Chapter 2 – Part 1
Forces

STATICS, AGE-1330

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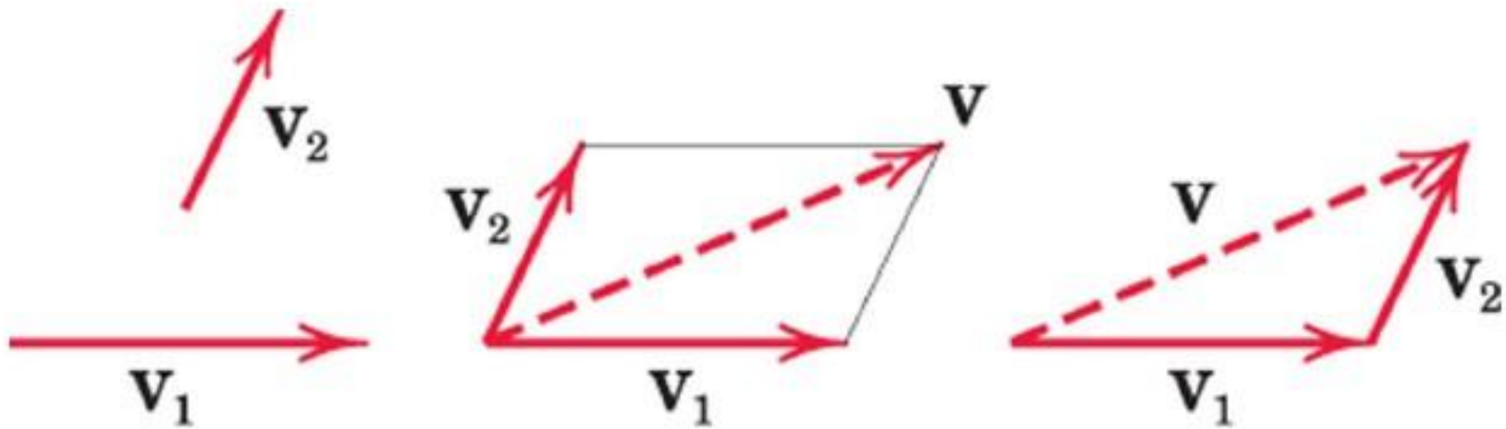
Mechanics: Scalars and Vectors

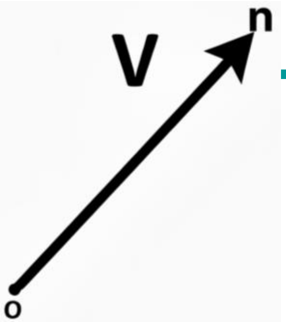
- Scalar
  - Only **magnitude** is associated with it
    - *e.g.*, time, volume, density, <u>speed</u>, energy, mass etc.
- Vector
  - Possess **direction** as well as **magnitude**
  - Parallelogram law of addition (and the triangle law)
  - *e.g.*, displacement, velocity, acceleration etc.

Mechanics: Scalars and Vectors

- Laws of vector addition
  - Equivalent vector $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ (Vector Sum)



Mechanics: Scalars and Vectors



A Vector **V** can be written as: $\mathbf{V} = V\mathbf{n}$

V = magnitude of **V**

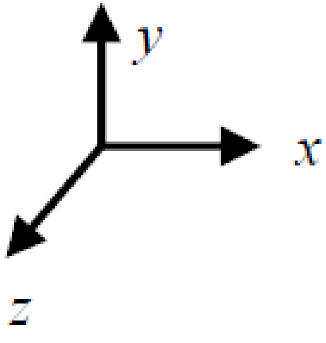
n = unit vector whose magnitude is one and whose direction coincides with that of **V**

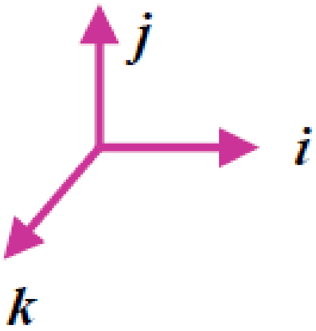
Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

$$\frac{\mathbf{V}}{V} = \mathbf{n}$$

Vectors represented by Bold and Non-Italic letters (**V**)

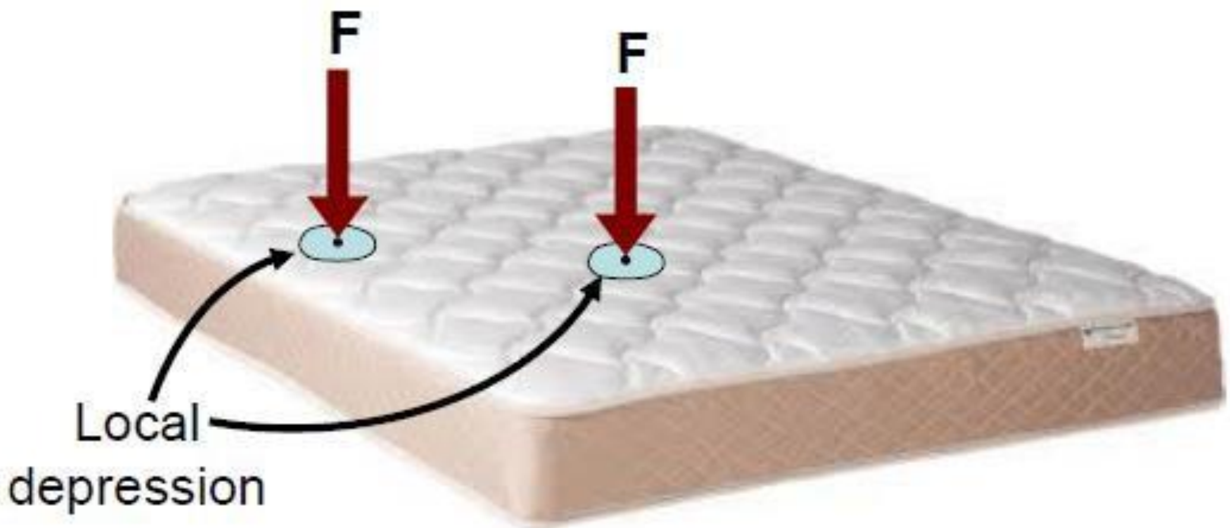
Magnitude of vectors represented by Non-Bold, Italic letters (V)





Types of Vectors: Fixed Vector

- **Fixed Vector**
  - Constant magnitude and direction
    - **Unique point of application**
  - *e.g.*, force on a deformable body

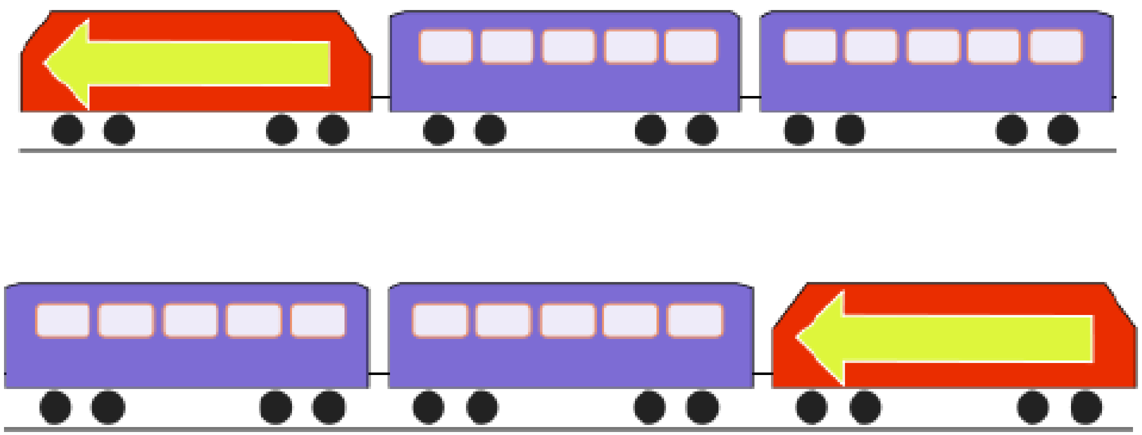


  - *e.g.*, force on a given particle

Types of Vectors: Sliding Vector

- **Sliding Vector**
  - Constant magnitude and direction
    - **Unique line of action**
      - "**Slide**" along the line of action
    - **No unique point of application**

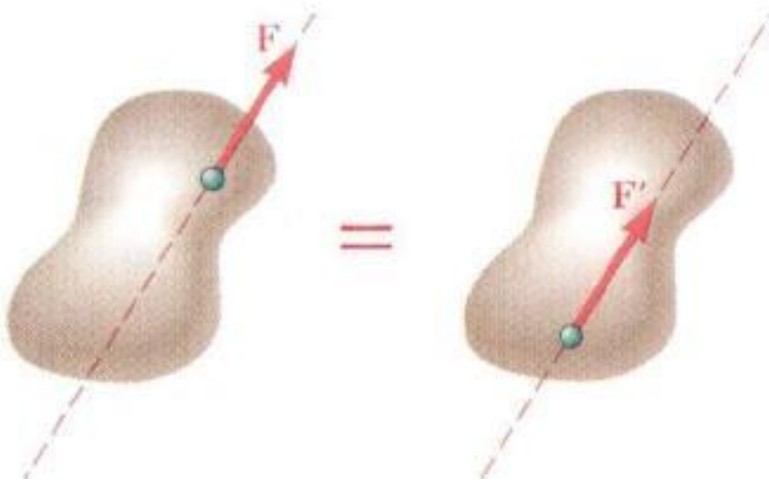
Force on coach **F**

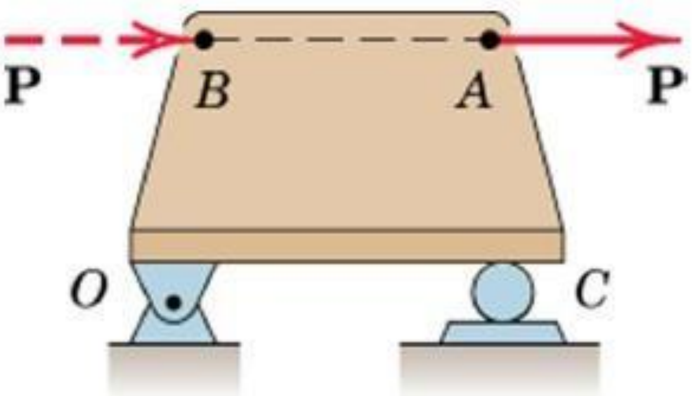


Force on coach **F**

Types of Vectors: Sliding Vector

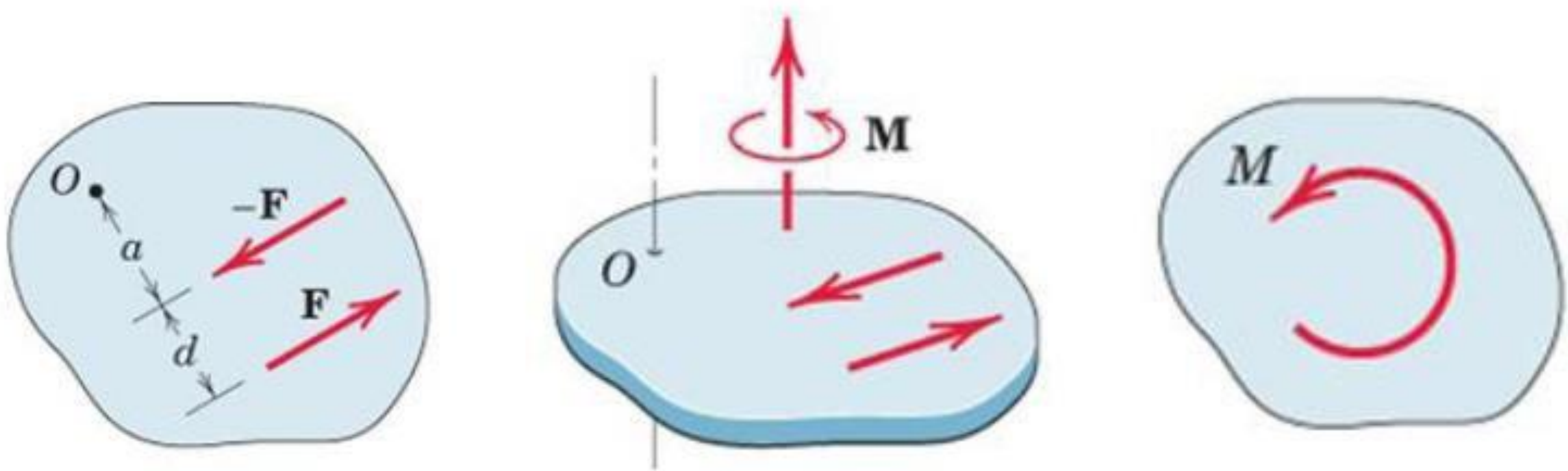
- **Sliding Vector**
  - Principle of Transmissibility
    - **Application of force** at **any point** along a **particular line of action**
      - **No change** in **resultant external effects** of the force





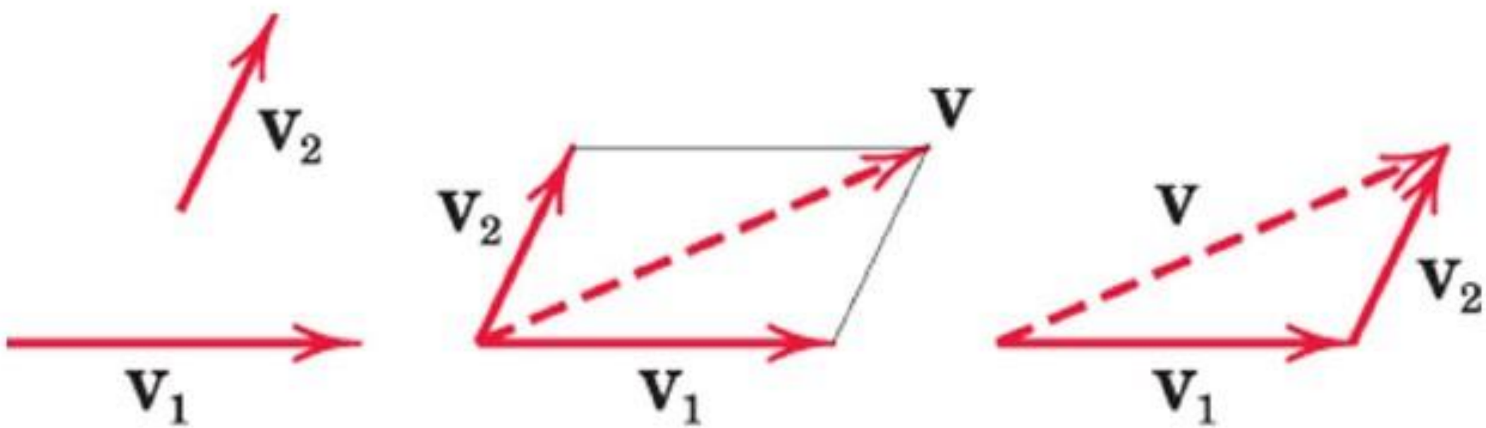
Types of Vectors: Free Vector

- **Free Vector**
  - Freely movable in space
    - **No unique line of action**
      - **No unique point of application**
  - *e.g.*, moment of a couple



Vectors: Rules of addition

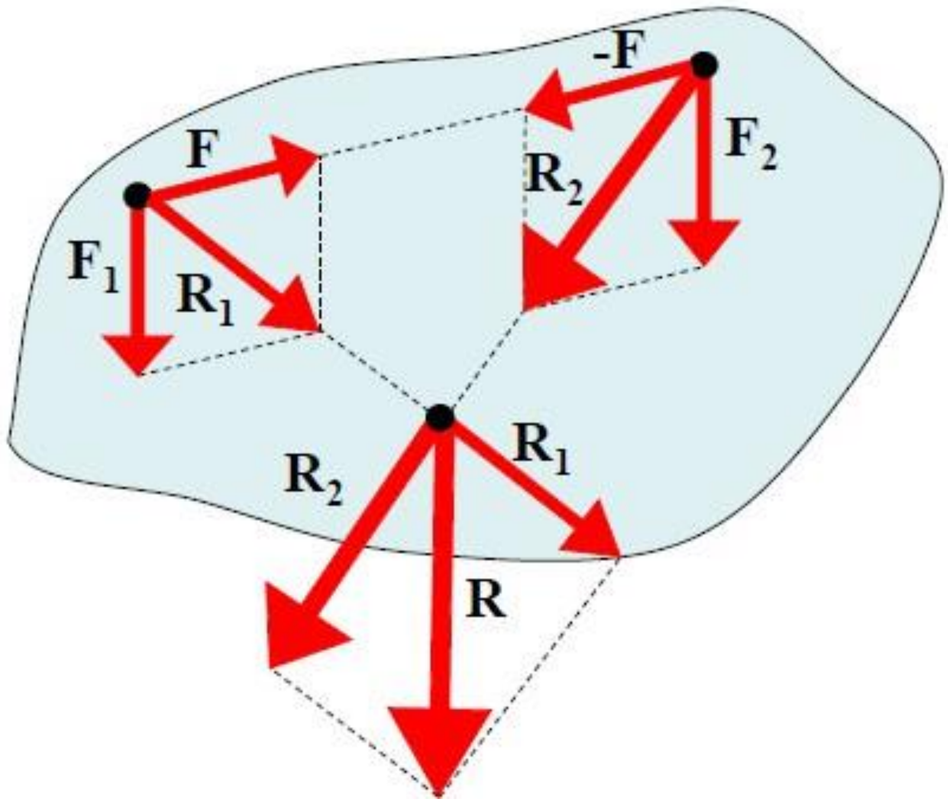
- **Parallelogram Law**
  - Equivalent vector represented by the diagonal of a parallelogram
    - $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ *(Vector Sum)*
    - $V \neq V_1 + V_2$ (Scalar sum)



Vectors: Parallelogram law of addition

- Addition of two parallel vectors

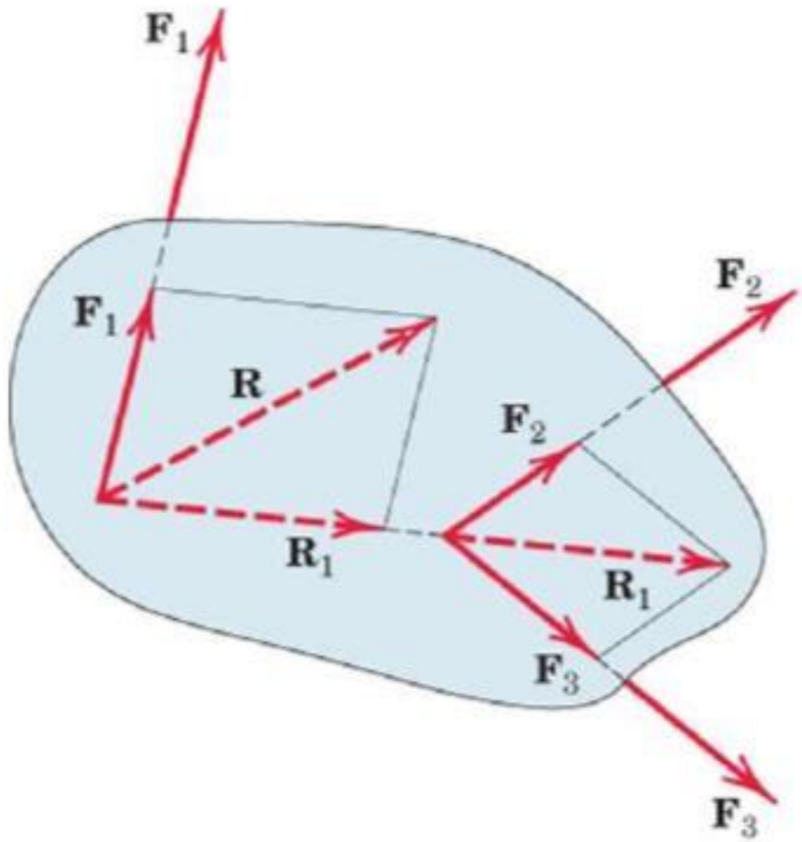
$$\mathbf{F_1 + F_2 = R}$$



Vectors: Parallelogram law of addition

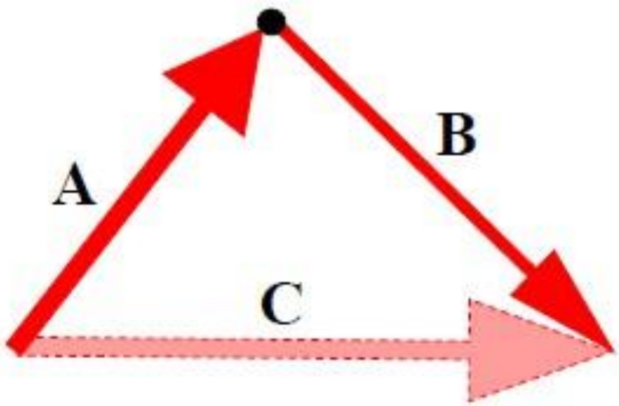
- Addition of 3 vectors

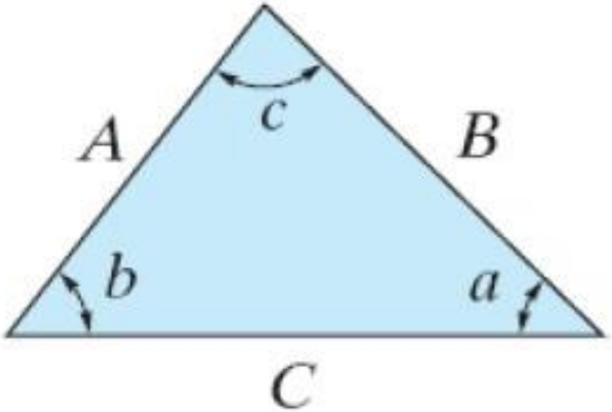
$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{R}$$



Vectors: Rules of addition

- **Trigonometric Rule**
  - Law of Sines
  - Law of Cosine





Sine law:

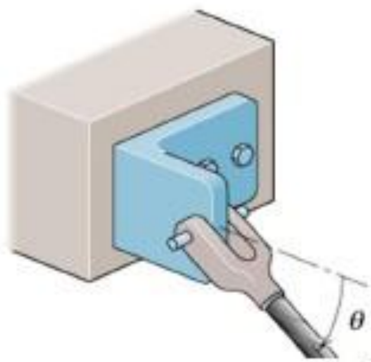
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

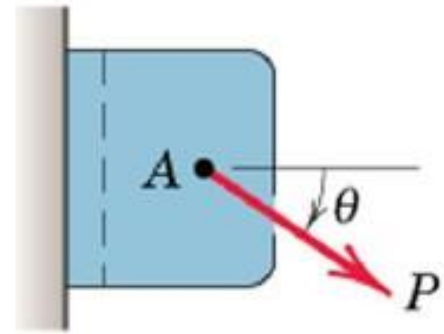
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Force Systems

Force Systems



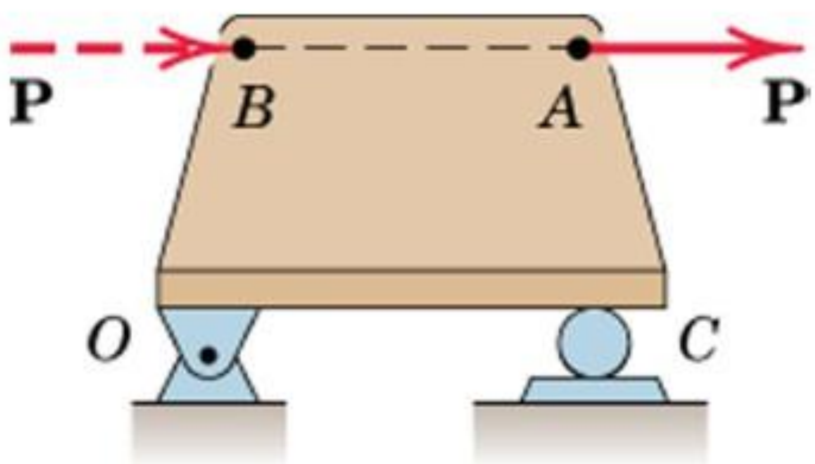


Cable Tension **P**

- **Force**: Represented by vector
  - Magnitude, direction, point of application
  - **P**: fixed vector (or sliding vector??)
  - External Effect
    - Applied force; Forces exerted by bracket, bolts, Foundation (reactive force)

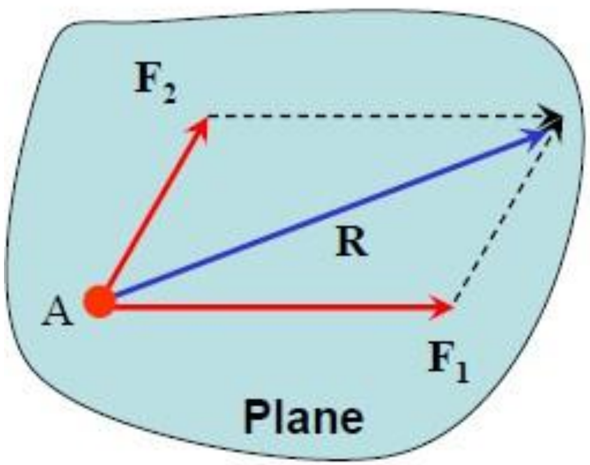
Force Systems

- **Rigid Bodies**
  - External effects only
    - **Line of action** of force is **important**
      - Not its point of application
      - Force as **sliding vector**



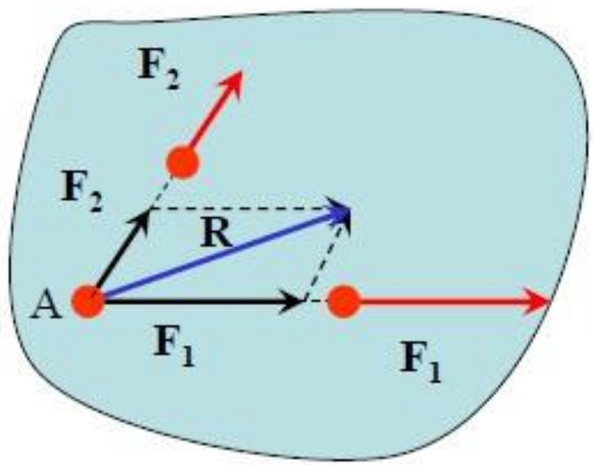
Force Systems

- **Concurrent forces**
  - Lines of action intersect at a point



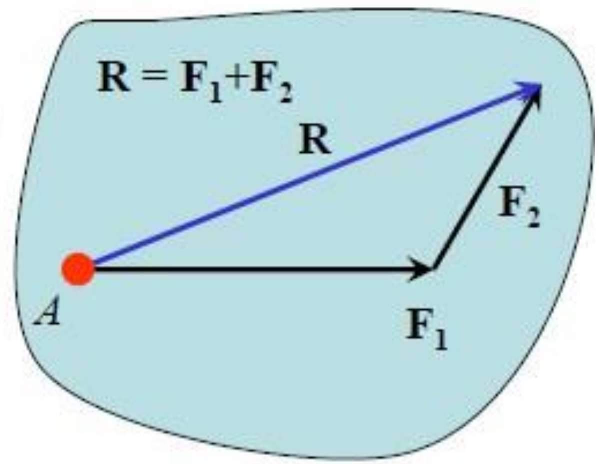
Concurrent Forces

$\mathbf{F_1}$ and $\mathbf{F_2}$



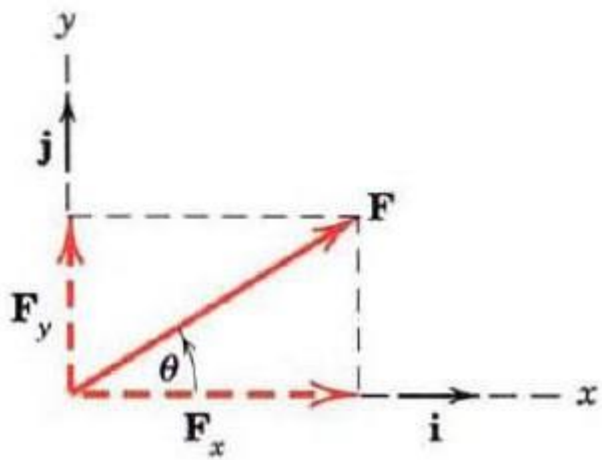
Principle of

Transmissibility



$\mathbf{R = F_1 + F_2}$

Rectangular components of 2-D Force System



$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

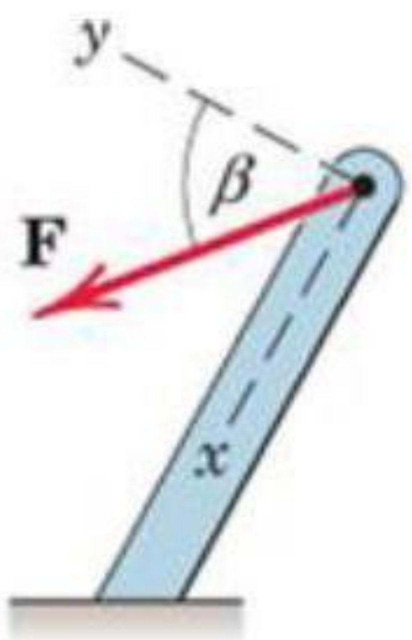
$$\mathbf{F} = F \angle \theta$$

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^2 + F_y^2}$$

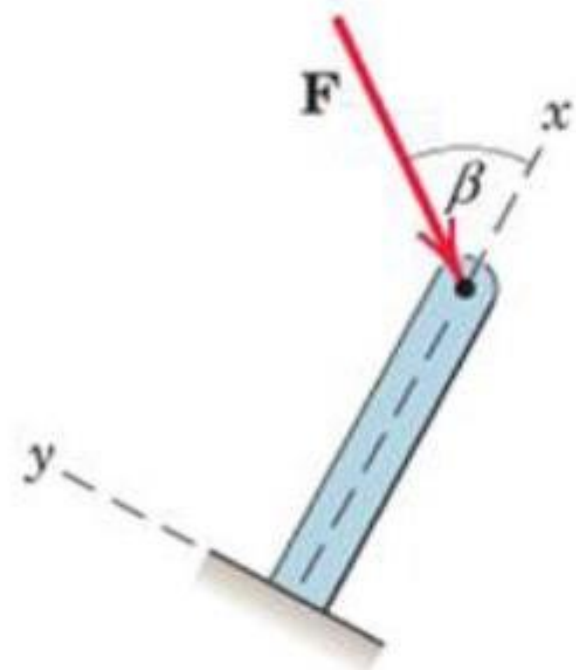
$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

Components of a Force

- Examples



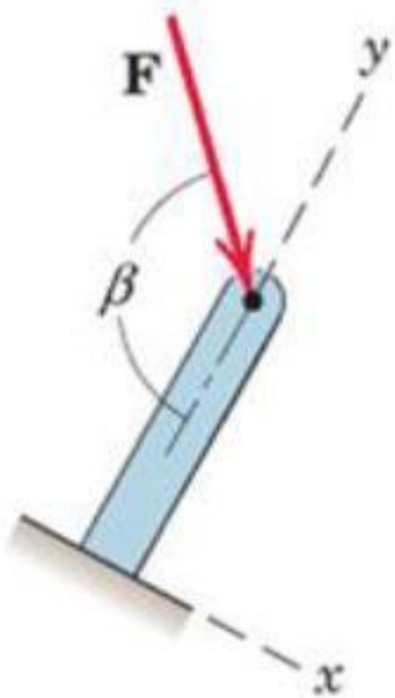
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



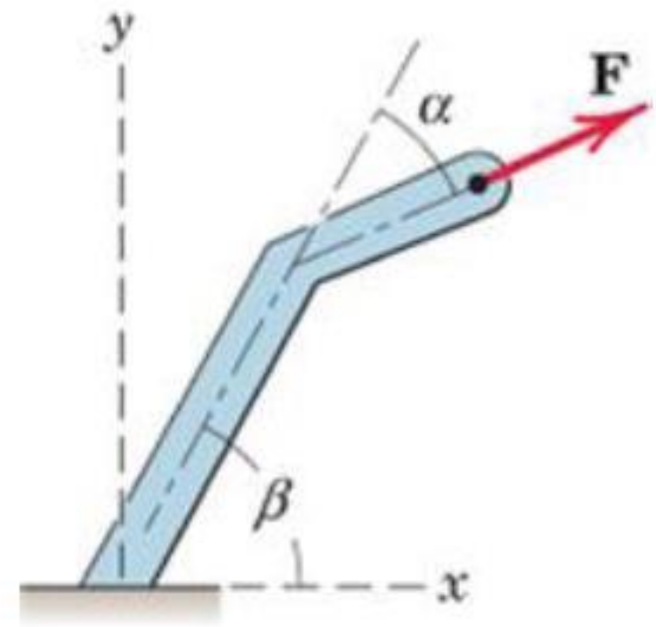
$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

Components of a Force

- Examples



$$F_x = F\sin(\pi - \beta)$$
$$F_y = -F\cos(\pi - \beta)$$

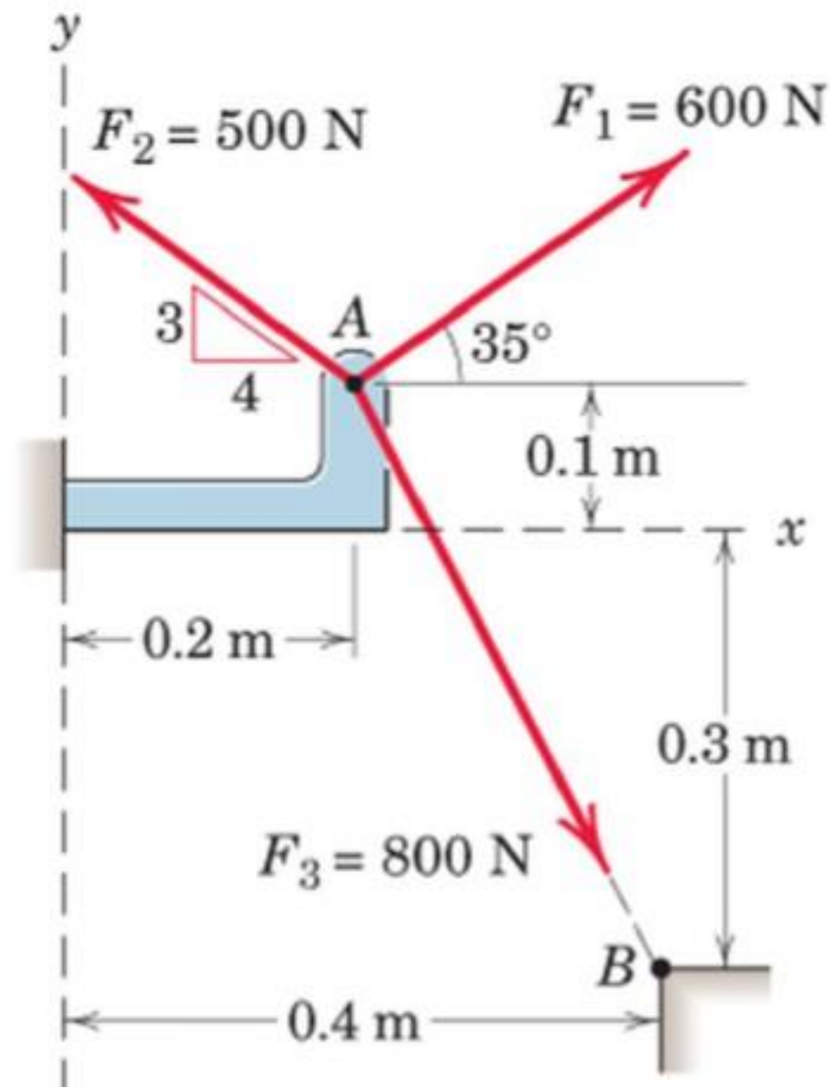


$$F_x = F\cos(\beta - \alpha)$$
$$F_y = F\sin(\beta - \alpha)$$

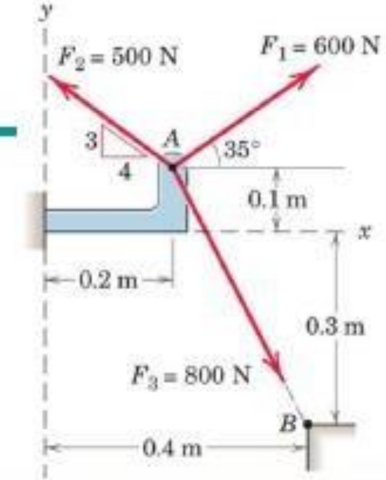
Components of a Force

Example 1:

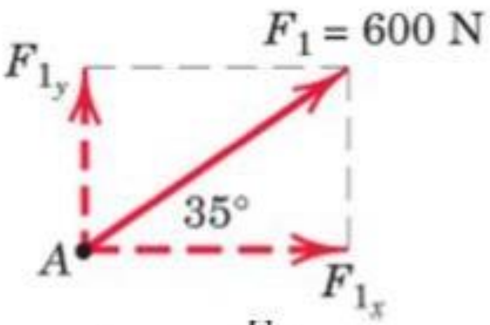
Determine the x and y scalar components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ acting at point A of the bracket

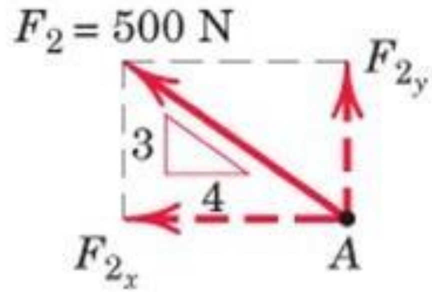


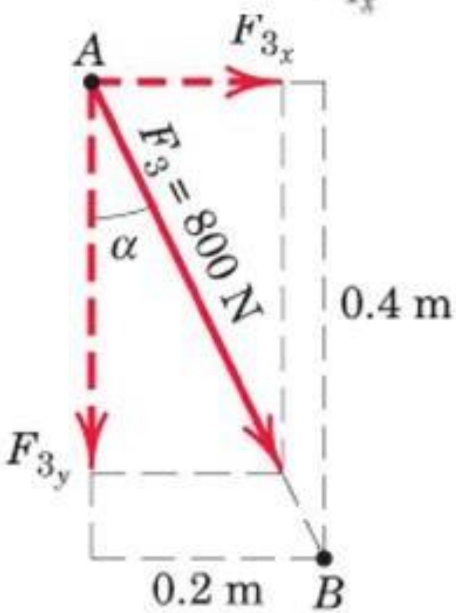
Components of Force



Solution:







$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

Components of Force

Alternative Solution: Scalar components of $\mathbf{F}_3$ can be obtained by writing $\mathbf{F}_3$ as a magnitude times a unit vector $\mathbf{n}_{\mathrm{AB}}$ in the direction of the line segment AB.

Unit vector can be formed by dividing any vector, such as the geometric position vector by its length or magnitude.

$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

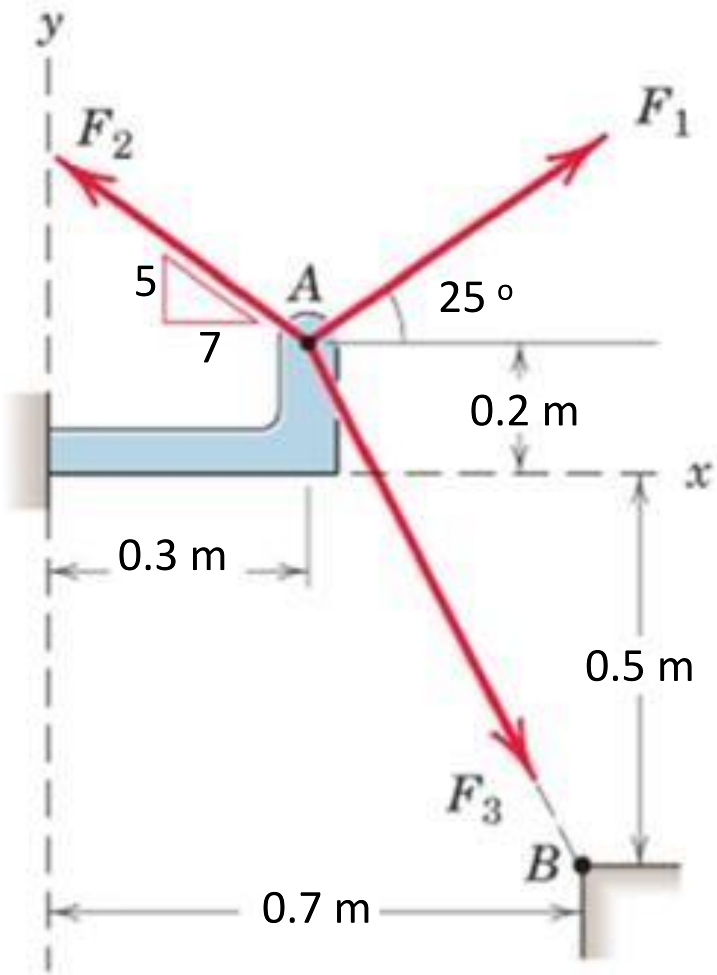
$$= 800[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j} \text{ N}$$

$$F_{3_x} = 358 \text{ N}$$

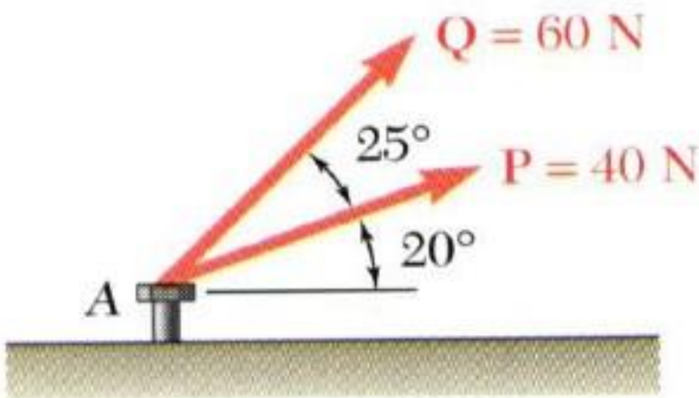
$$F_{3_y} = -716 \text{ N}$$

Determine the components of each force
Evaluate the resultant in x and y directions



Components of Force

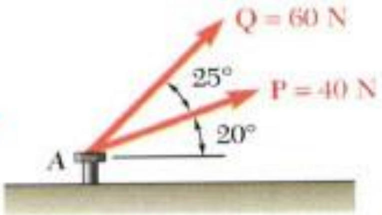
Example 2: The two forces act on a bolt at A. Determine their resultant.



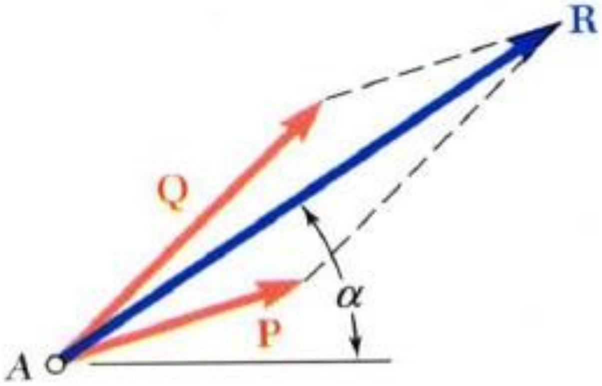
- **Graphical solution –**
- Construct a parallelogram with sides in the same direction as P and Q and lengths in proportion.
- Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

- **Trigonometric solution**
- Use the law of cosines and law of sines to find the resultant.

Components of Force

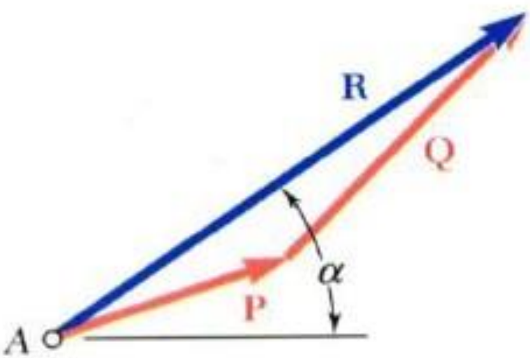


Solution:



- **Graphical solution** - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$$\mathbf{R} = 98\,\mathrm{N} \quad \alpha = 35^\circ$$

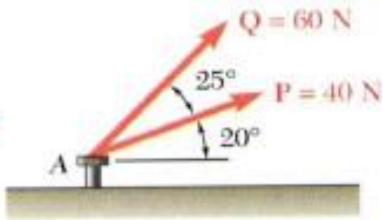


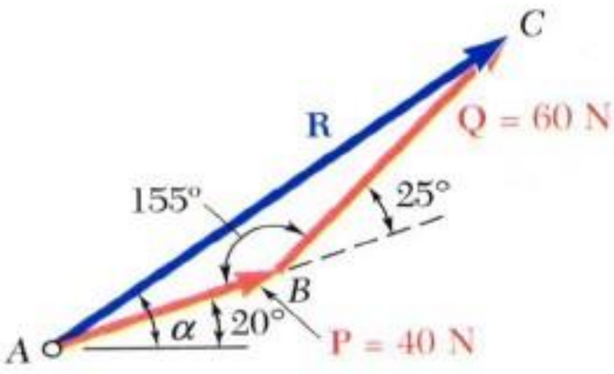
- **Graphical solution** - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\mathbf{R} = 98\,\mathrm{N} \quad \alpha = 35^\circ$$

Components of Force

Trigonometric Solution:



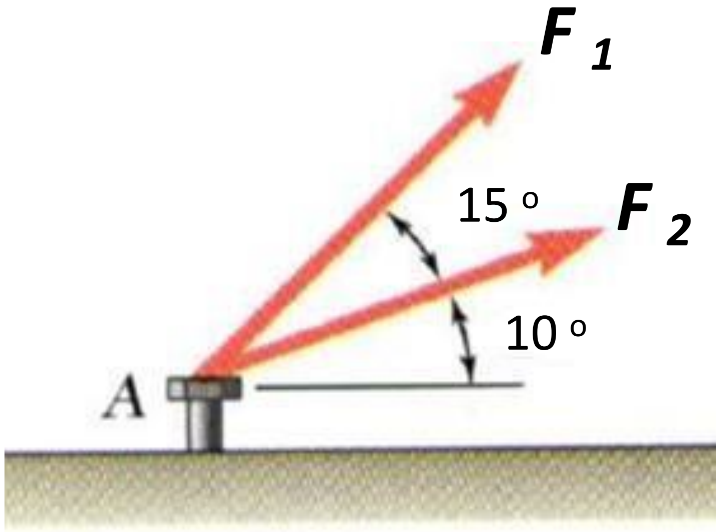


$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

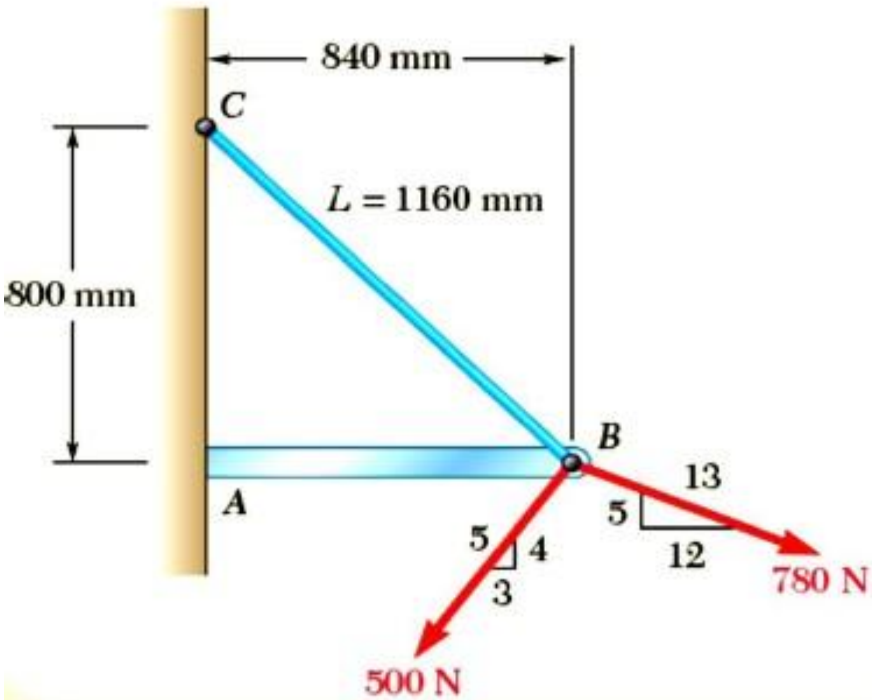
$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Determine the resultant force if F1 = F2 = 200 N.



Components of Force

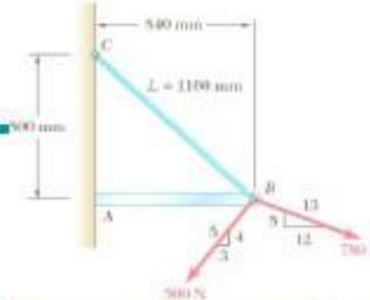
Example 3: Tension in cable *BC* is 725 N; determine the resultant of the three forces exerted at point *B* of beam *AB*.



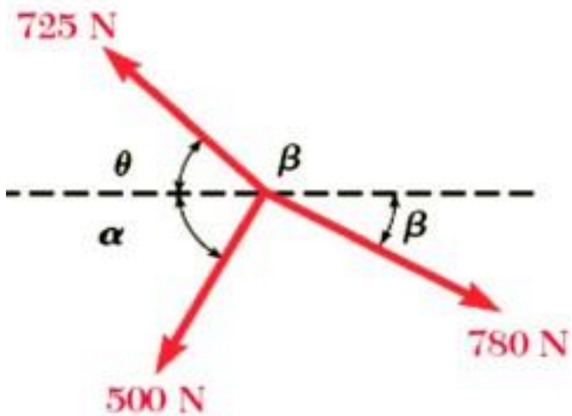
Solution:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force



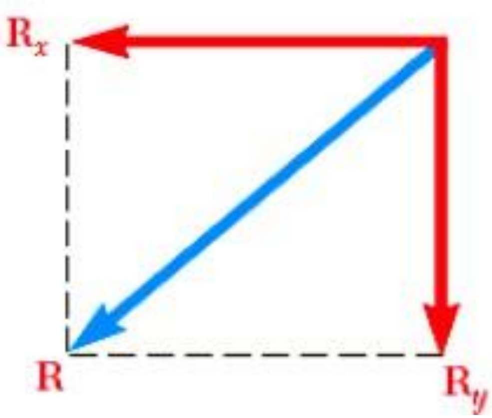
Solution



- Resolve each force into rectangular components.

Magnitude, N	x Component, N	y Component, N
725	−525	500
500	−300	− 400
780	720	− 300
	$R_x = -105$	$R_y = -200$

$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$ $\mathbf{R} = (-105\ \text{N})\mathbf{i} + (-200\ \text{N})\mathbf{j}$

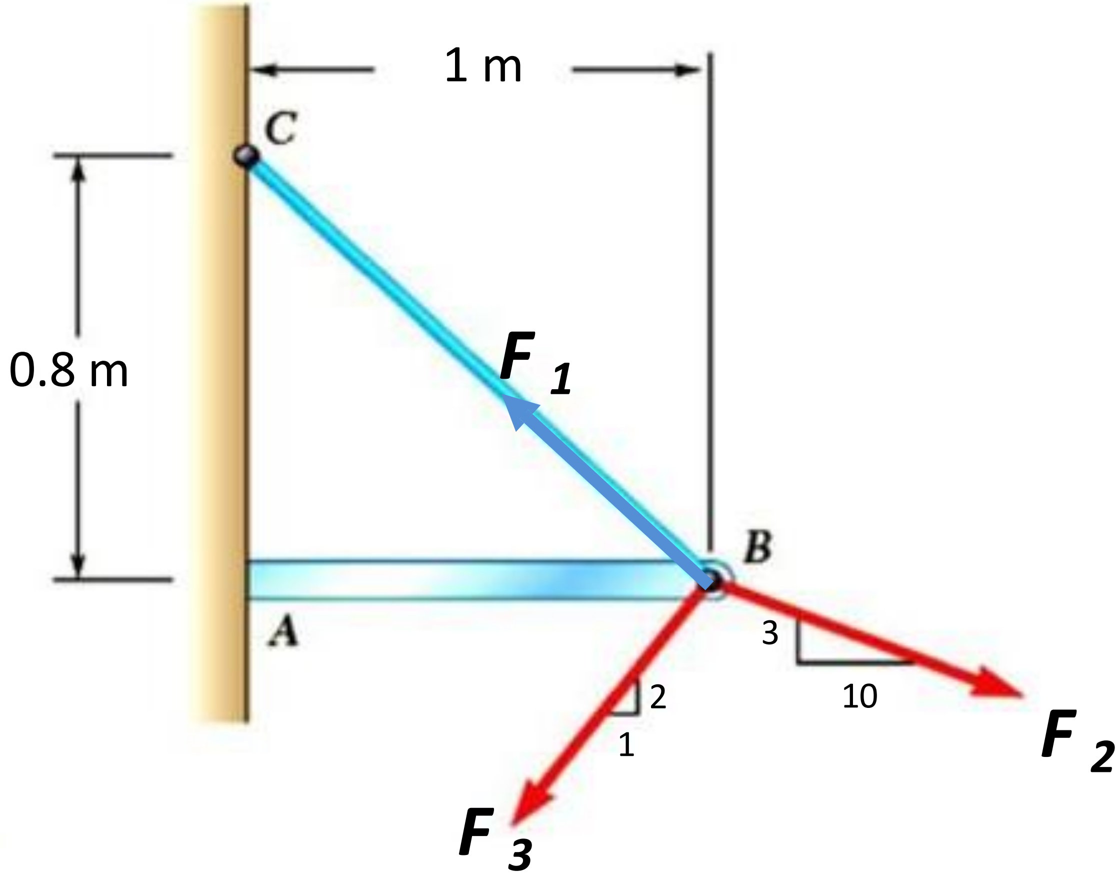


- Calculate the magnitude and direction.

$$\tan\alpha = \frac{-R_y}{-R_x} = \frac{200\,\text{N}}{105\,\text{N}} \quad \alpha = 62.3^\circ$$

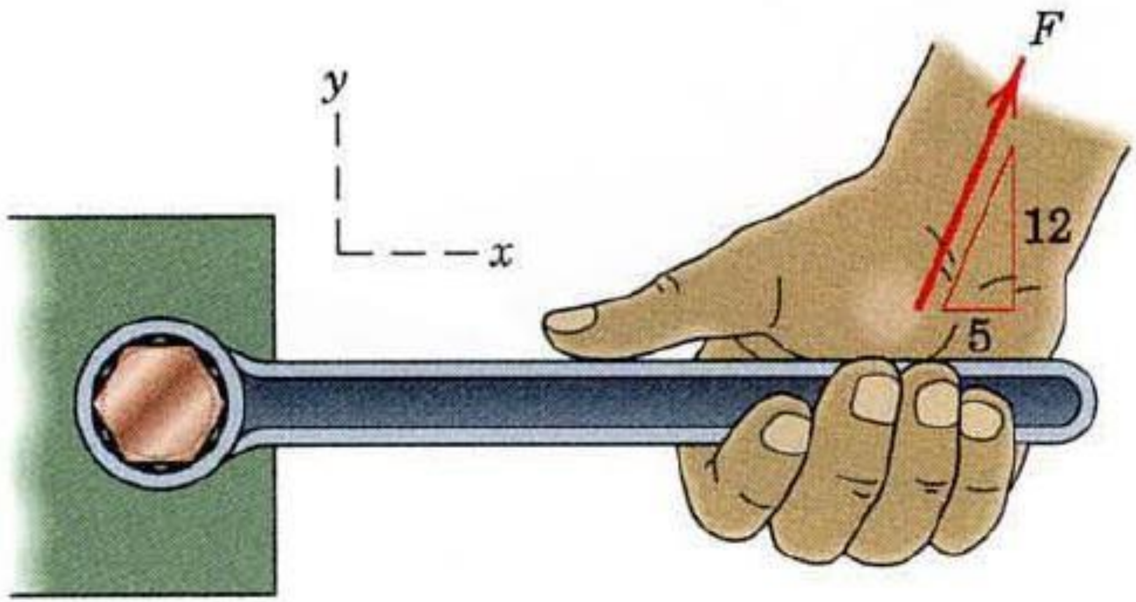
$$R = \sqrt{R_{x^2} + R_{y^2}} = 225.9\,\text{N} \qquad \measuredangle\ 62.3^\circ$$

If F1 = 1 kN, F2 = 900 N, F3 = 1.2 kN, compute the resultatnt force acting on point B.

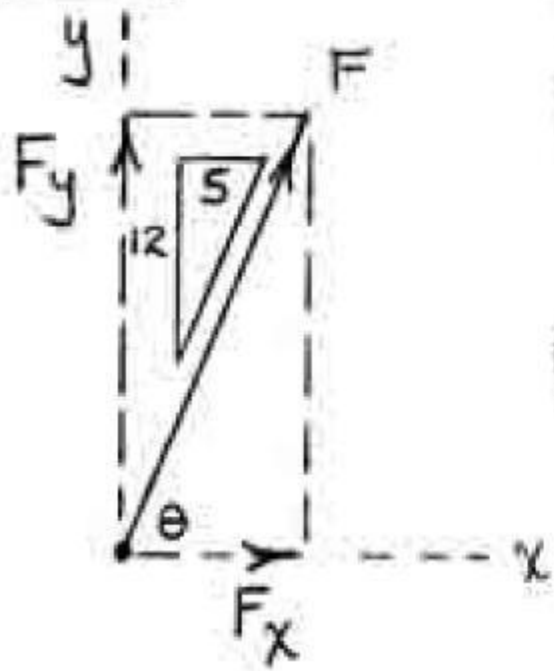


2/7 The y-component of the force **F** which a person exerts on the handle of the box wrench is known to be 70 lb. Determine the x-component and the magnitude of **F**.

Ans. $F_x = 29.2$ lb, $F = 75.8$ lb



2/7



$$\cos\theta = \frac{5}{13} , \quad \sin\theta = \frac{12}{13}$$

$$F_y = F\sin\theta = F\frac{12}{13} = 70$$

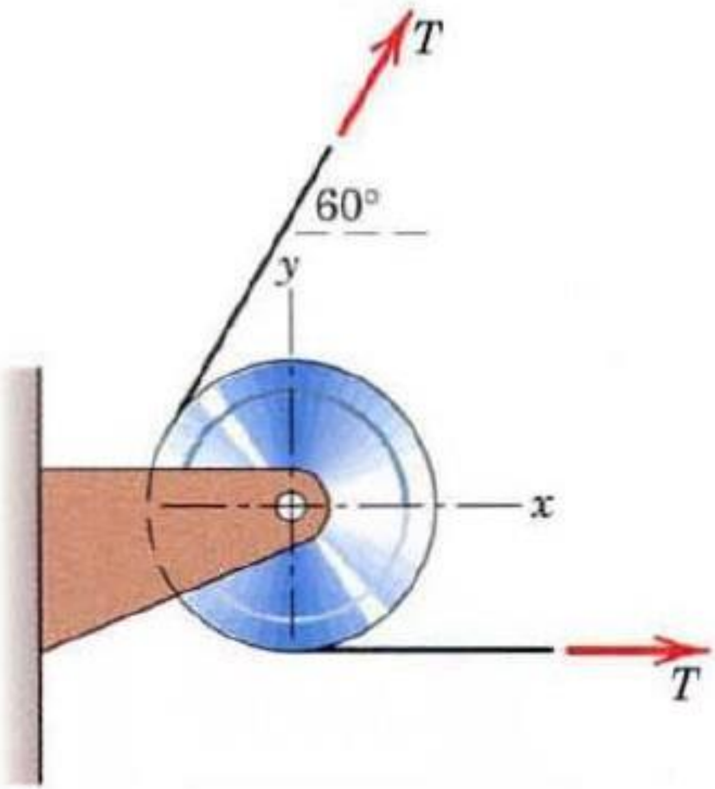
$$\underline{F = 75.8 \text{ lb}}$$

$$F_x = F\cos\theta = 75.8\left(\frac{5}{13}\right)$$

$$= \underline{29.2 \text{ lb}}$$

2/13 If the equal tensions T in the pulley cable are 400 N, express in vector notation the force $\mathbf{R}$ exerted on the pulley by the two tensions. Determine the magnitude of $\mathbf{R}$.

Ans. $\mathbf{R} = 600\mathbf{i} + 346\mathbf{j}$ N, $R = 693$ N



2/13

$$R_x = \Sigma F_x = 400 + 400 \cos 60^\circ = 600 \text{ N}$$

$$R_y = \Sigma F_y = 400 \sin 60^\circ = 346 \text{ N}$$

$$\Rightarrow \underline{R} = \underline{600\,\underline{i} + 346\,\underline{j} \text{ N}}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$