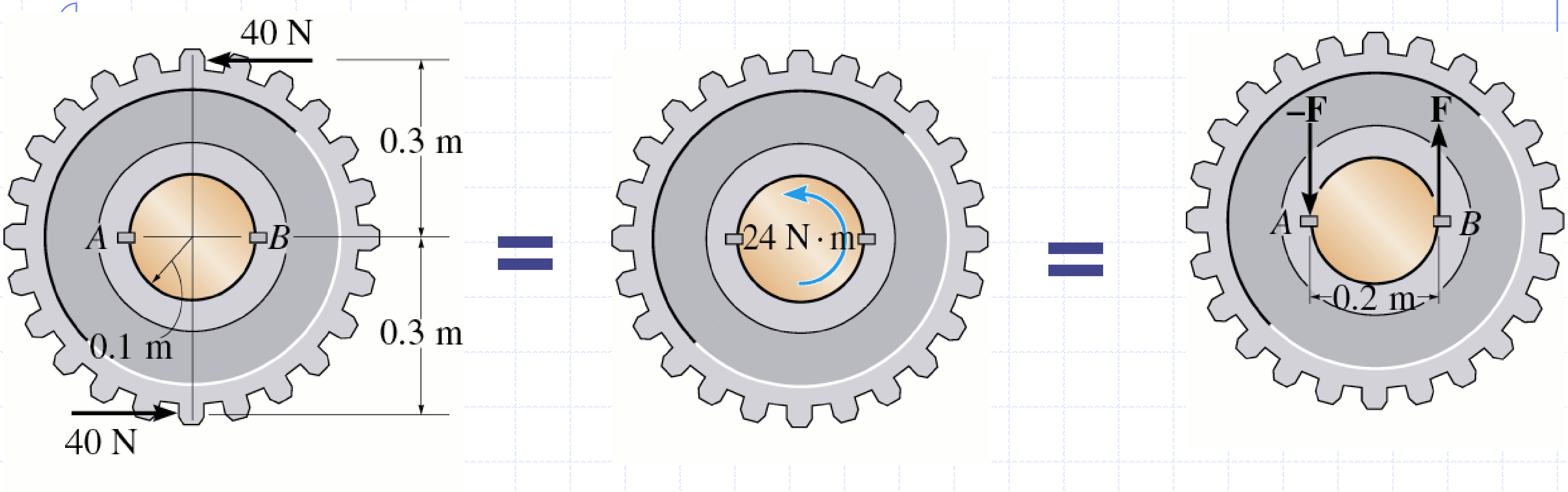
$$= -98.6\,\text{N}\cdot\text{m}$$

$$M_O = 98.6\,\text{N}\cdot\text{m}\,(+\text{cw})$$

$$\vec{M}_O = \left[-98.6\ \hat{k}\right]\text{N}\cdot\text{m}$$


y
O
x
0.2 m
400 sin 30° N
0.4 m
400 cos 30° N

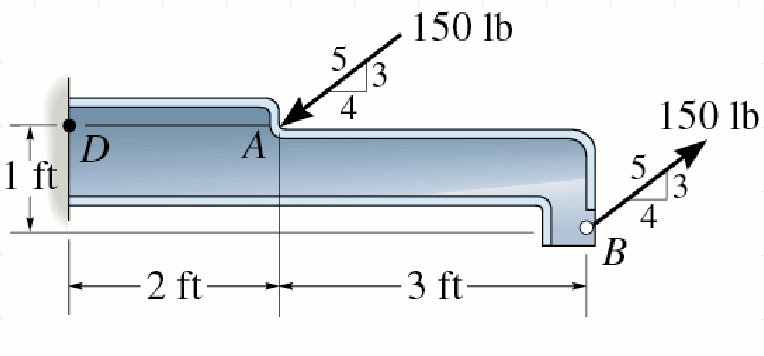
Example 3

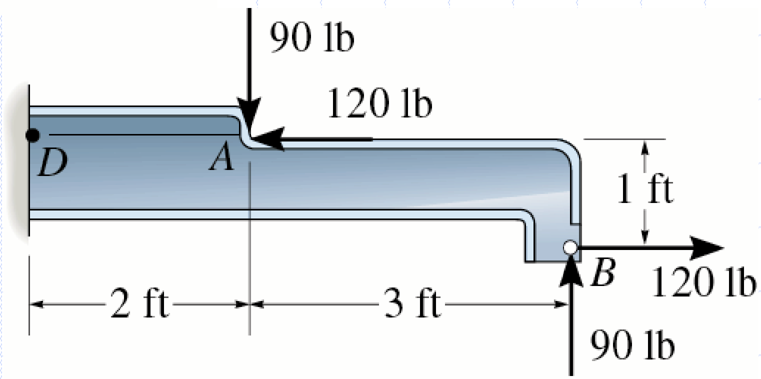


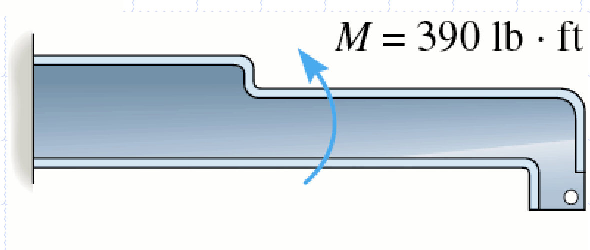
Determine the magnitude of **F**

Example 4

Determine the resultant of the two forces acting on the shown part

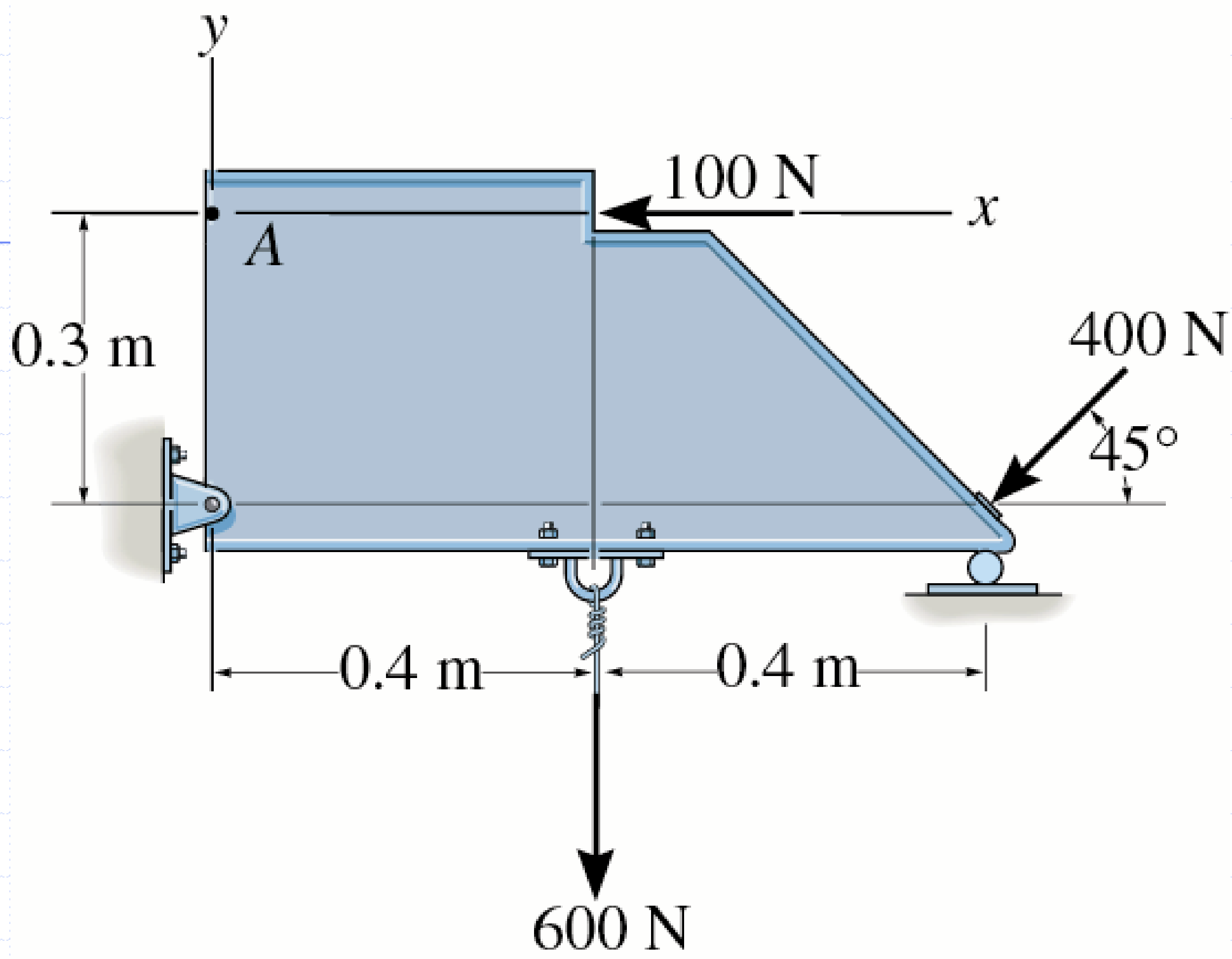






Example 5

Replace the forces acting on the brace shown below with an equivalent resultant force and couple moment at point A.


y
100 N
x
A
0.3 m
400 N
45°
0.4 m
0.4 m
600 N

$$F_{R_x} = \sum F_x$$

$$F_{R_x} = -100\ \text{N} - 400\cos 45^{o} = -382.8\ \text{N}$$

$$F_{R_x} = 382.8\ \text{N} \longleftarrow$$

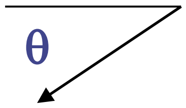
$$F_{R_y} = \sum F_y$$

$$F_{R_y} = -600\ \text{N} - 400\sin 45^{o} = -882.8\ \text{N}$$

$$F_{R_y} = 882.8\ \text{N} \downarrow$$

$$F_R = \sqrt{(382.8)^2 + (882.8)^2} = 962\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_y}}\right) = \tan^{-1}\left(\frac{-882.8}{-382.8}\right) = 66.6^{o}$$



$$(+\ \text{ccw})\quad M_{R_A} = \sum M_A \quad (+\ \text{ccw})$$

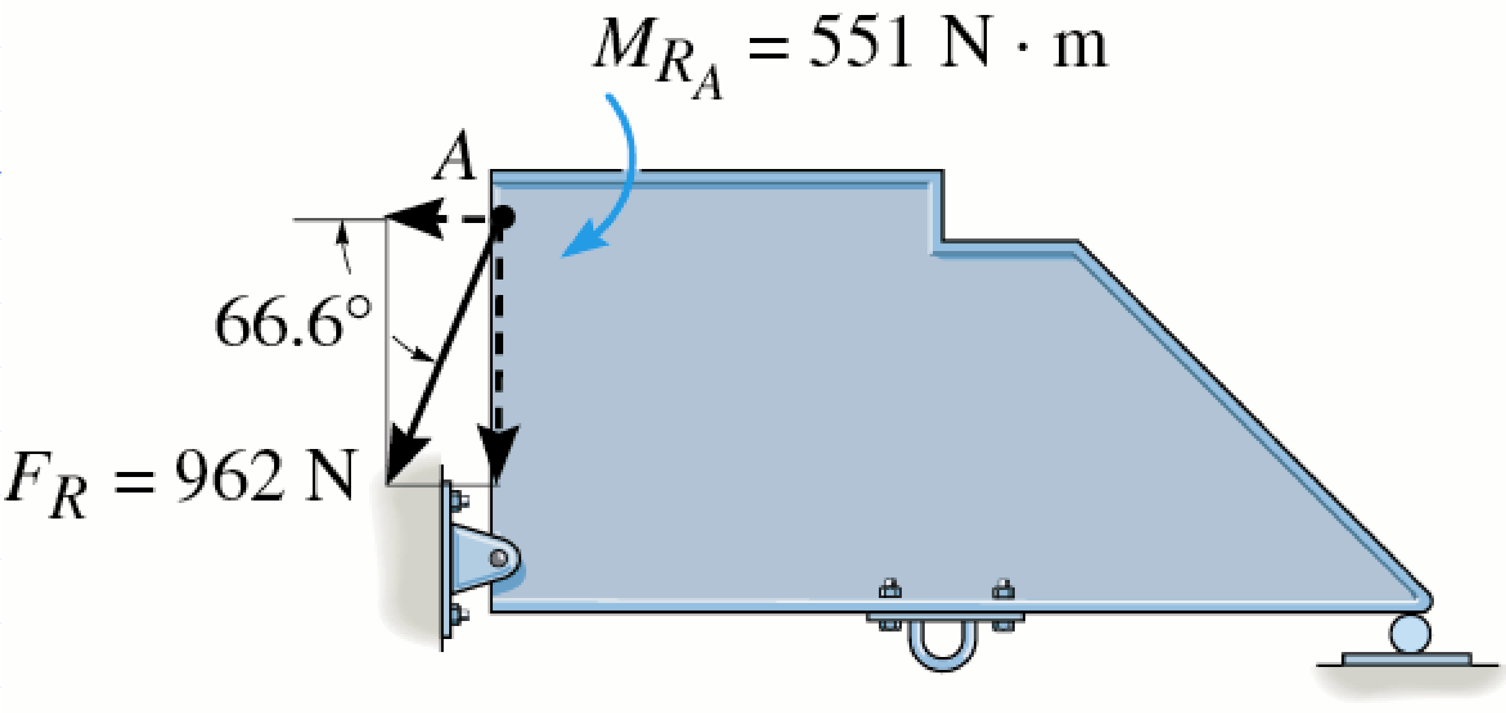
$$M_{R_A} = (100\ \text{N})(0) - (600\ \text{N})(0.4\ \text{m}) - (400\ \sin 45^{\circ}\ \text{N})(0.8\ \text{m})$$

$$-(400\ \sin 45^{\circ}\ \text{N})(0.8\ \text{m})$$

$$M_{R_A} = -551\ \text{N}\cdot\text{m} = 551\ \text{N}\cdot\text{m}\ (\text{cw})$$

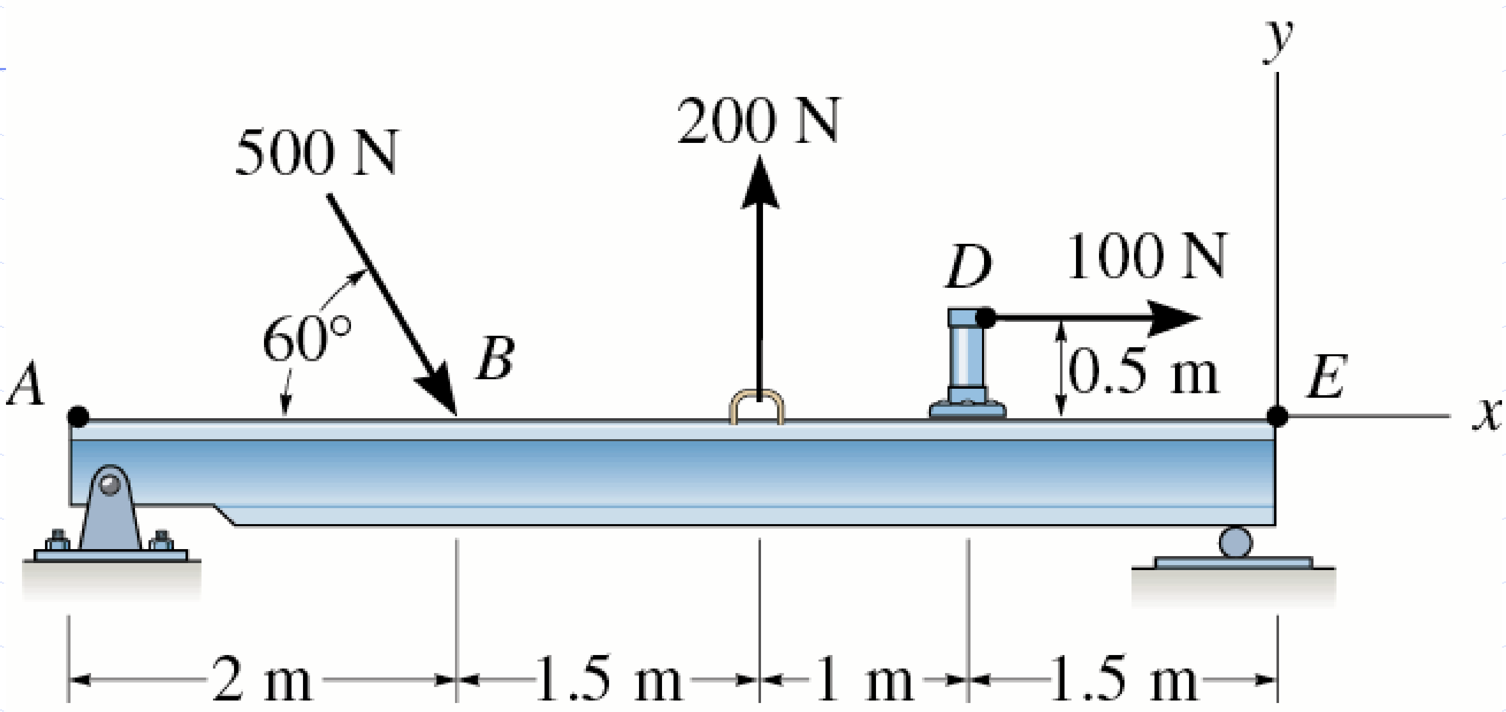
$$F_R = \sqrt{(382.8)^2 + (882.8)^2} = 962\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{-882.8}{-382.8}\right) = 66.6^{\circ}$$



$M_{R_A} = 551$ N · m
A
66.6°
$F_R = 962$ N

Example 6



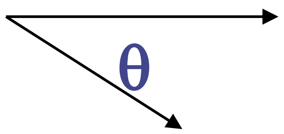
Determine the magnitude, direction, and location on the beam of the resultant force that is equivalent to the system of forces shown.

$$F_{R_x} = \sum F_x = 500\cos 60^{\circ}\ \text{N} + 100\ \text{N} = 350\ \text{N}$$

$$F_{R_y} = \sum F_y = -500\sin 60^{\circ}\ \text{N} + 200\ \text{N} = -233\ \text{N}$$

$$F_R = \sqrt{(350)^2 + (-233)^2} = 420.5\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{233}{350}\right) = 33.7^{\circ}$$



$$(+\text{ccw})\quad M_{RE} = \sum M_E$$

$$= \left(500 \ \sin 60^{o}\right)(4) + \left(500 \ \cos 60^{o}\right)(0) -$$

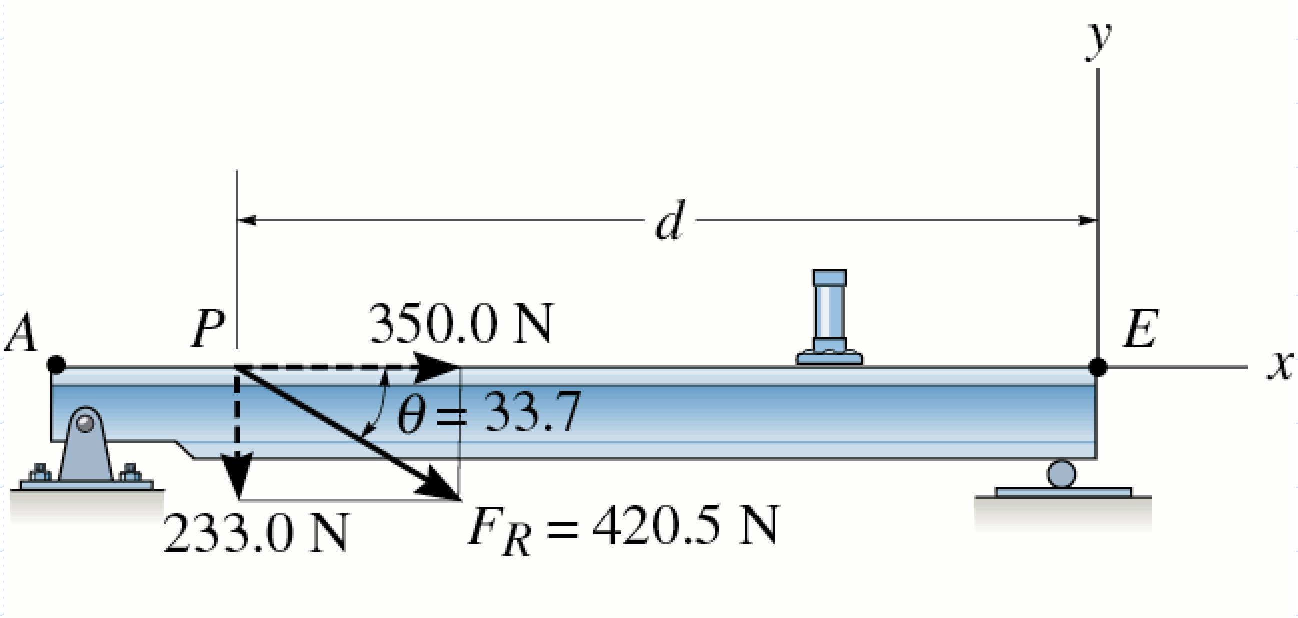
$$(100)(0.5) - (200)(2.5)$$

$$= 1182.1 \ \text{N} \cdot \text{m}$$

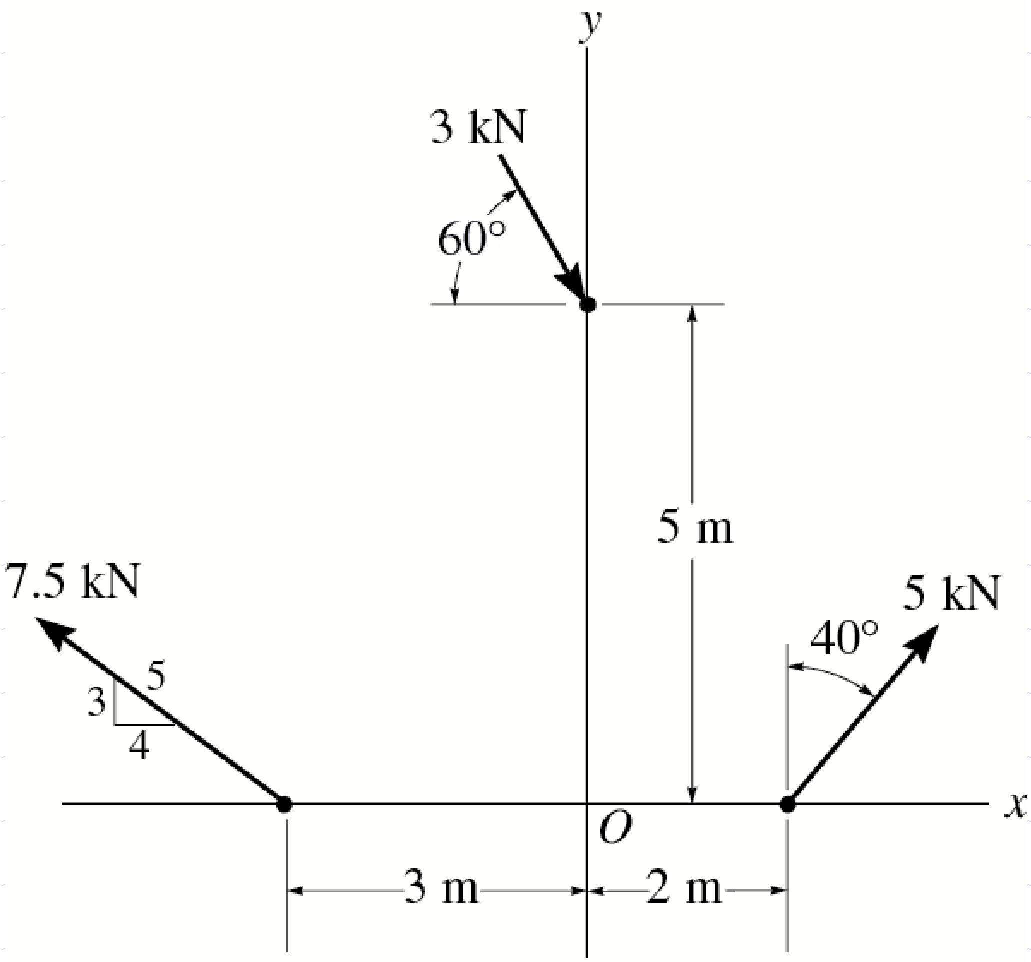
$$(+ccw)\quad M_{RE} = \sum M_E = (500 \sin 60^o)(4) + (500 \cos 60^o)(0) - (100)(0.5) - (200)(2.5) = 1182.1 \text{ N}\cdot\text{m}$$

$$233\,d + 350\,(0) = 1182.1 \text{ N}\cdot\text{m}$$

$$d = 5.07 \text{ m}$$



Example 7



Replace the force system with an equivalent force system and specify a location (0,y) for a single equivalent force to be applied.

$$\sum F_x = 5(\sin 40^o) + 3\cos(60^o) - \tfrac{4}{5}\left(7.5\right) = -1.286 \quad kN$$

$$\sum F_y = 5(\cos 40^o) - 3\sin(60^o) + \tfrac{3}{5}\left(7.5\right) = 5.732 \quad kN$$

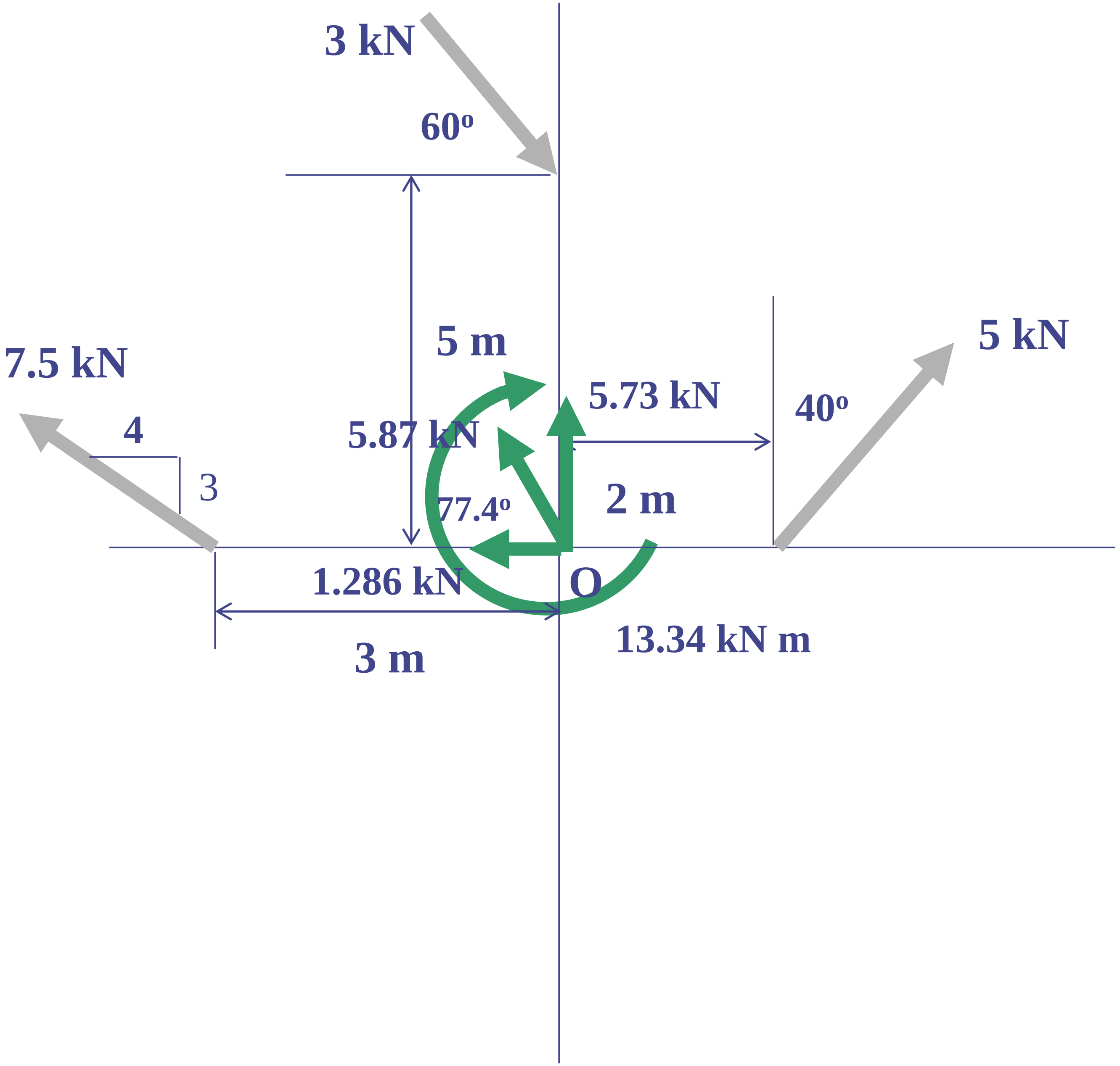
$$\sum M_O = -\tfrac{3}{5}\left(7.5\right)(3) + 5(\cos 40^o)(2)$$

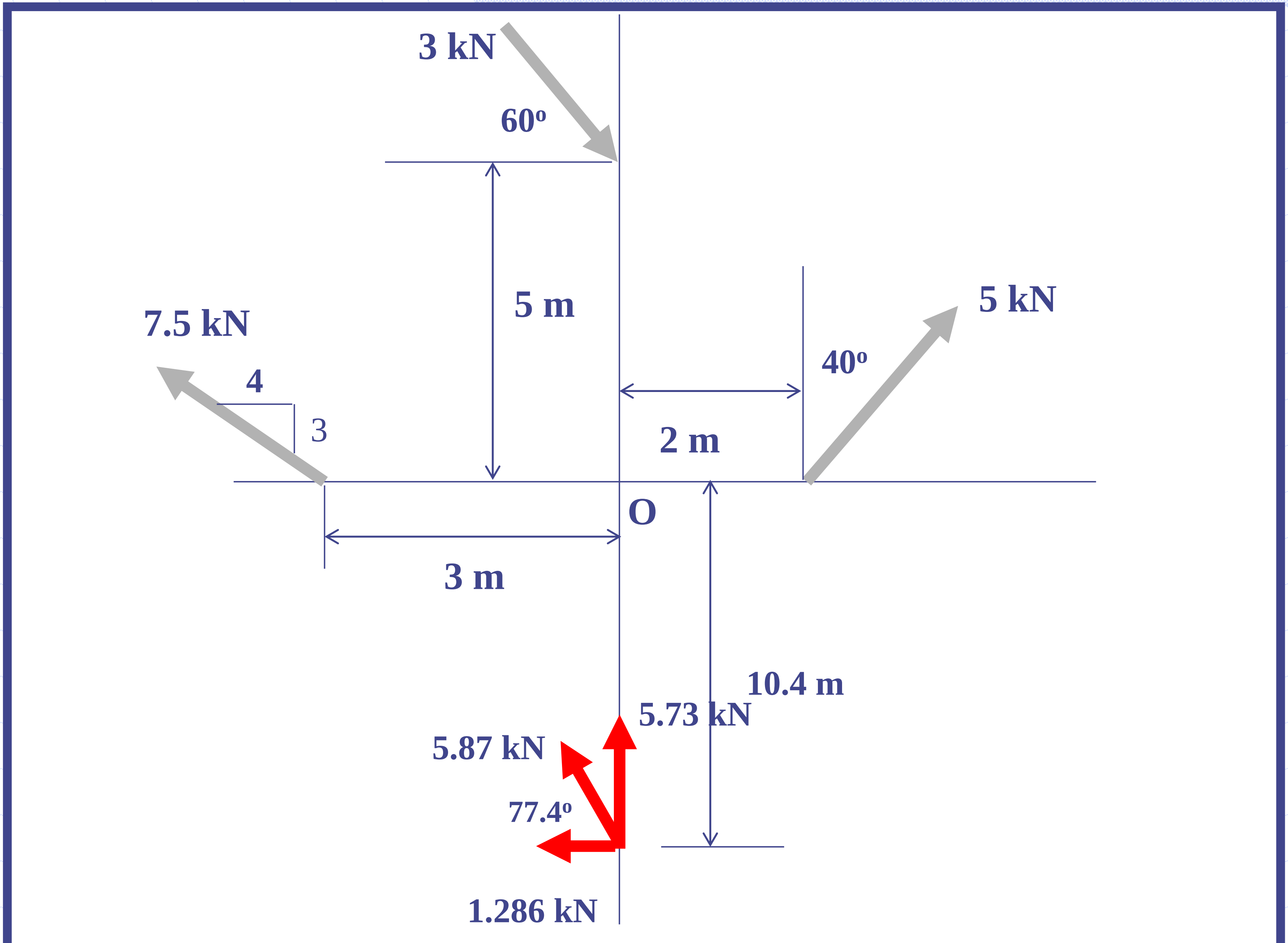
$$-3\cos(60^o)(5) = -13.34 \quad kN \cdot m$$

$$\left(1.286 \quad kN\right) y = 13.34 \quad kN \cdot m$$

$$y = 10.4 \quad m \; \textit{down}$$

$$y = \text{-}10.4m$$


3 kN
60º
5 m
7.5 kN
4
3
5.87 kN
5.73 kN
40º
5 kN
77.4º
2 m
1.286 kN
O
13.34 kN m
3 m



3 kN
60º
5 m
7.5 kN
4
3
5 kN
40º
2 m
O
3 m
10.4 m
5.73 kN
5.87 kN
77.4º
1.286 kN