# Chapter 2 – Part 1
# Forces

***STATICS, AGE-1330***

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# 1. Introduction

# Definitions

- ***Scalar*** - A quantity characterized by a positive or negative number is called a scalar. Examples of scalars used in Statics are mass, volume or length.

# Definitions

***Vector*** - A quantity that has both magnitude and a direction. Examples of vectors used in Statics are position, force, and moment.

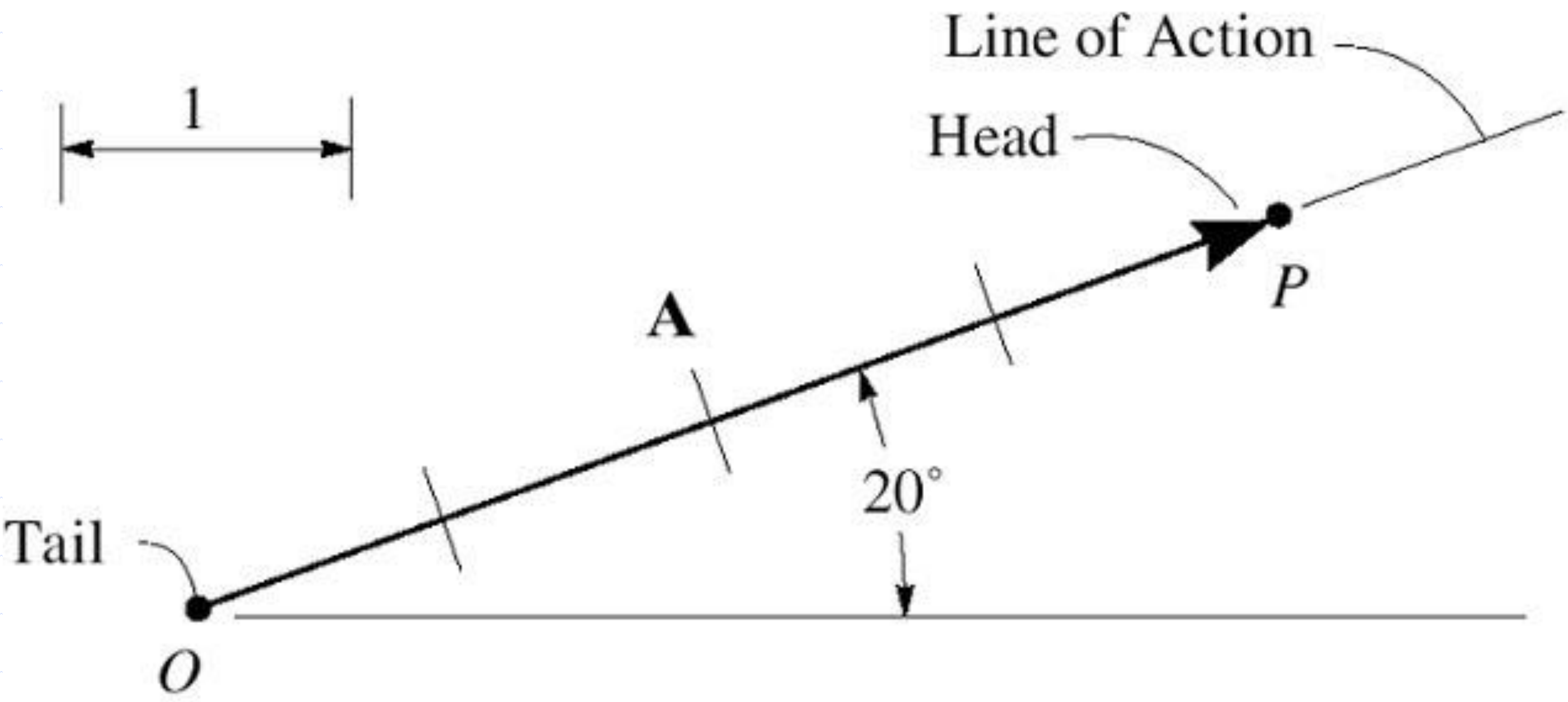
# Mechanics: Scalars and Vectors

- Scalar
  - Only **magnitude** is associated with it
    - *e.g.*, time, volume, density, <u>speed</u>, energy, mass etc.
- Vector
  - Possess **direction** as well as **magnitude**
  - Parallelogram law of addition (and the triangle law)
  - *e.g.*, displacement, velocity, acceleration etc.

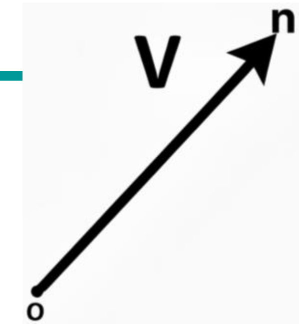
# Symbols

- Vectors are denoted by a letter with an arrow over it or a boldface letter such as **A**.

# Vector Definitions



# Mechanics: Scalars and Vectors




A Vector **V** can be written as: $\mathbf{V} = V\mathbf{n}$

$V$ = magnitude of **V**

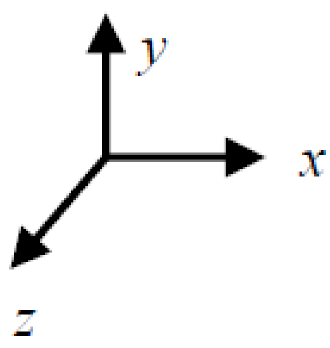
**n** = unit vector whose magnitude is one and whose direction coincides with that of **V**

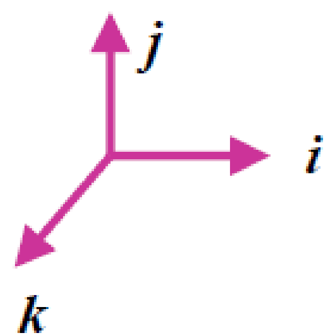
Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

$$\frac{\mathbf{V}}{V} = \mathbf{n}$$

Vectors represented by Bold and Non-Italic letters (**V**)

Magnitude of vectors represented by Non-Bold, Italic letters ($V$)

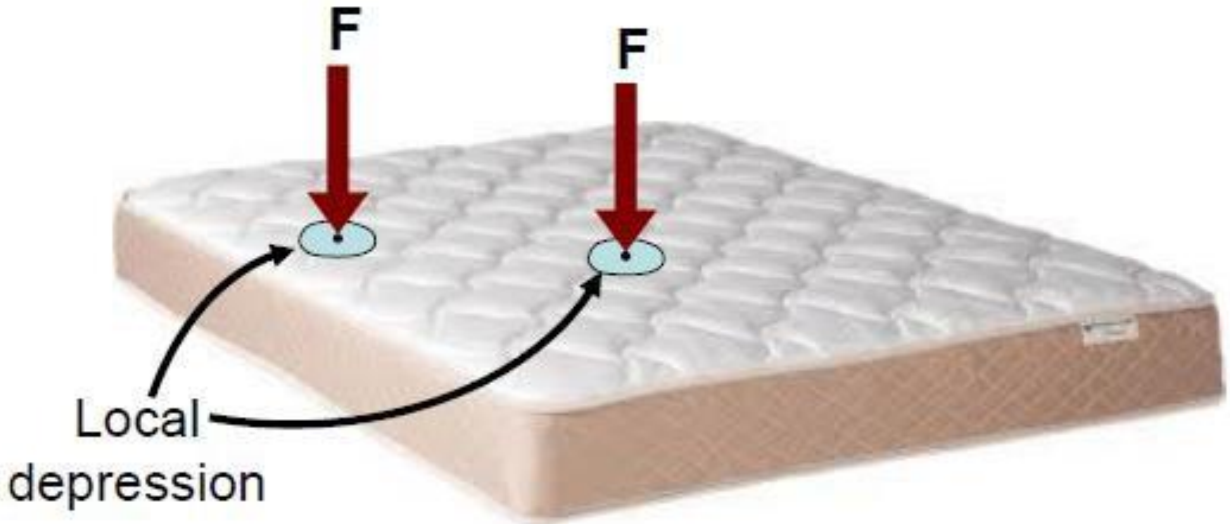


# 2. Types of Vectors

# Types of Vectors: Fixed Vector

- **Fixed Vector**
  - Constant magnitude and direction
    - **Unique point of application**
  - *e.g.*, force on a deformable body

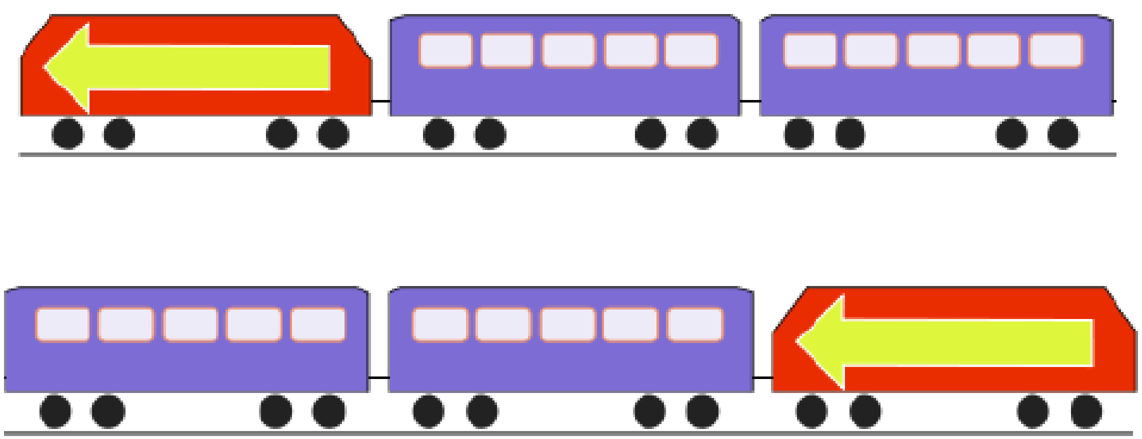



  - *e.g.*, force on a given particle

# Types of Vectors: Sliding Vector

- **Sliding Vector**
  - Constant magnitude and direction
    - **Unique line of action**
      - “**Slide**” along the line of action
    - **No unique point of application**

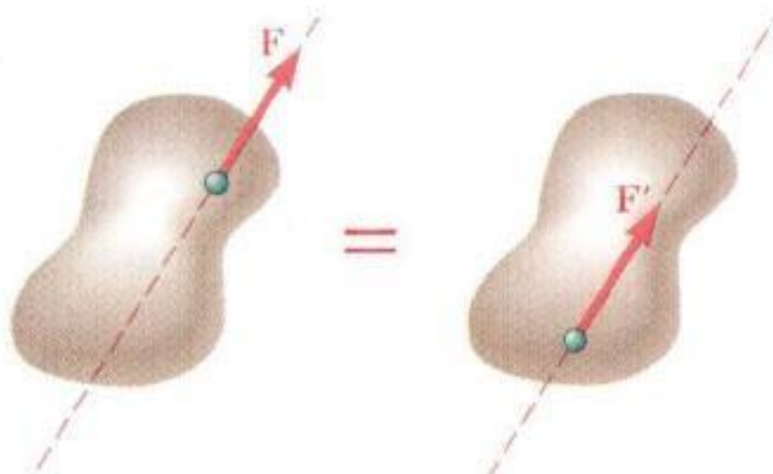
Force on coach **F**

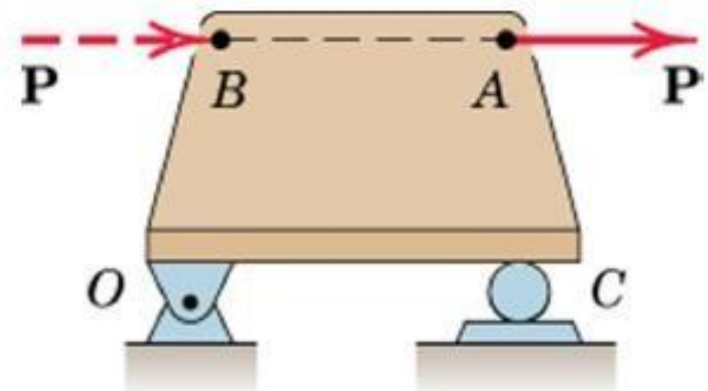


Force on coach **F**

# Types of Vectors: Sliding Vector

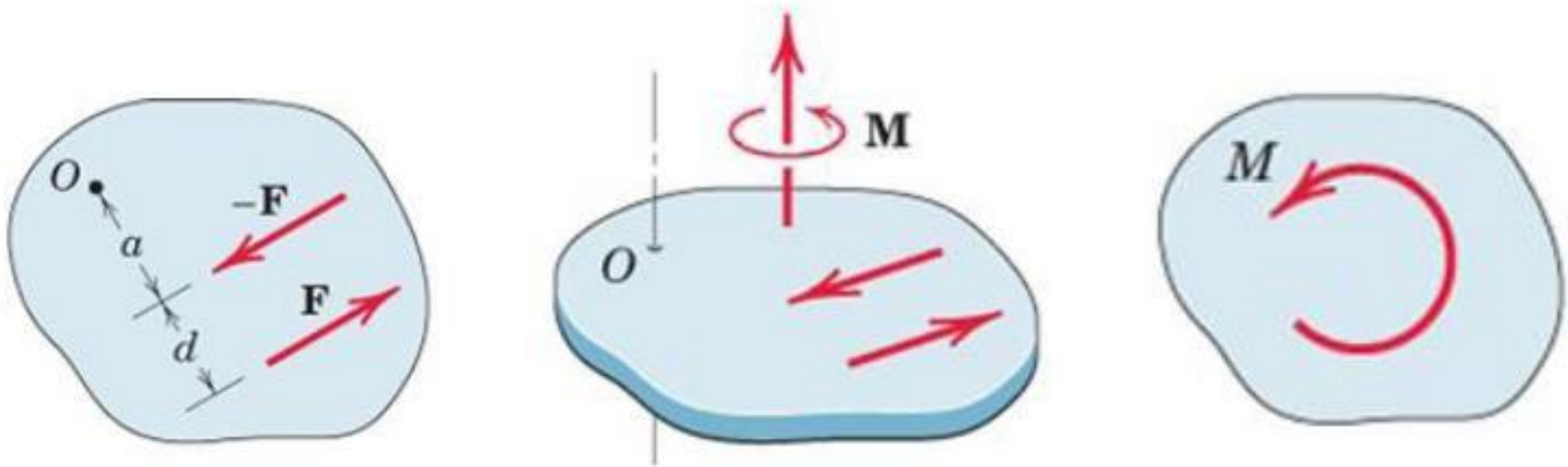
- **Sliding Vector**
  - Principle of Transmissibility
    - **Application of force** at **any point** along a **particular line of action**
      - **No change** in **resultant external effects** of the force



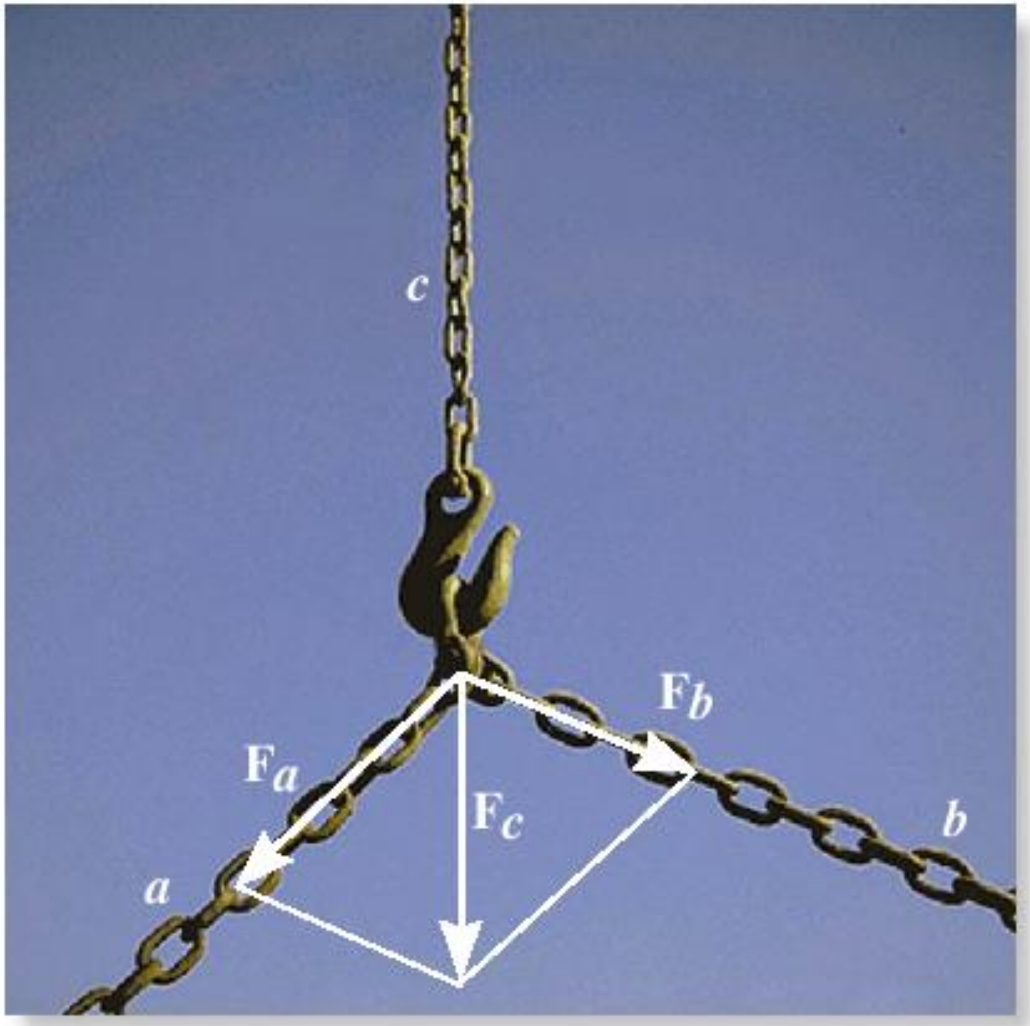



# Types of Vectors: Free Vector

- **Free Vector**
  - Freely movable in space
    - **No unique line of action**
      - **No unique point of application**
  - *e.g.*, moment of a couple

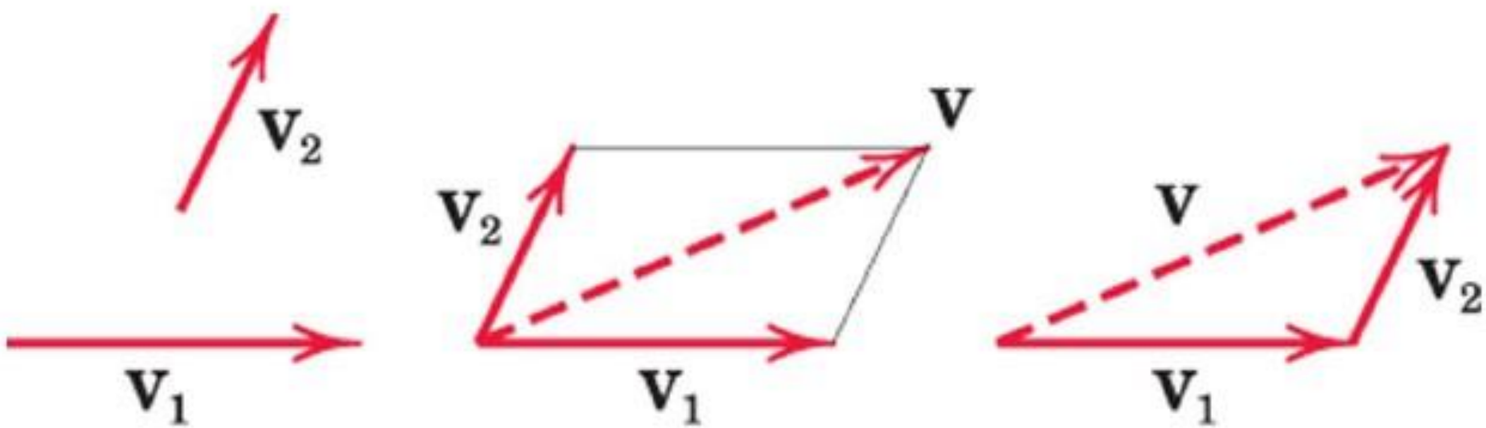


# 3. Vector Addition

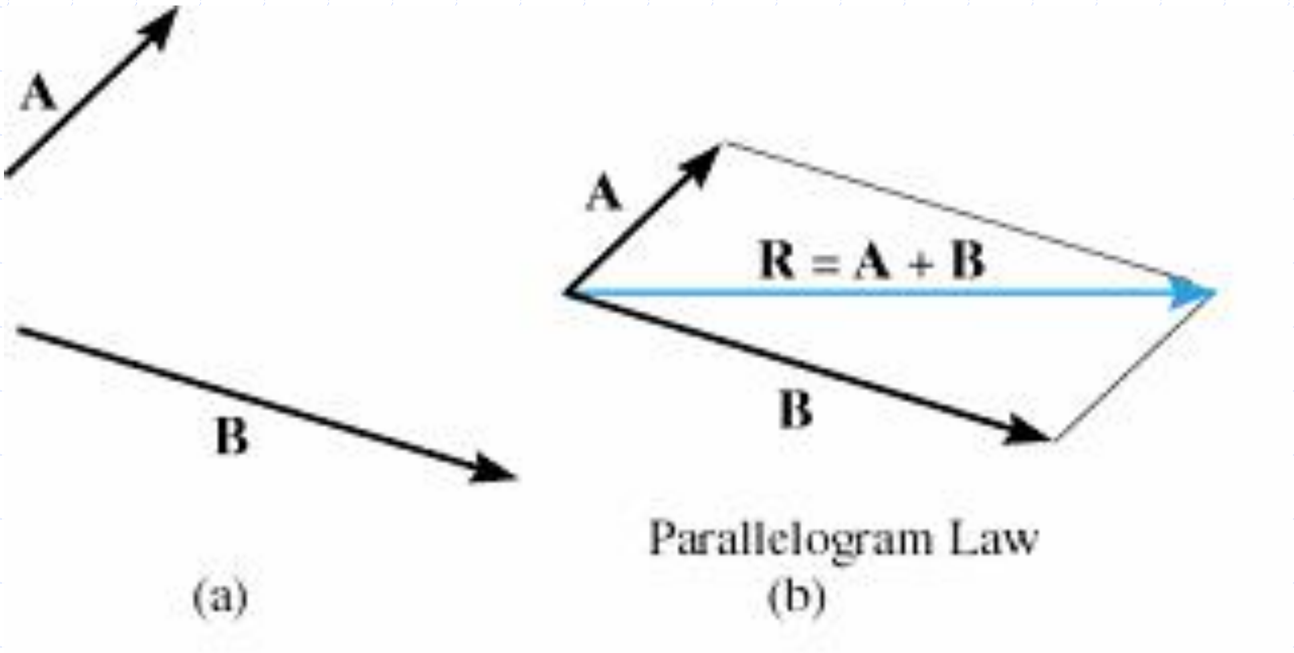

c
Fa
Fb
Fc
a
b

# Vectors: Rules of addition

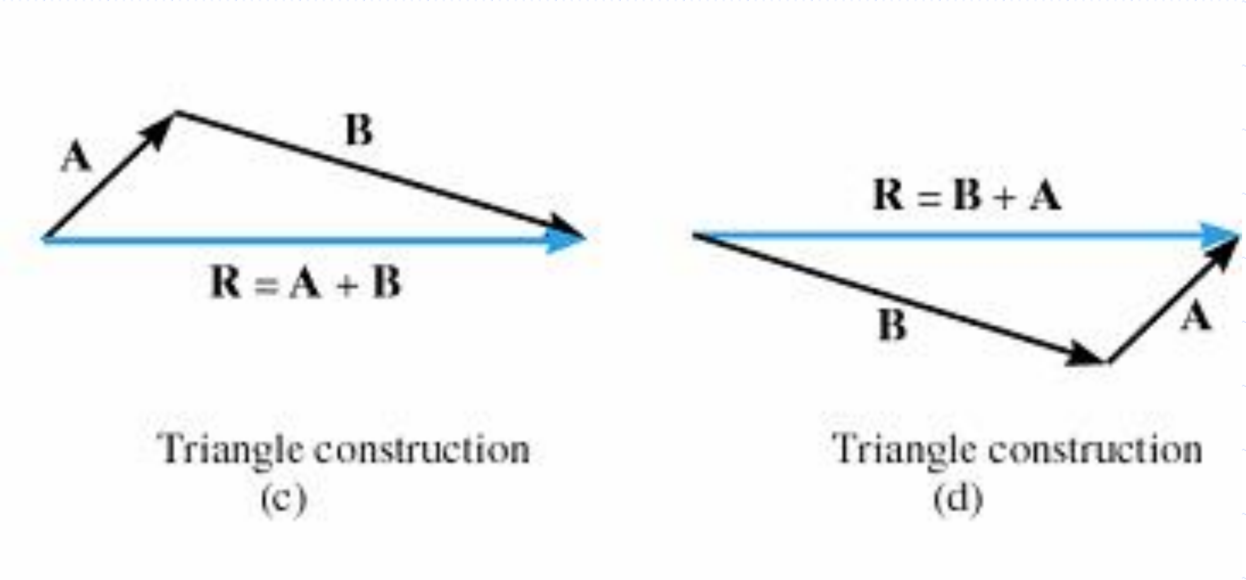
- **Parallelogram Law**
  - Equivalent vector represented by the diagonal of a parallelogram
    - $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ *(Vector Sum)*
    - $V \neq V_1 + V_2$ (Scalar sum)



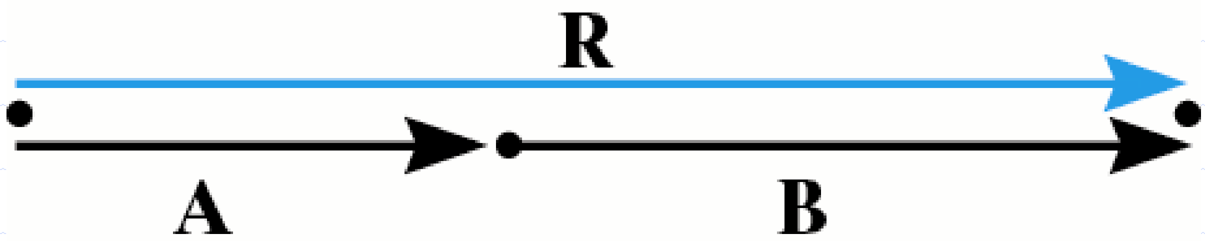
# Vector Addition



# Vector Addition



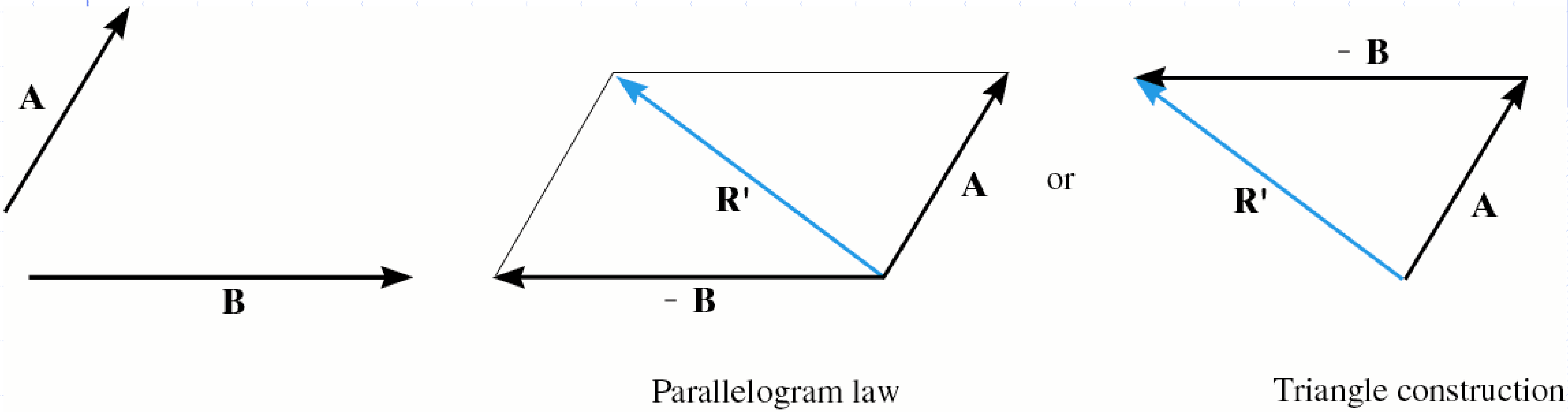
# Vector Addition




$$R = A + B$$

Addition of collinear vectors

# Vector Subtraction

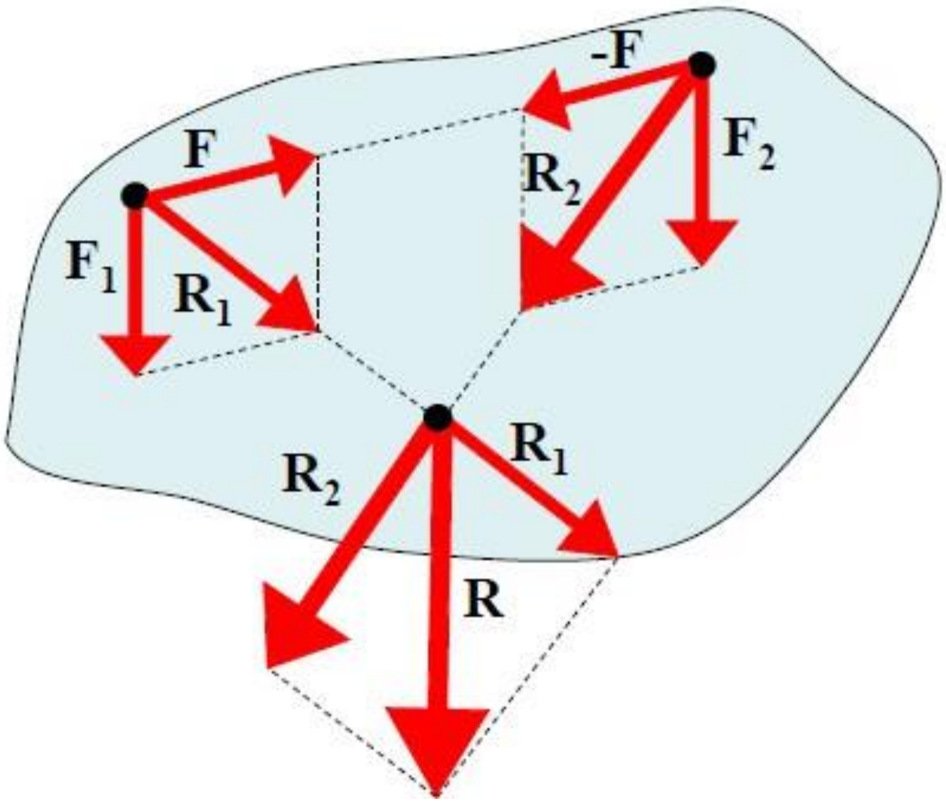



Vector Subtraction

# Vectors: Parallelogram law of addition

- Addition of two parallel vectors

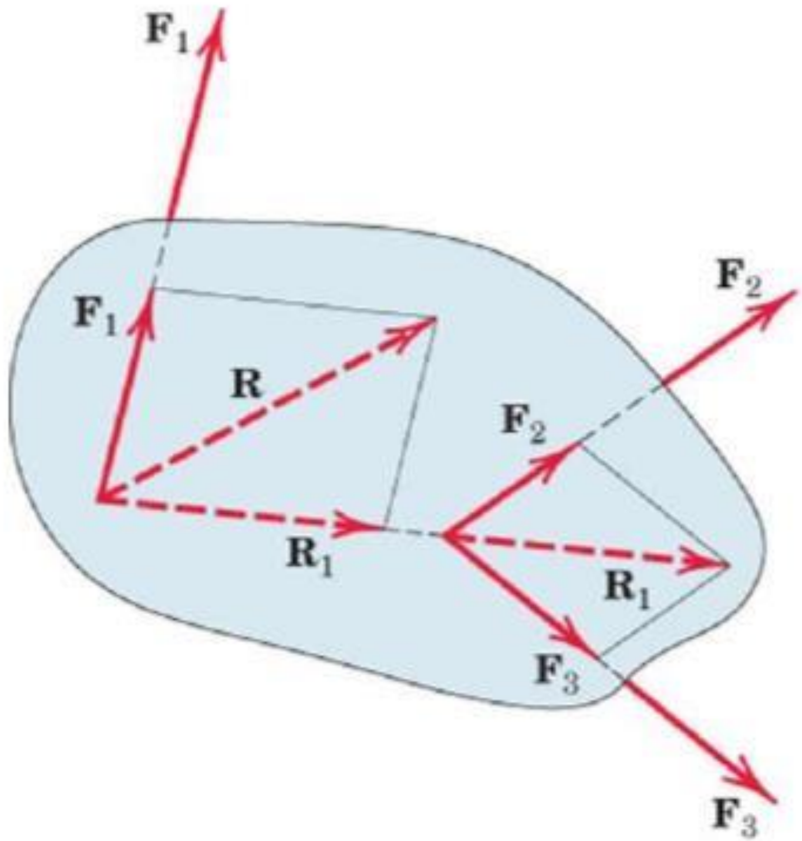
$$\mathbf{F_1} + \mathbf{F_2} = \mathbf{R}$$

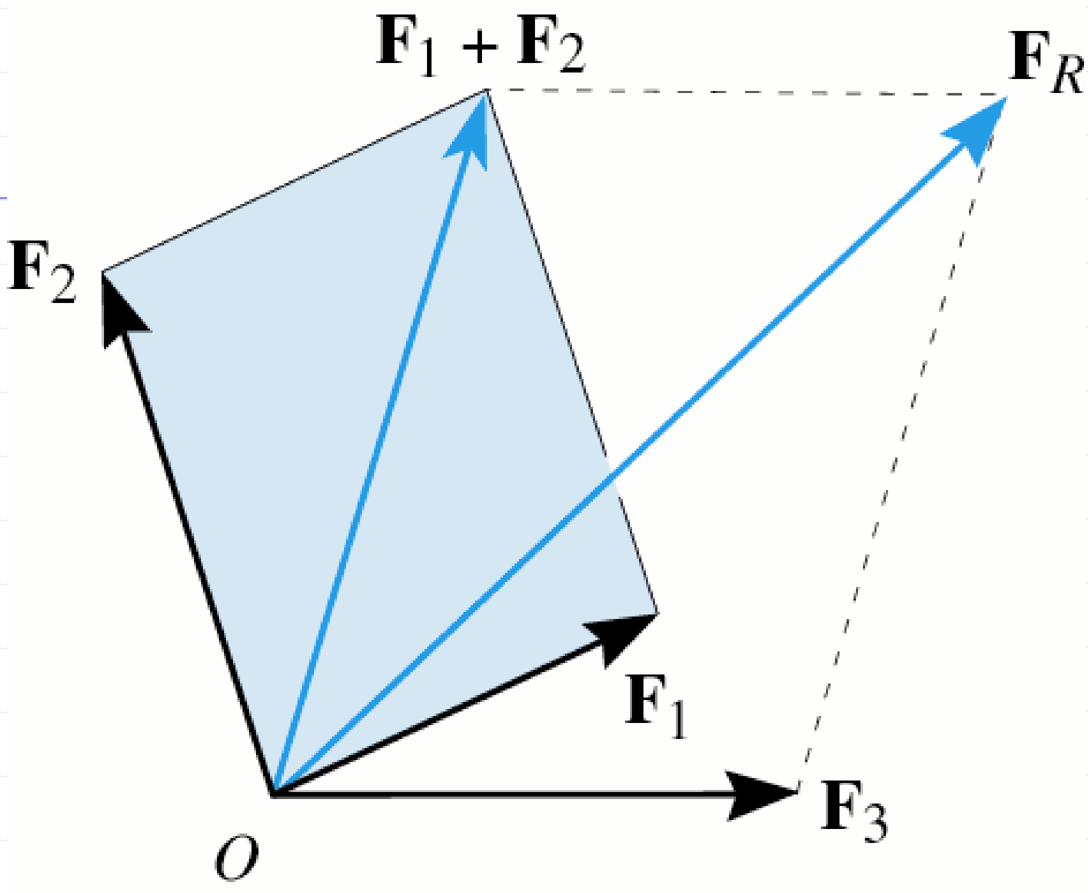


# Vectors: Parallelogram law of addition

- Addition of 3 vectors

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{R}$$

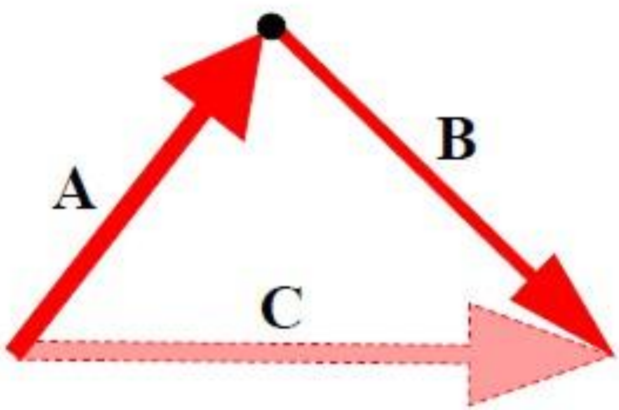


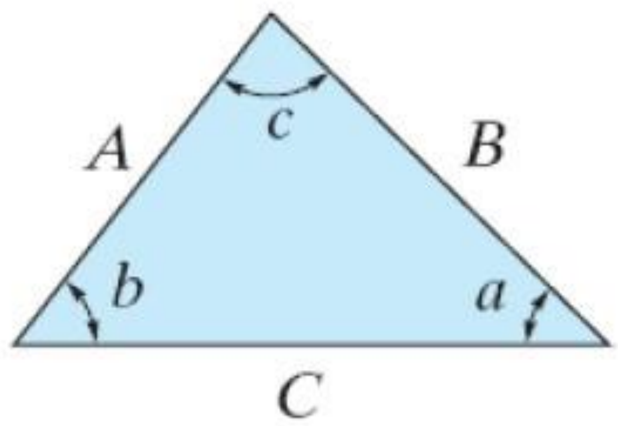


$F_1 + F_2$
$F_R$
$F_2$
$F_1$
$F_3$
$O$

# Vectors: Rules of addition

- **Trigonometric Rule**
  - Law of Sines
  - Law of Cosine



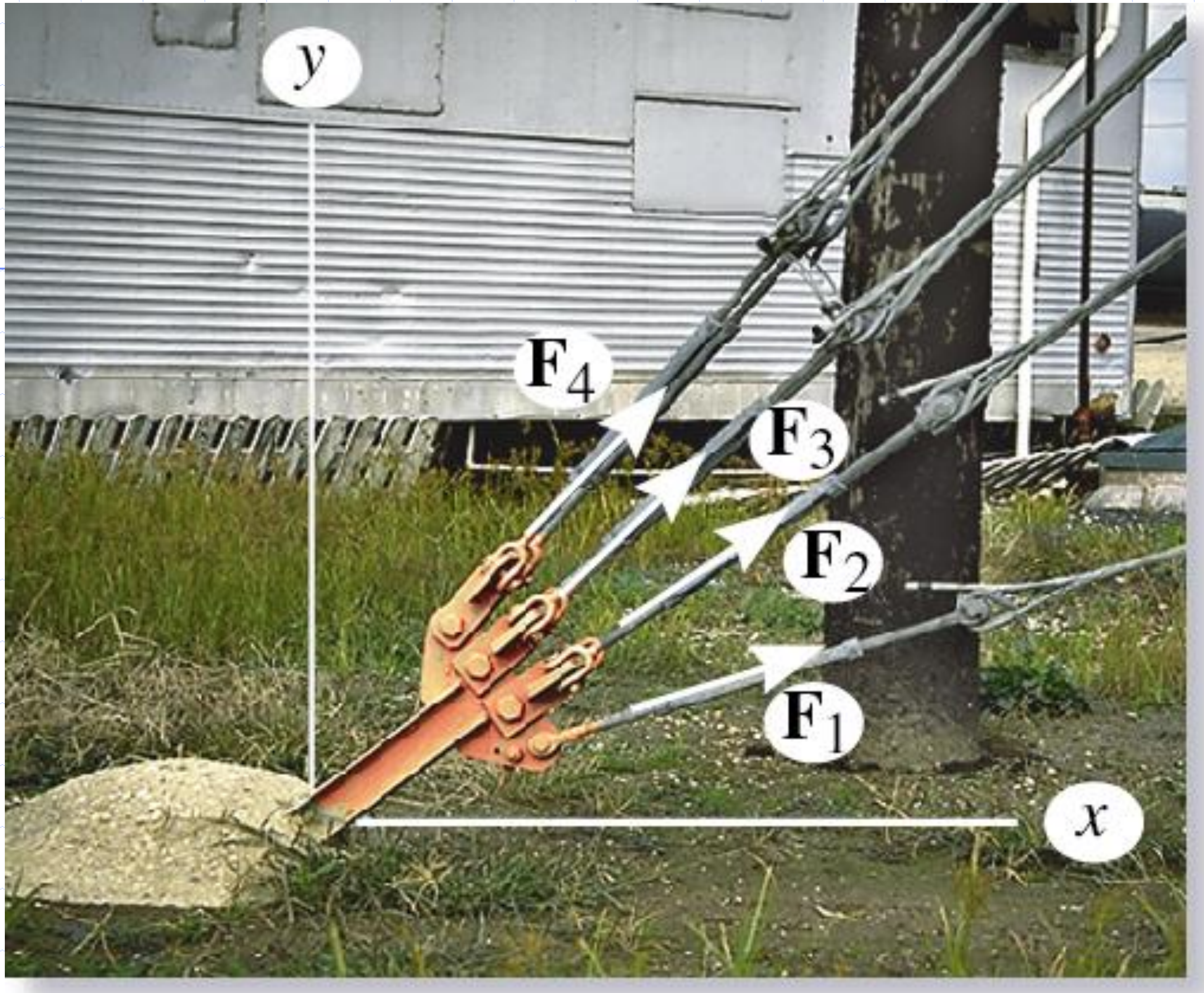

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

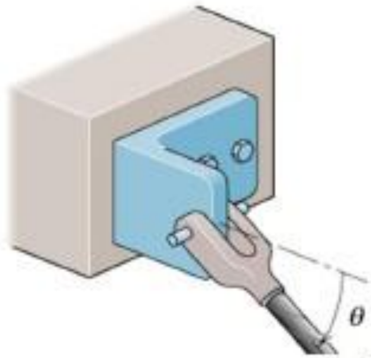
Cosine law:

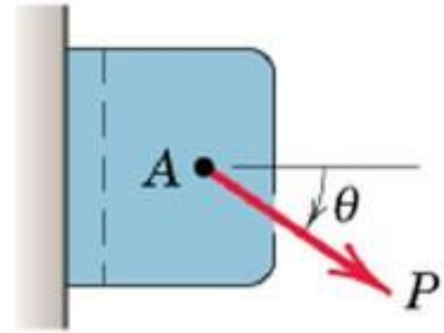
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

# 4. Force Systems


y
F4
F3
F2
F1
x

# Force Systems



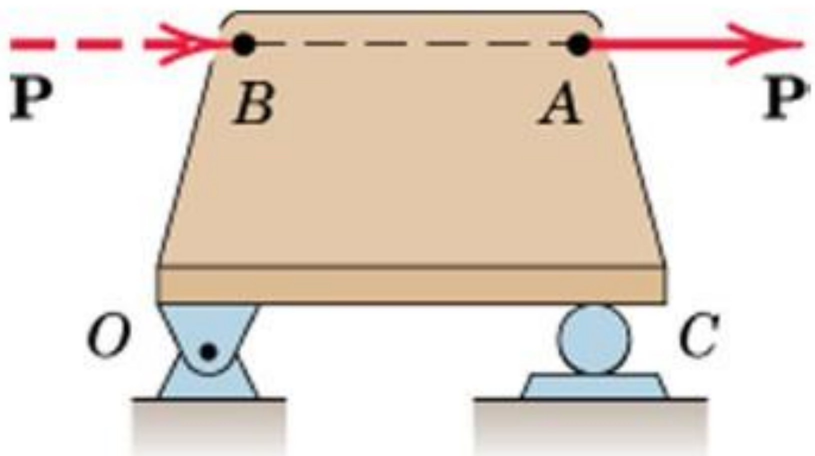




Cable Tension **P**

- **Force**: Represented by vector
  - Magnitude, direction, point of application
  - **P**: fixed vector (or sliding vector??)
  - External Effect
    - Applied force; Forces exerted by bracket, bolts, Foundation (reactive force)

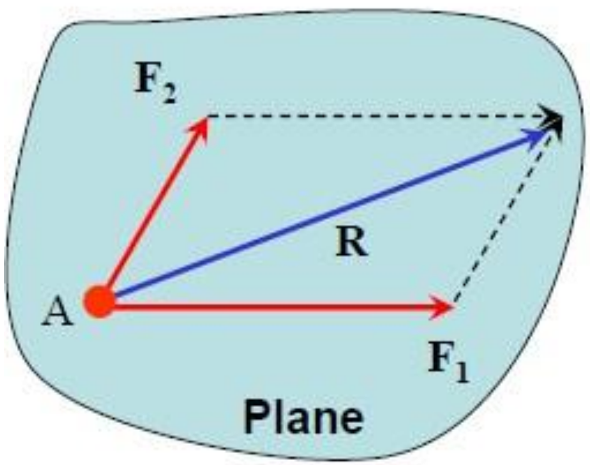
# Force Systems

- **Rigid Bodies**
  - External effects only
    - **Line of action** of force is **important**
      - Not its point of application
      - Force as **sliding vector**



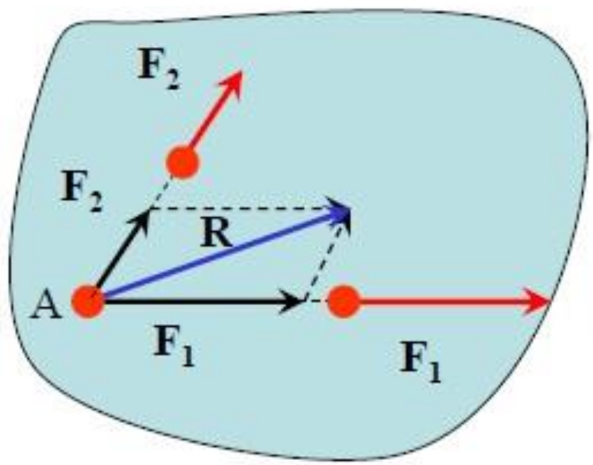
# Force Systems

- **Concurrent forces**
  - Lines of action intersect at a point



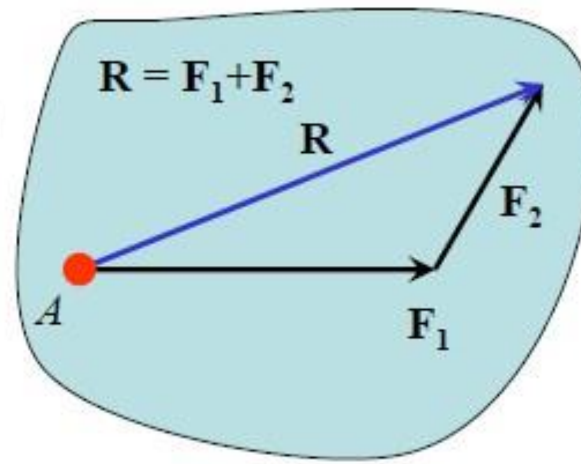

Concurrent Forces

$\mathbf{F_1}$ and $\mathbf{F_2}$




Principle of

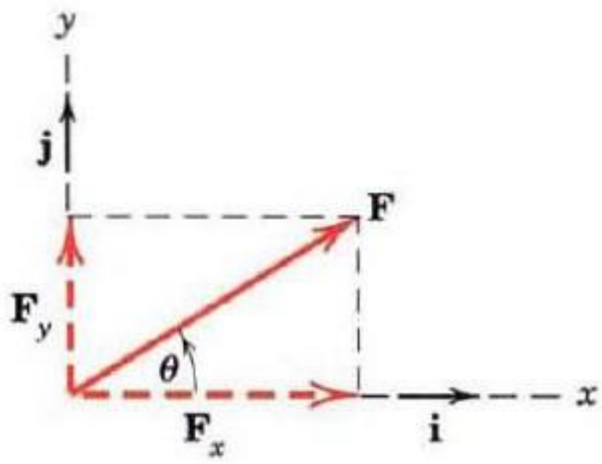
Transmissibility




$\mathbf{R = F_1 + F_2}$

# 5. Components of a Force

# Rectangular components of 2-D Force System




$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

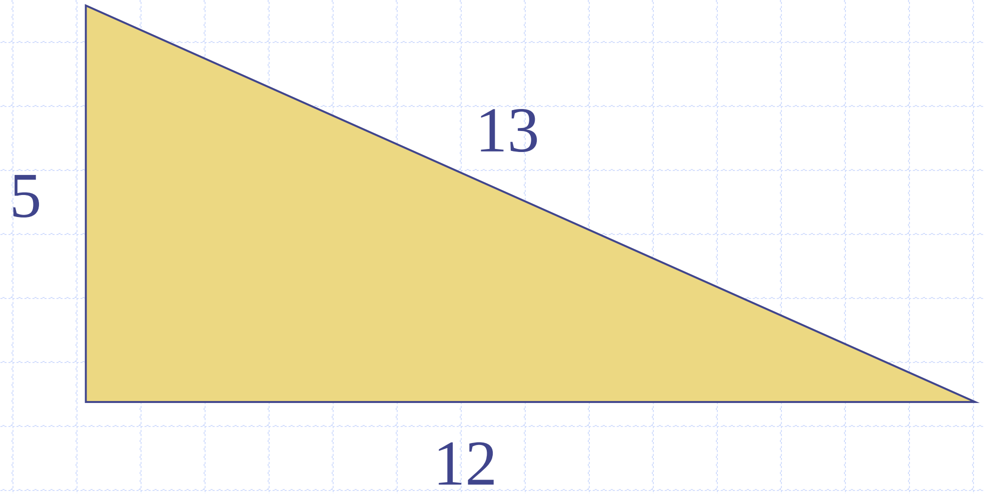
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

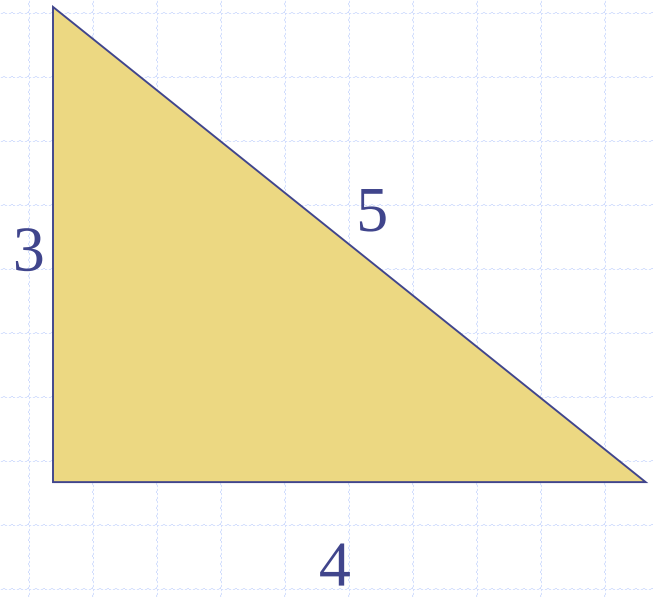
$$\mathbf{F} = F \angle \theta$$

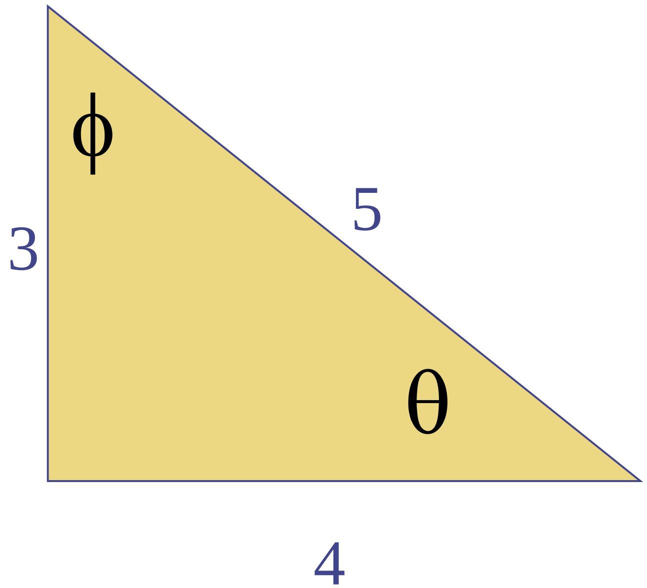
$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

# Special Triangles



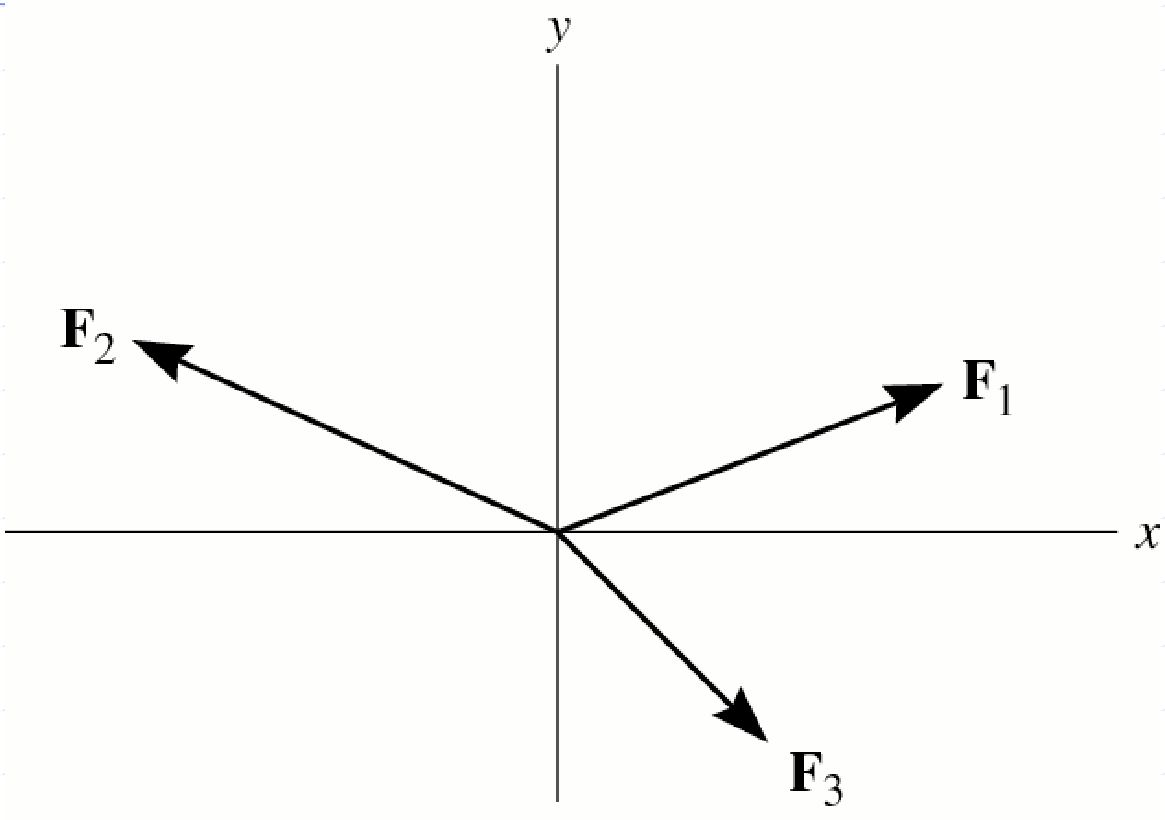


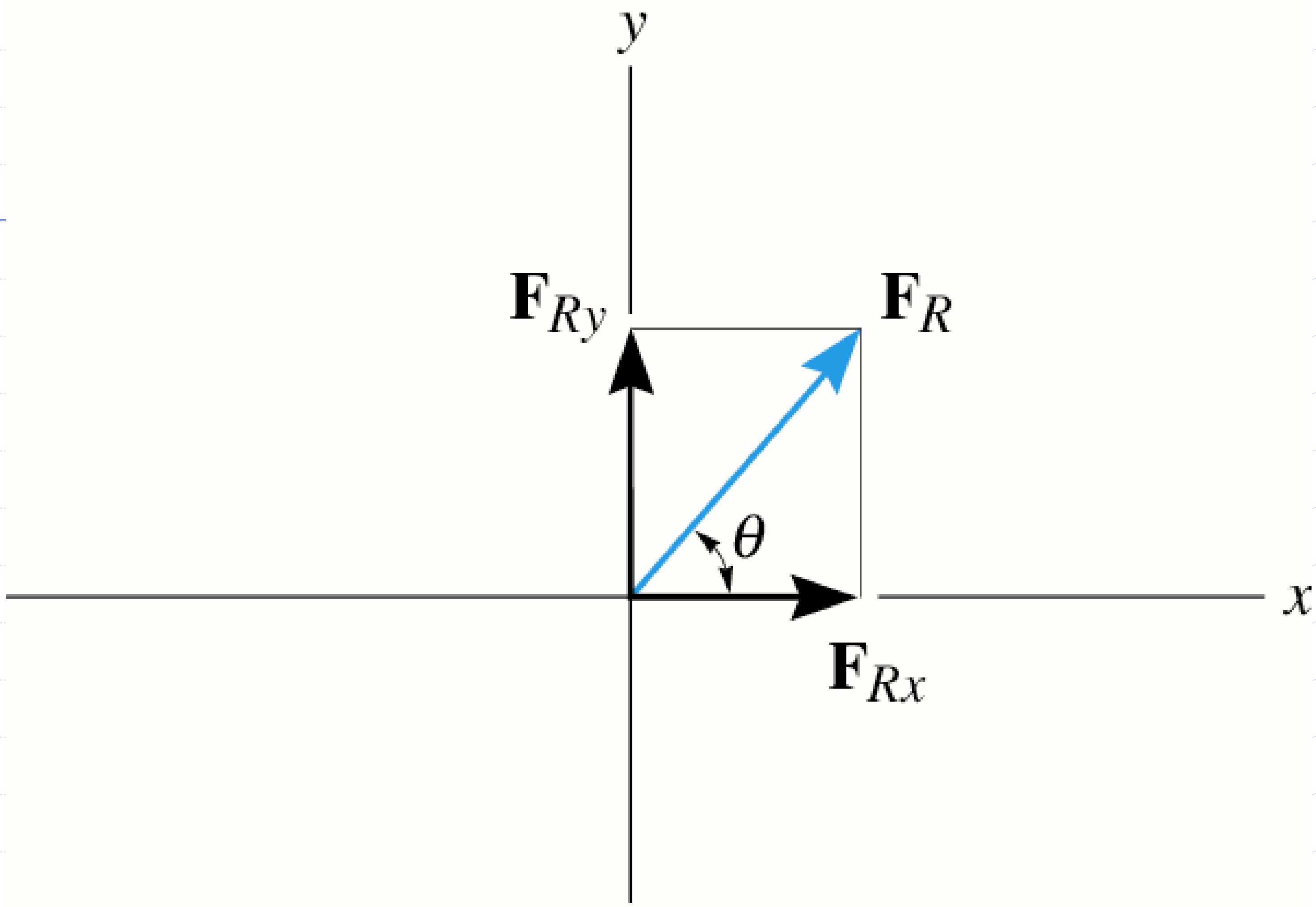

$$5 = \sqrt{3^2 + 4^2}$$

$$\cos\theta = \frac{4}{5} = 0.8 \qquad \sin\theta = \frac{3}{5} = 0.6$$

$$\cos\phi = \frac{3}{5} = 0.6 \qquad \cos\phi = \frac{4}{5} = 0.8$$

# Coplanar Force Resultants




y
F_Ry
F_R
θ
x
F_Rx

$$F_{Rx} = \sum F_x$$

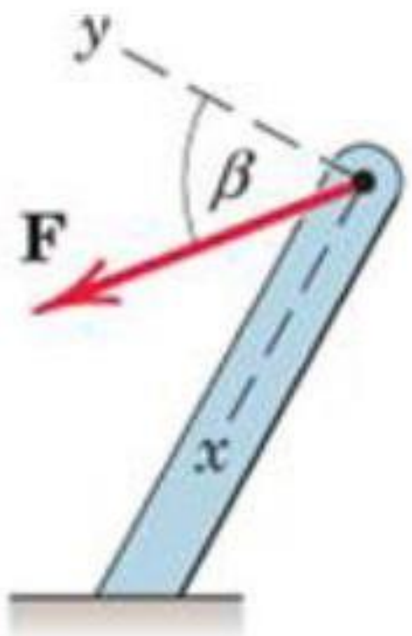
$$F_{Ry} = \sum F_y$$

$$\left|\vec{F}_R\right| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

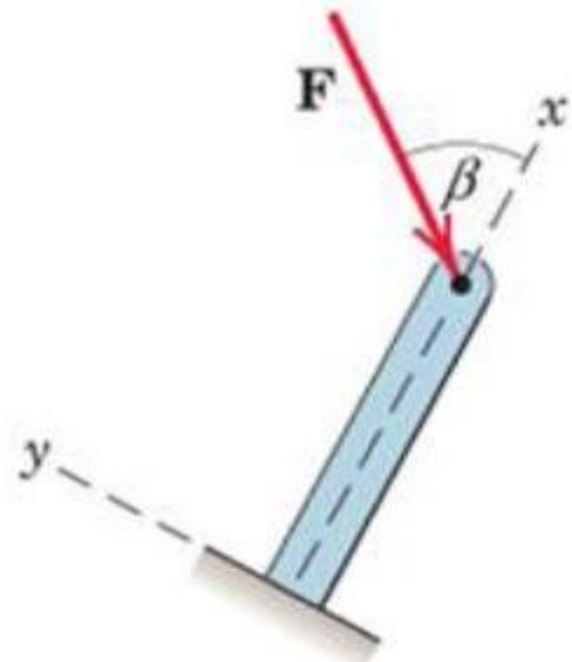
$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

# Components of a Force

- Examples



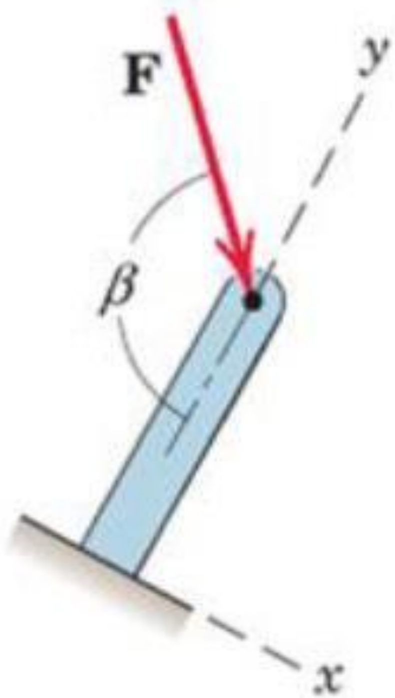

$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



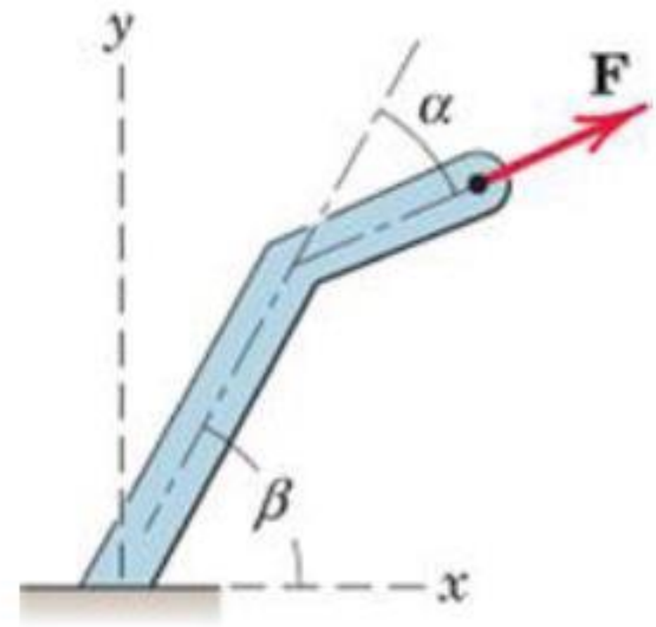

$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

# Components of a Force

- Examples




$$F_x = F\sin(\pi - \beta)$$
$$F_y = -F\cos(\pi - \beta)$$

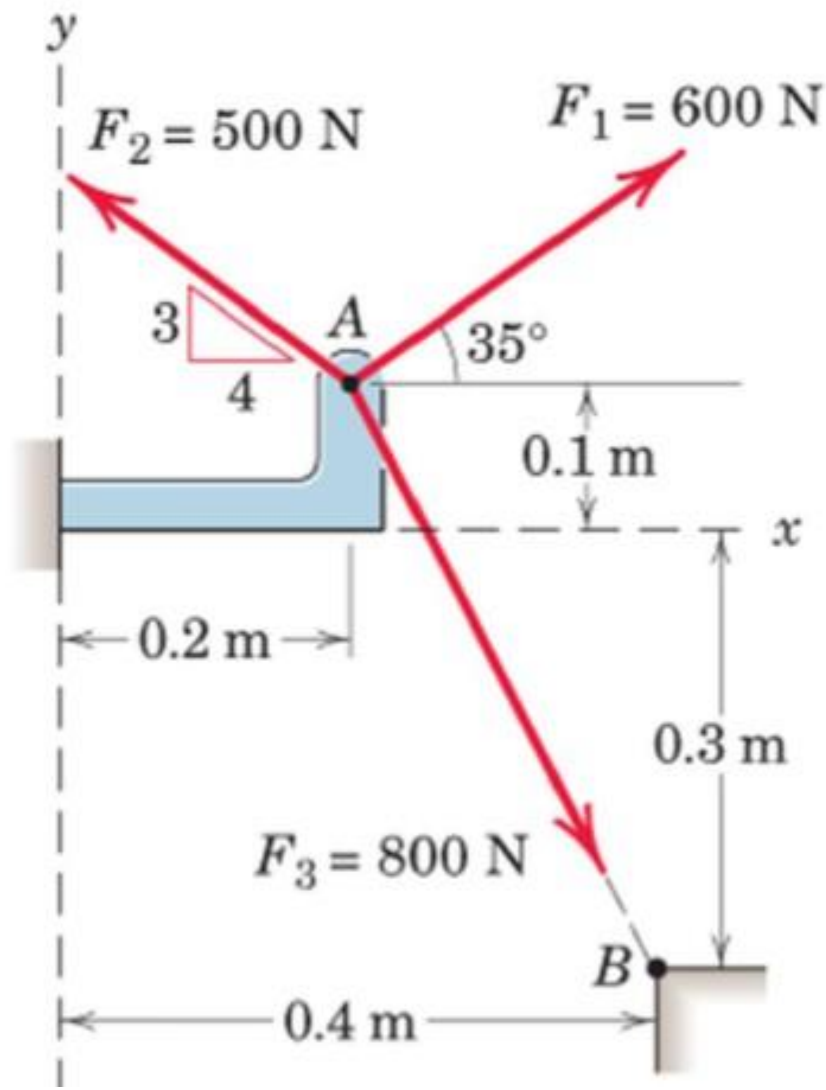



$$F_x = F\cos(\beta - \alpha)$$
$$F_y = F\sin(\beta - \alpha)$$

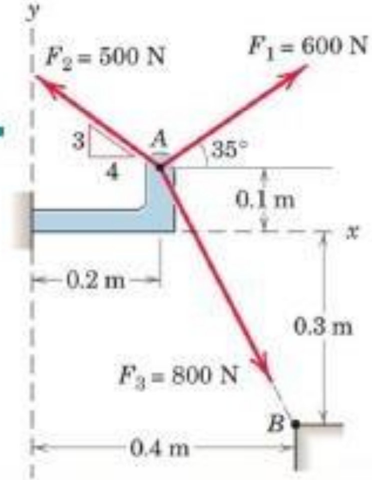
# Components of a Force

**Example 1:**

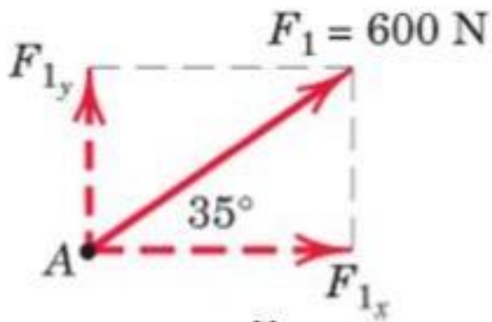
Determine the x and y scalar components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ acting at point A of the bracket

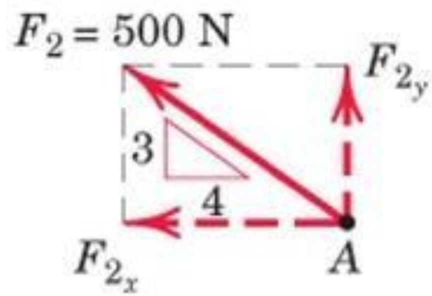


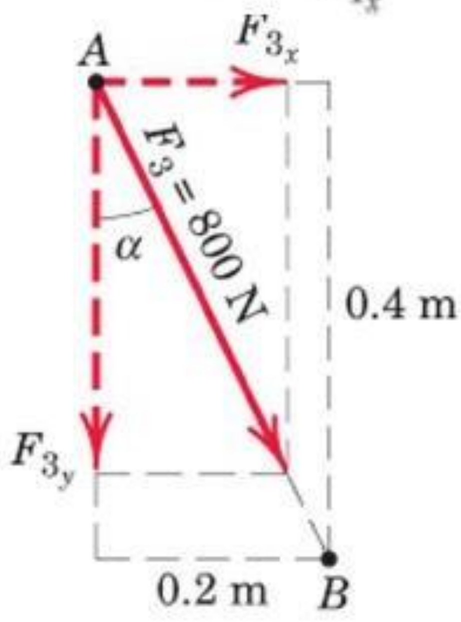
# Components of Force




Solution:




$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

# Components of Force

Alternative Solution: Scalar components of $\mathbf{F}_3$ can be obtained by writing $\mathbf{F}_3$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment AB.

Unit vector can be formed by dividing any vector, such as the geometric position vector by its length or magnitude.

$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

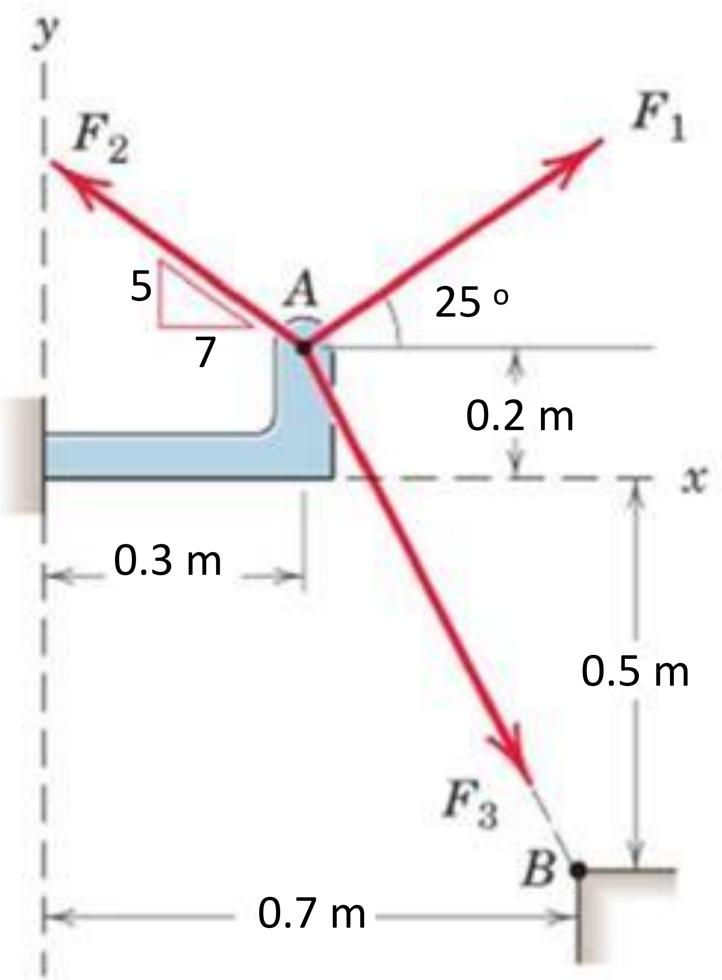
$$= 800[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j}\ \text{N}$$

$$F_{3_x} = 358\ \text{N}$$

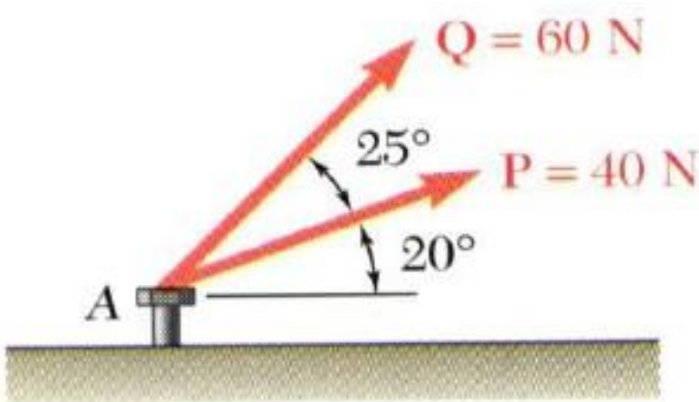
$$F_{3_y} = -716\ \text{N}$$

Determine the components of each force
Evaluate the resultant in x and y directions



# Components of Force

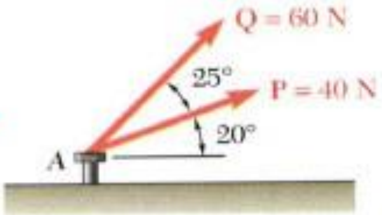
**Example 2:** The two forces act on a bolt at A. Determine their resultant.



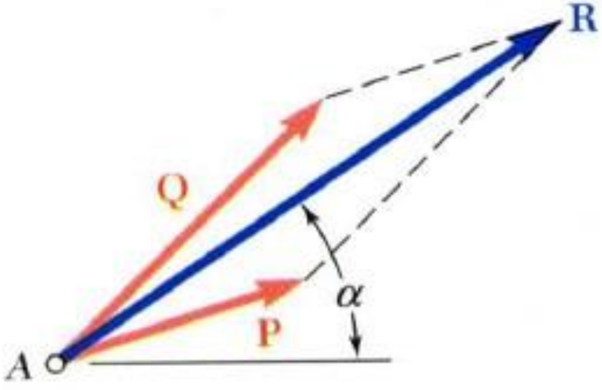

- **Graphical solution –**
- Construct a parallelogram with sides in the same direction as P and Q and lengths in proportion.
- Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

- **Trigonometric solution**
- Use the law of cosines and law of sines to find the resultant.

# Components of Force

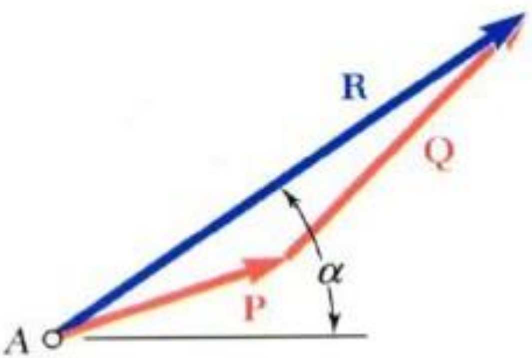



Solution:




- **Graphical solution** - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

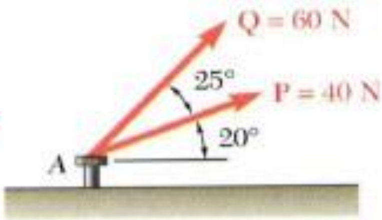
$$\mathbf{R} = 98\,\mathrm{N} \quad \alpha = 35^{\circ}$$



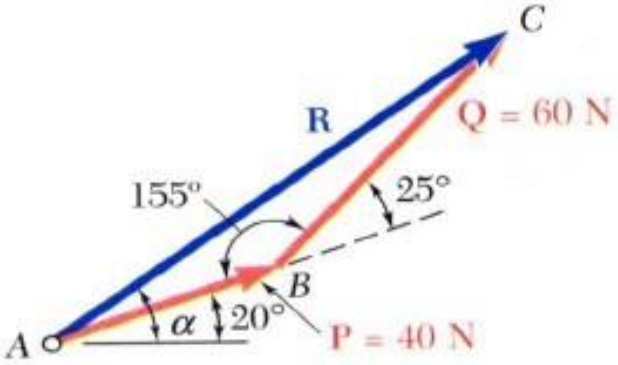

- **Graphical solution** - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\mathbf{R} = 98\,\mathrm{N} \quad \alpha = 35^{\circ}$$

# Components of Force




**Trigonometric Solution:**

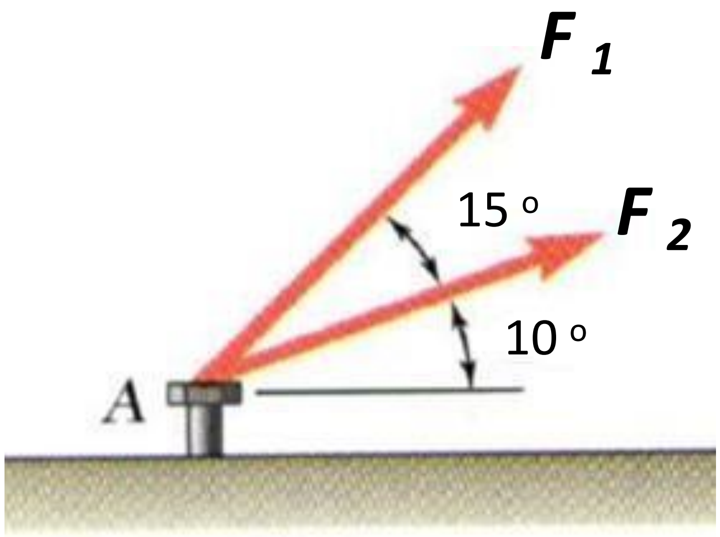



$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

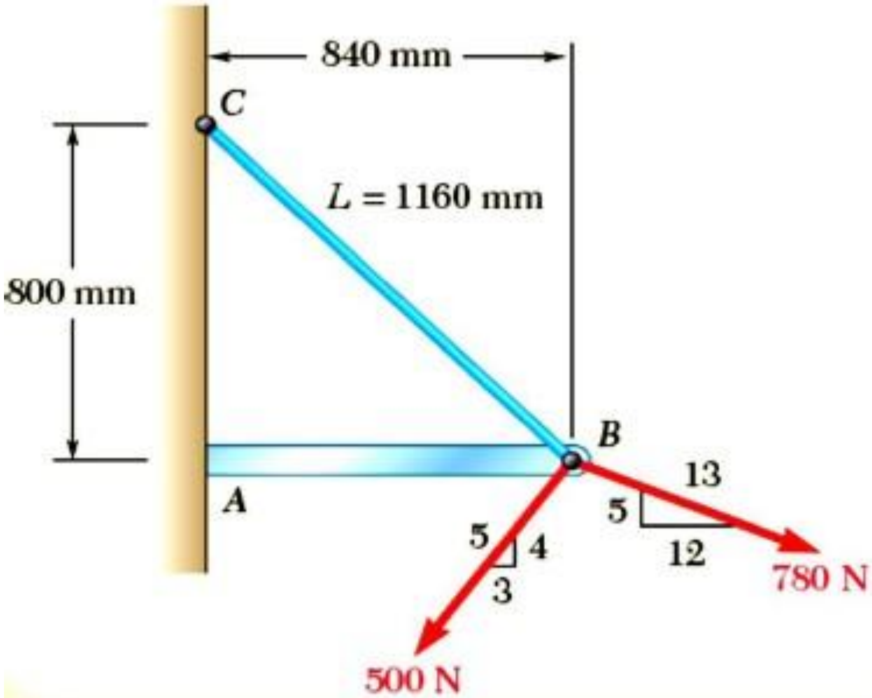
$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Determine the resultant force if F1 = F2 = 200 N.



# Components of Force

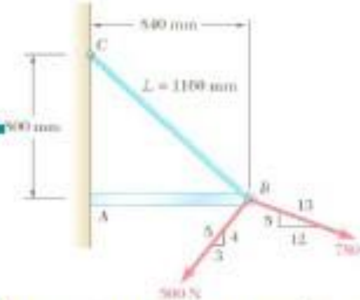
**Example 3:** Tension in cable *BC* is 725 N; determine the resultant of the three forces exerted at point *B* of beam *AB*.



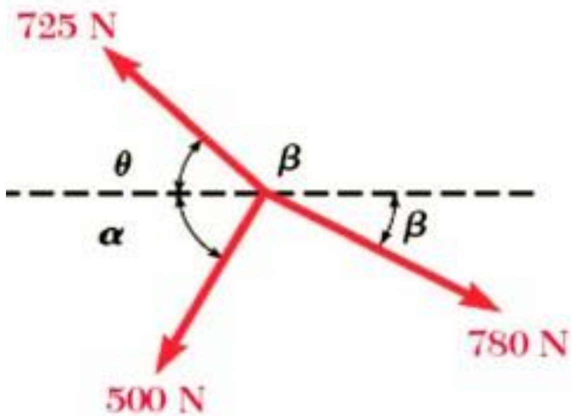

Solution:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

# Components of Force



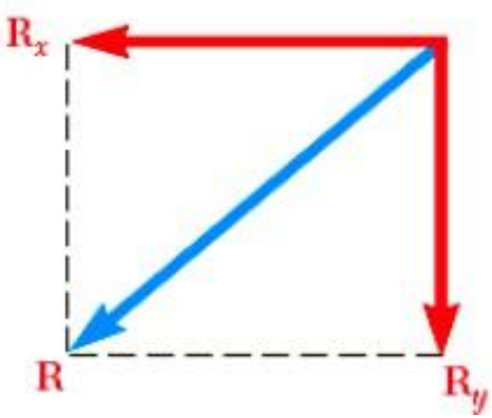
## Solution




- Resolve each force into rectangular components.

| Magnitude, N | $x$ Component, N | $y$ Component, N |
|---|---|---|
| 725 | −525 | 500 |
| 500 | −300 | −400 |
| 780 | 720 | −300 |
| | $R_x = -105$ | $R_y = -200$ |

$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$ $\mathbf{R} = (-105\ \text{N})\mathbf{i} + (-200\ \text{N})\mathbf{j}$



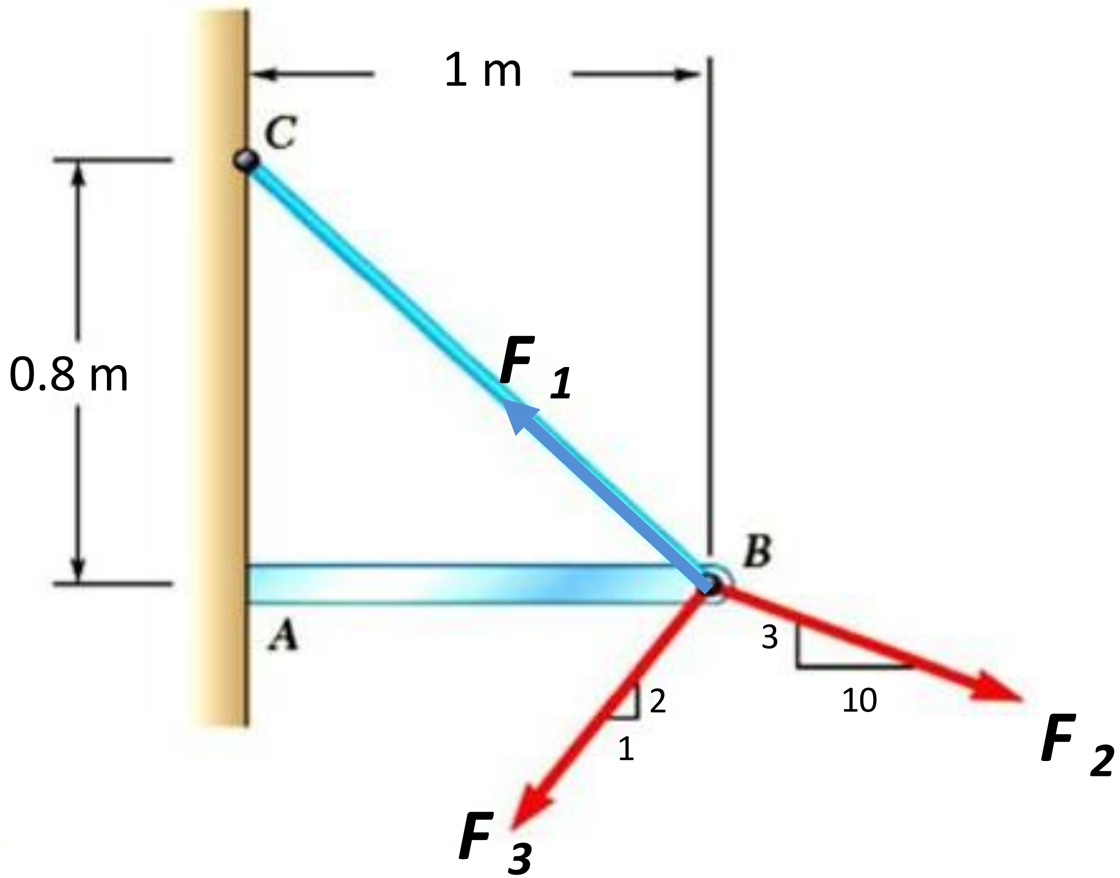

- Calculate the magnitude and direction.

$$\tan\alpha = \frac{-R_y}{-R_x} = \frac{200\,\text{N}}{105\,\text{N}} \qquad \alpha = 62.3^\circ$$

$$R = \sqrt{R_{x^2} + R_{y^2}} = 225.9\,\text{N} \qquad \measuredangle\ 62.3^\circ$$

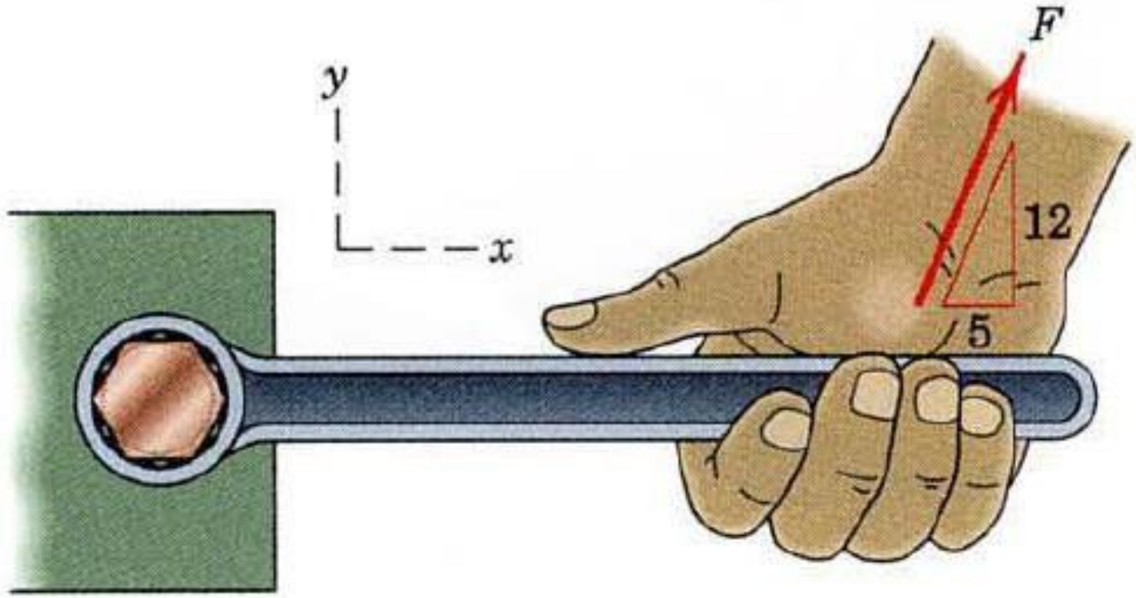
# 6. Additional Exercises

If F1 = 1 kN, F2 = 900 N, F3 = 1.2 kN, compute the resultatnt force acting on point B.

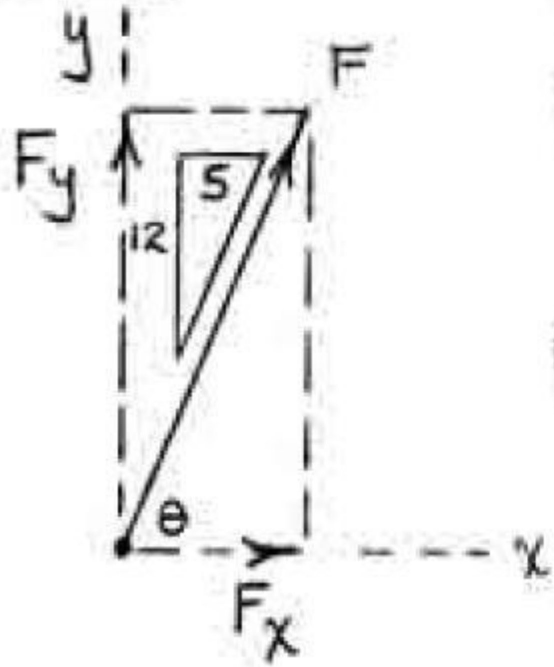


**2/7** The $y$-component of the force **F** which a person exerts on the handle of the box wrench is known to be 70 lb. Determine the $x$-component and the magnitude of **F**.

*Ans.* $F_x = 29.2$ lb, $F = 75.8$ lb



2/7




$$\cos\theta = \frac{5}{13}, \quad \sin\theta = \frac{12}{13}$$

$$F_y = F\sin\theta = F\,\frac{12}{13} = 70$$

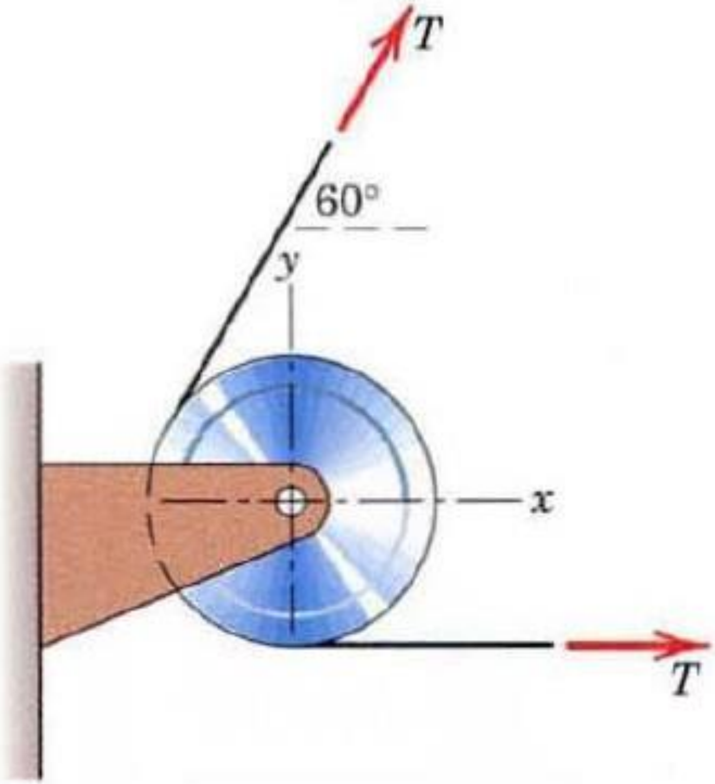
$$\underline{F = 75.8 \text{ lb}}$$

$$F_x = F\cos\theta = 75.8\left(\frac{5}{13}\right)$$

$$= \underline{29.2 \text{ lb}}$$

**2/13** If the equal tensions $T$ in the pulley cable are 400 N, express in vector notation the force **R** exerted on the pulley by the two tensions. Determine the magnitude of **R**.

*Ans.* $\mathbf{R} = 600\mathbf{i} + 346\mathbf{j}$ N, $R = 693$ N



**2/13**

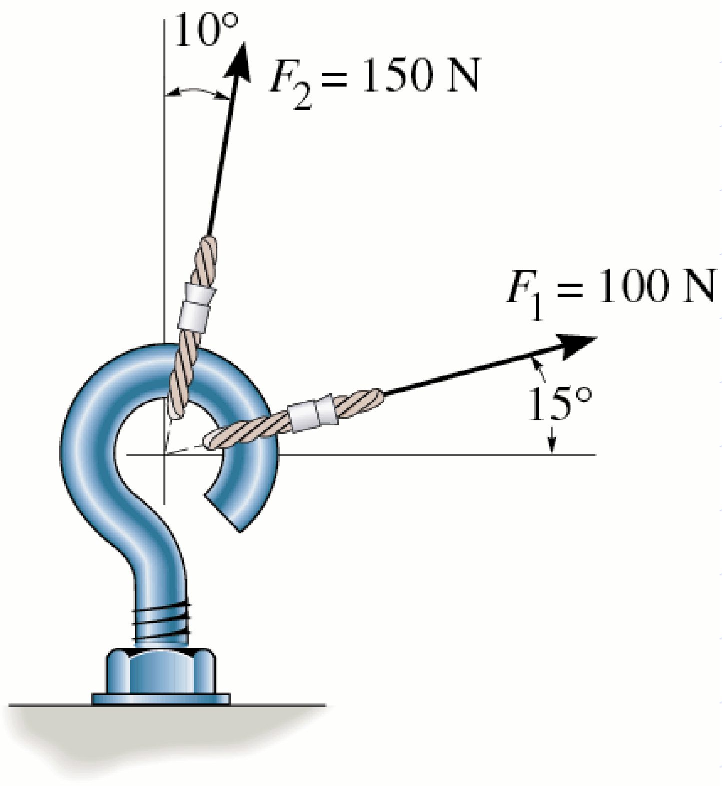
$$R_x = \Sigma F_x = 400 + 400 \cos 60^\circ = 600 \text{ N}$$

$$R_y = \Sigma F_y = 400 \sin 60^\circ = 346 \text{ N}$$

$$\Rightarrow \underline{R} = \underline{600\,\underline{i} + 346\,\underline{j} \text{ N}}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

# Example 1



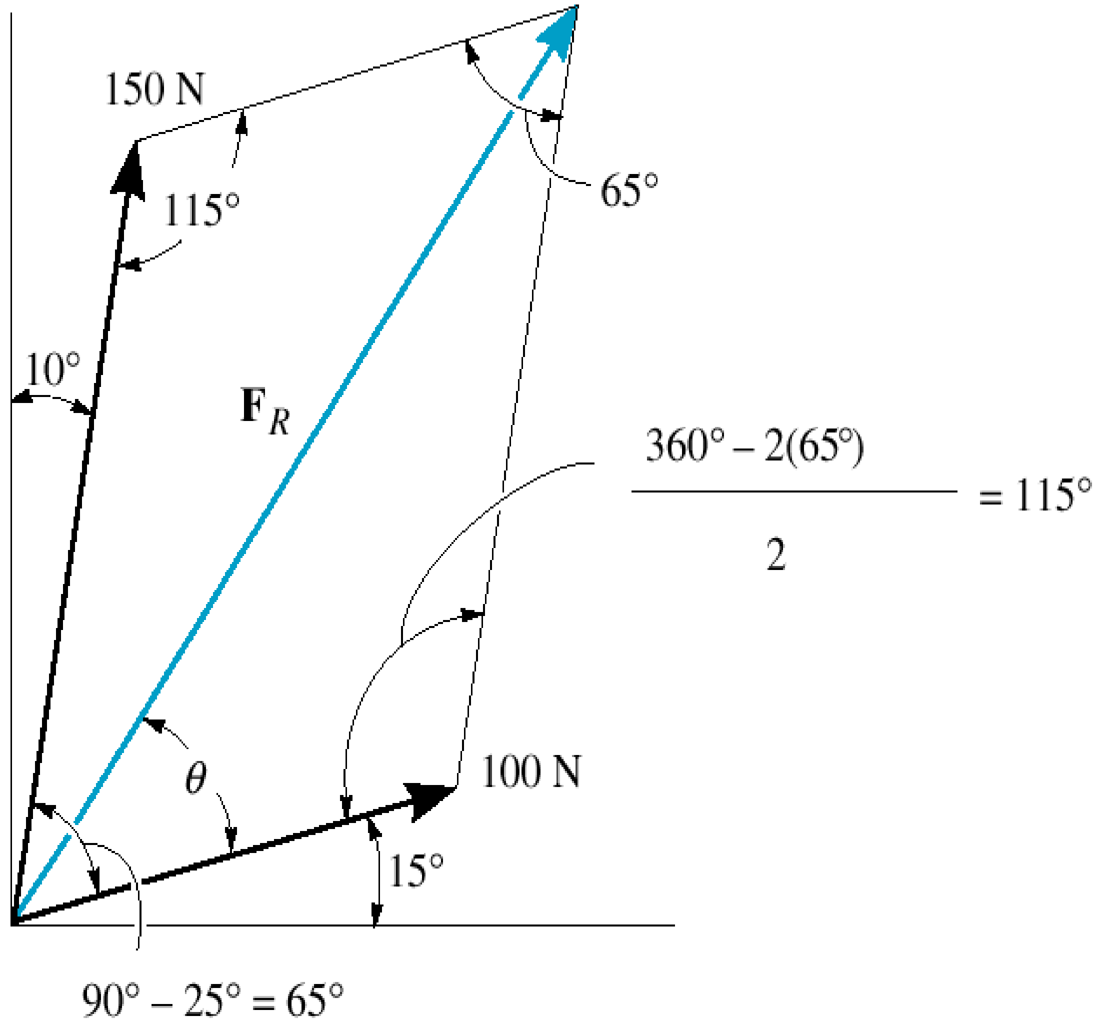

The screw eye in the figure at the left is subjected to two forces $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the magnitude and direction of the resultant force.

# Parallelogram Law

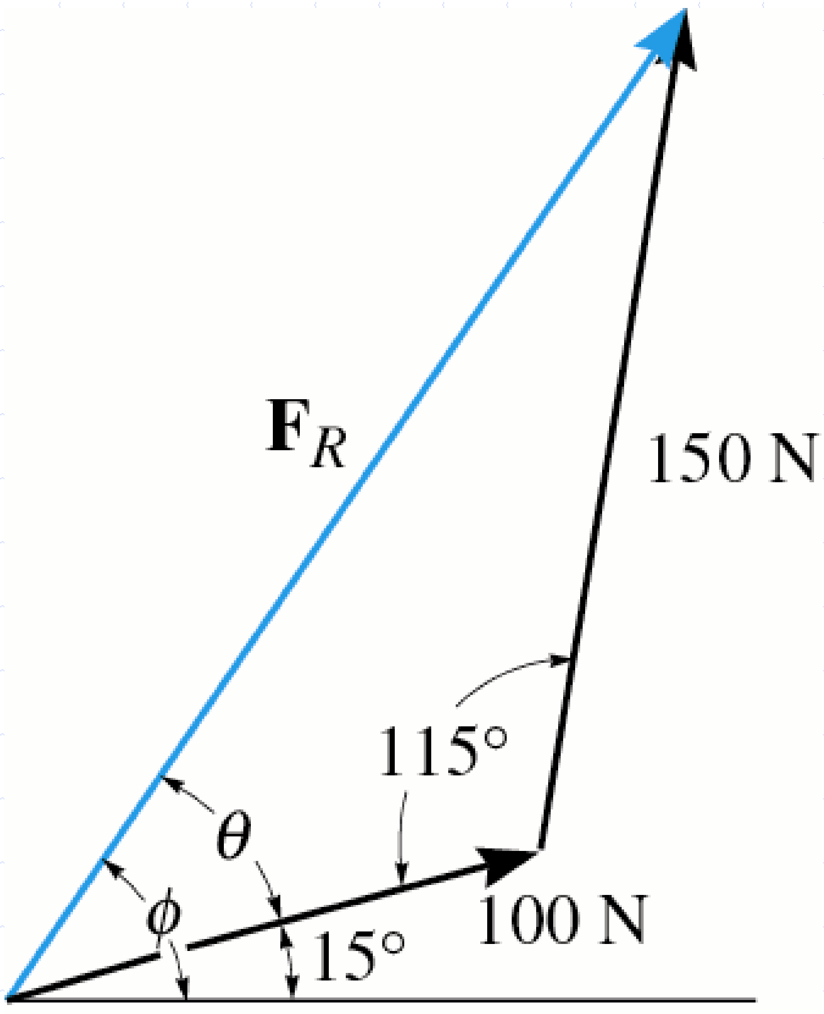
<u>Calculate angles</u>

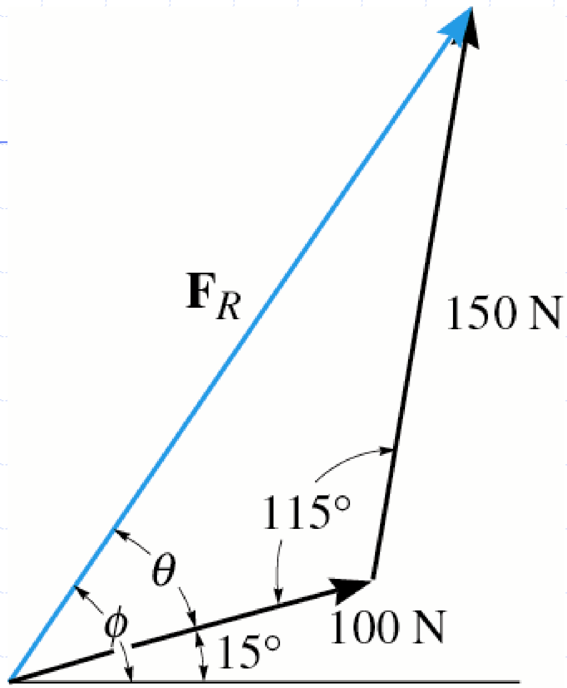
Angle COA = $90^0 - 15^0 - 10^0 = 65^0$

Angle OAB = $180^0 - 65^0 = 115^0$



# Triangular Construction






**Find $F_R$ from law of cosines.**

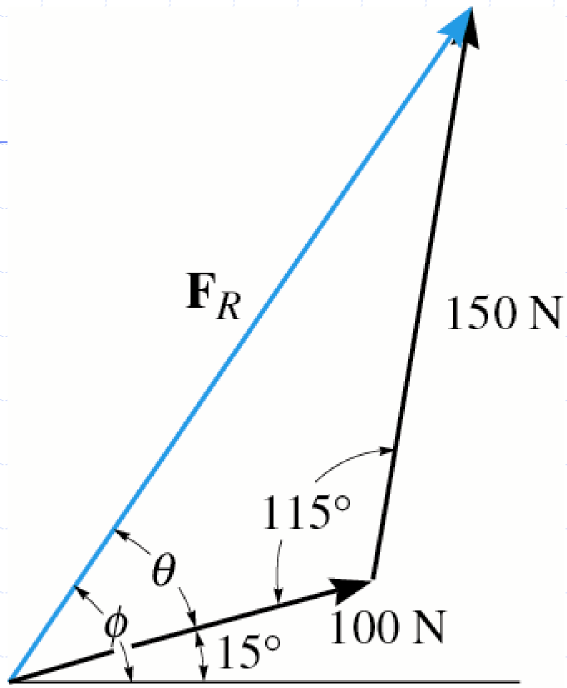
**Find θ from law of sines.**

**Angle $\phi = \theta + 15^0$**

$$F_R = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115^0}$$

$$F_R = \sqrt{10000 + 22500 - 30000(-0.4226)}$$

$$F_R = 212.6N = 213N$$




$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115^0}$$

$$\sin\theta = \frac{150}{212.6}(0.9063) = 0.6394$$

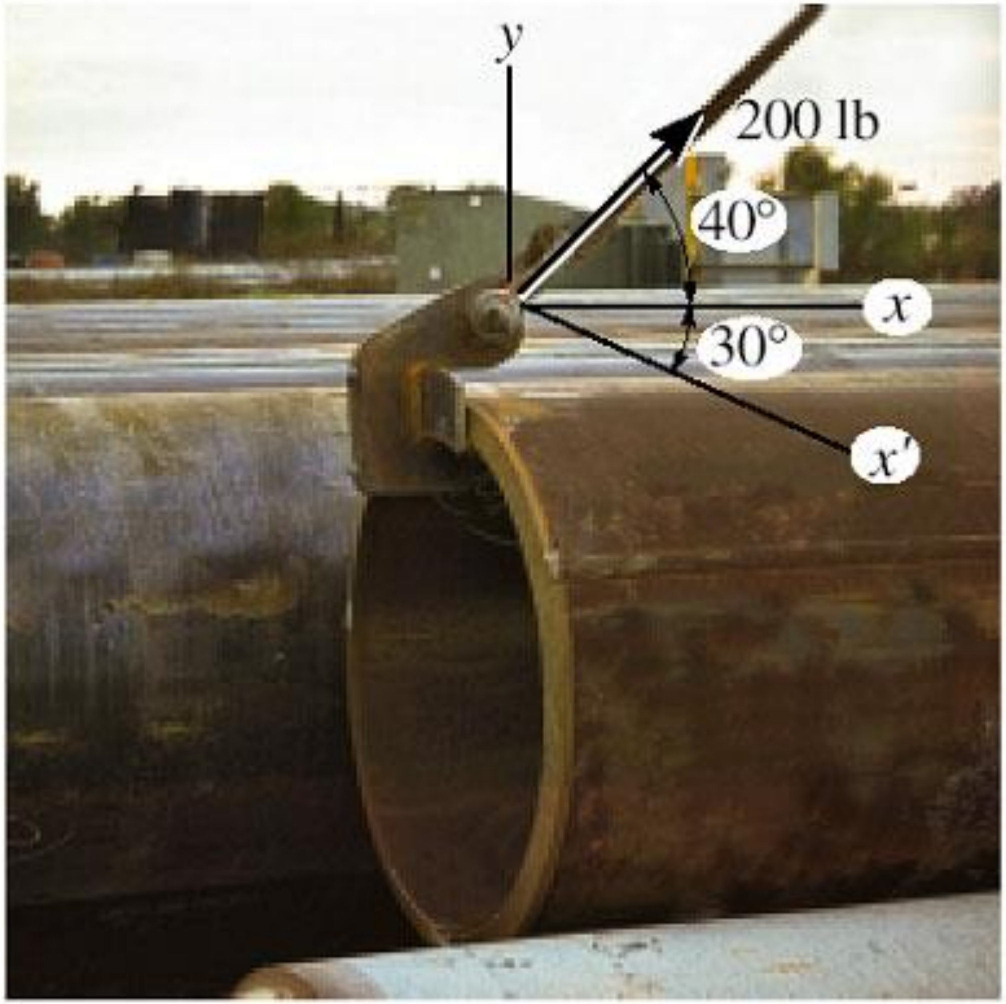
$$\theta = \sin^{-1}(0.6394) = 39.75^0 = 39.8^0$$

$$\phi = \theta + 15^0$$

# Answer

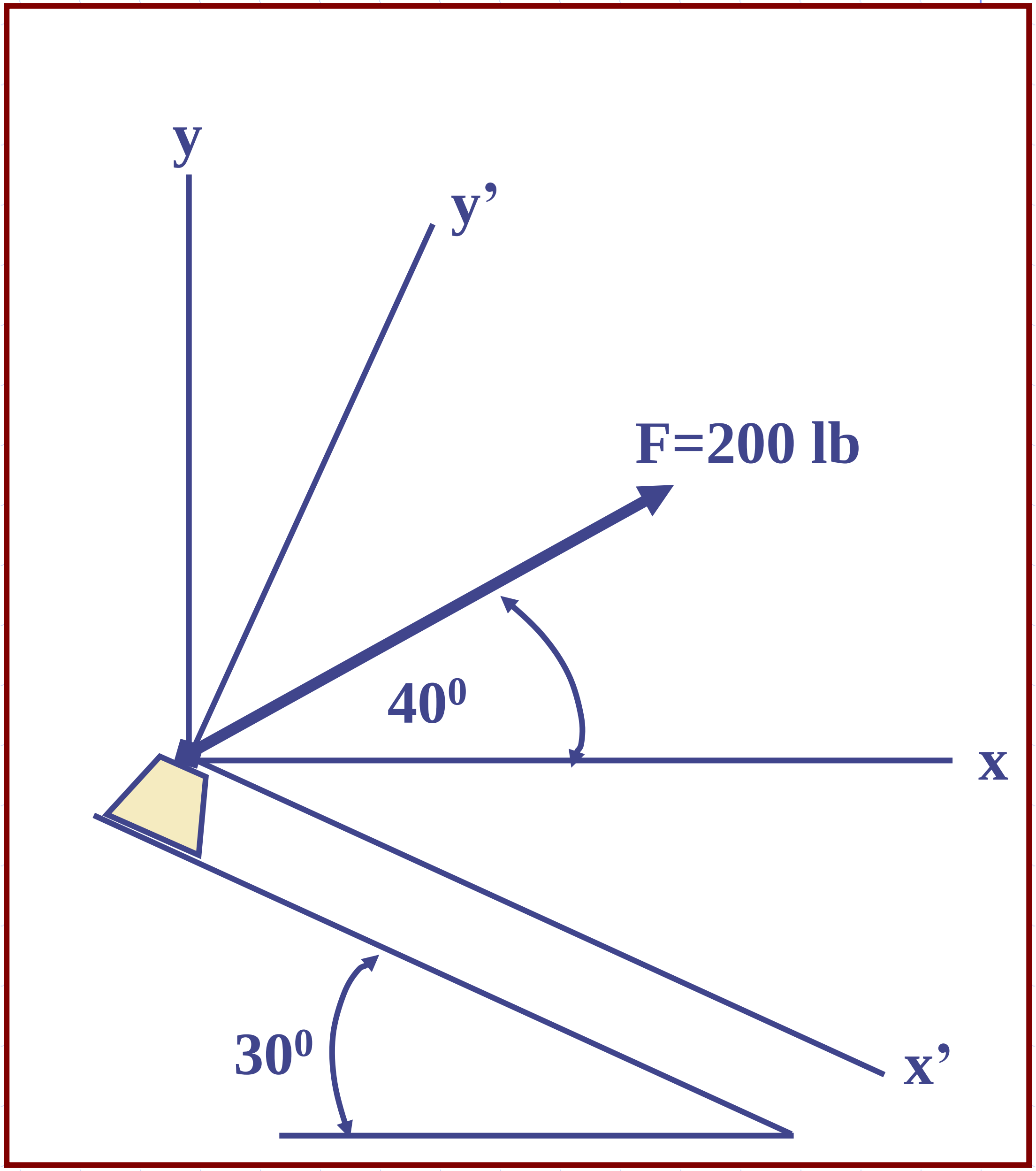
**The resultant force has a magnitude of 213 N and is directed 54.8º from the horizontal.**

# Example 2

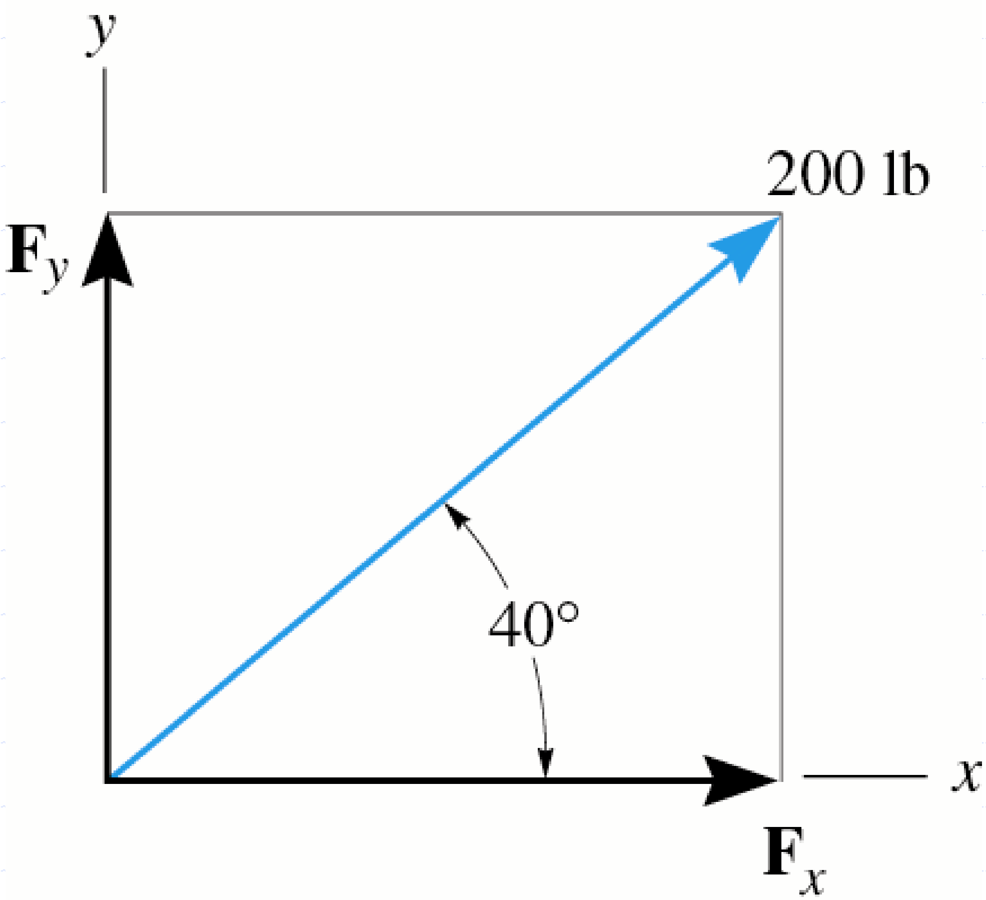



**Resolve the 200 lb force into components in the x and y directions and in the x' and y directions**

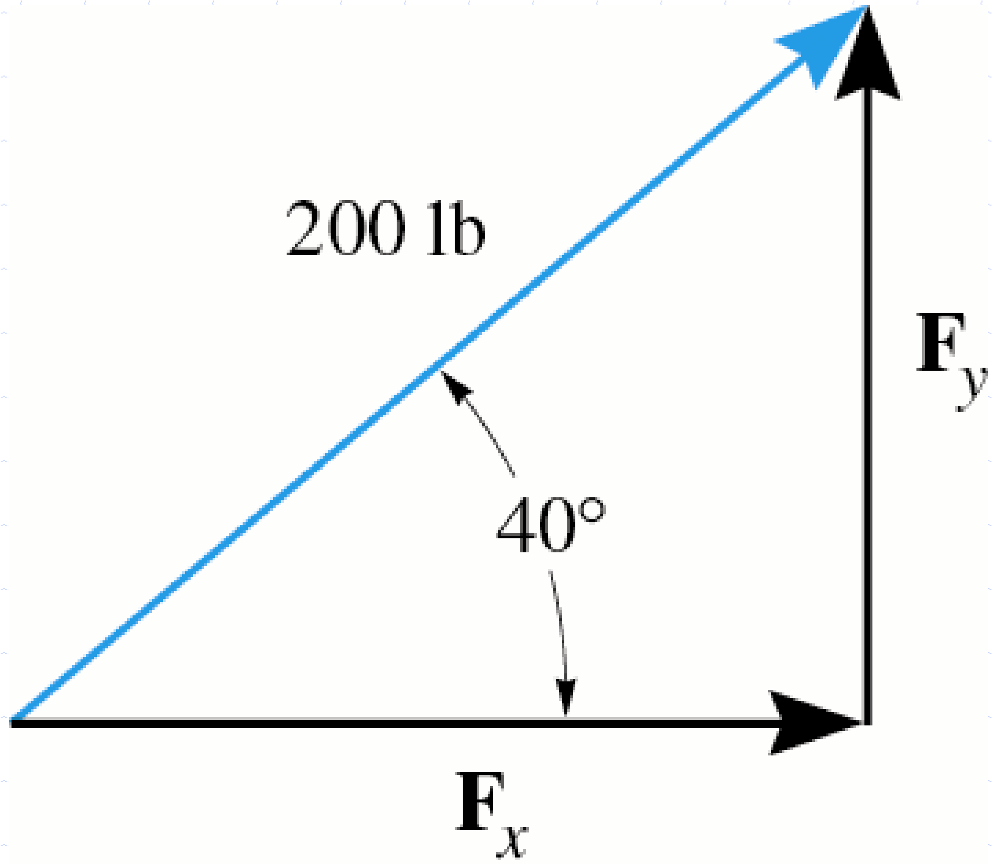
Resolve the 200 lb force into components in the x and y directions and in the x' and y directions



# Parallelogram Law



# Triangular Construction



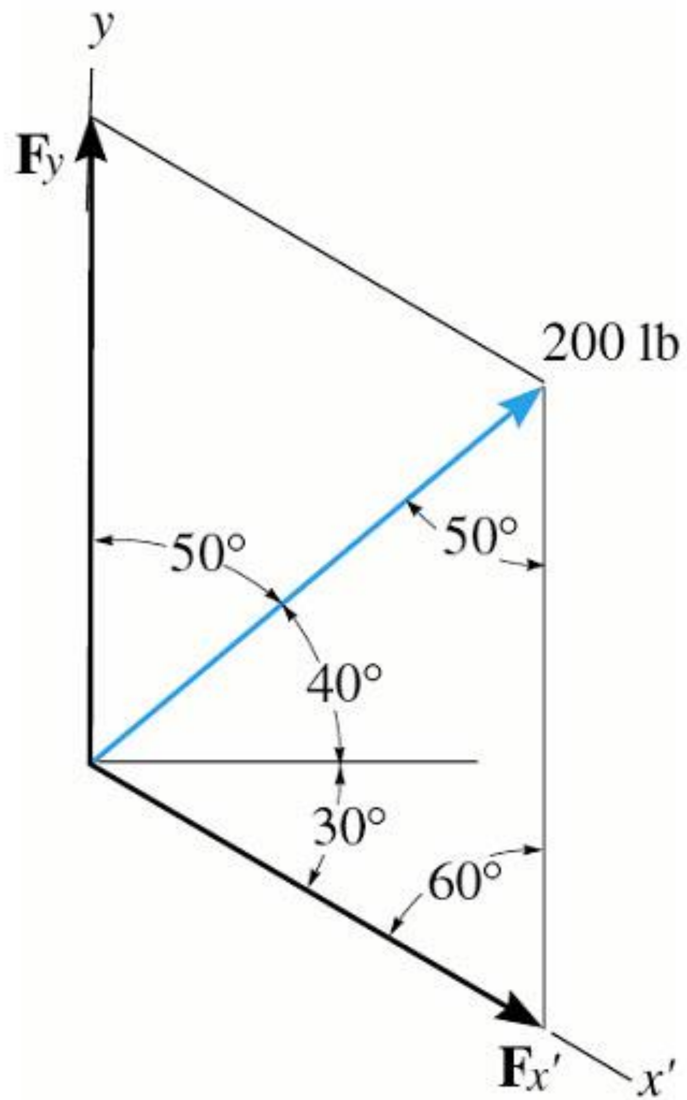
# Solution – Part (a)

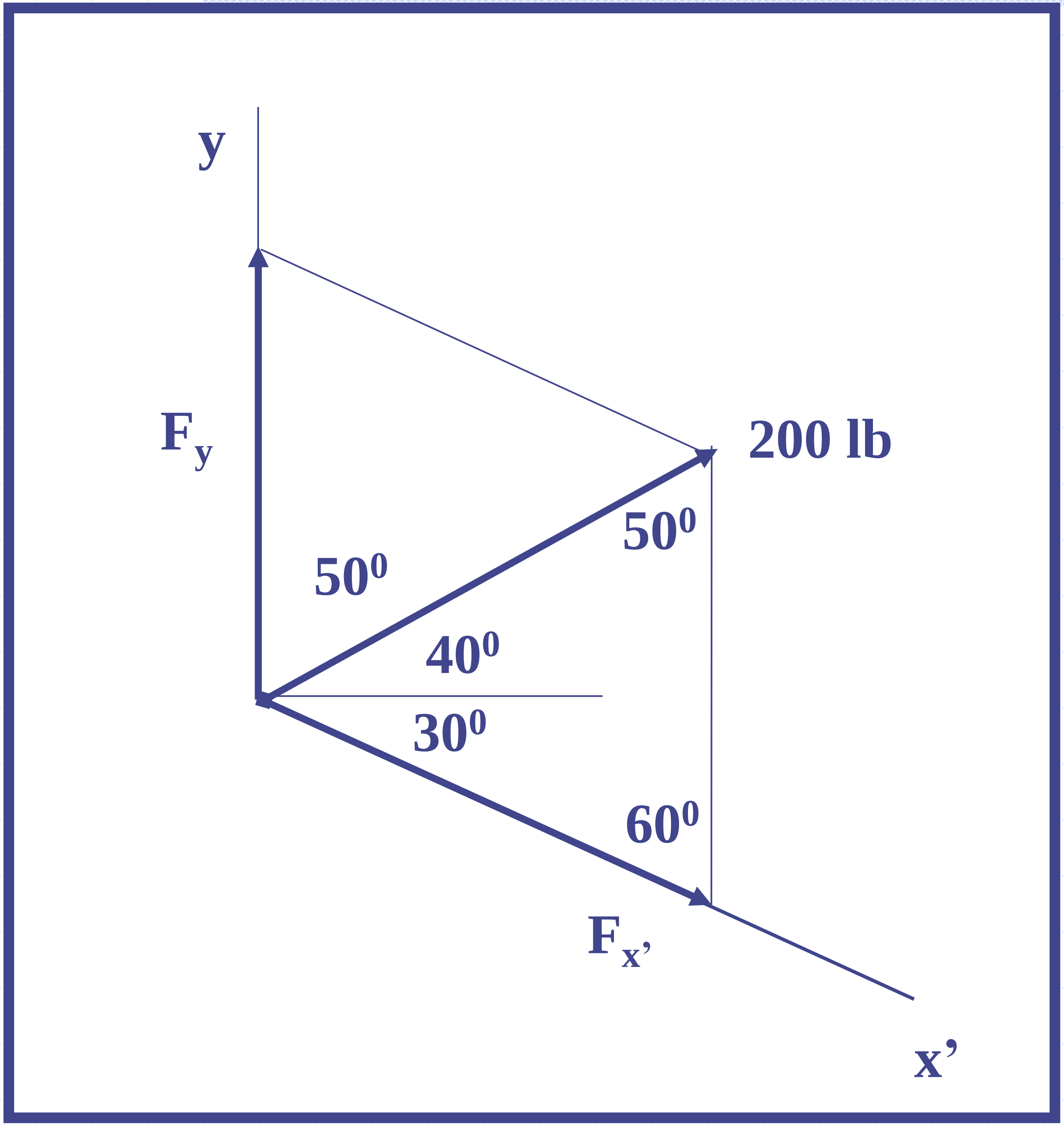
$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

$$F_x = 200\,\text{lb} \cos 40^o = 153\,\text{lb}$$

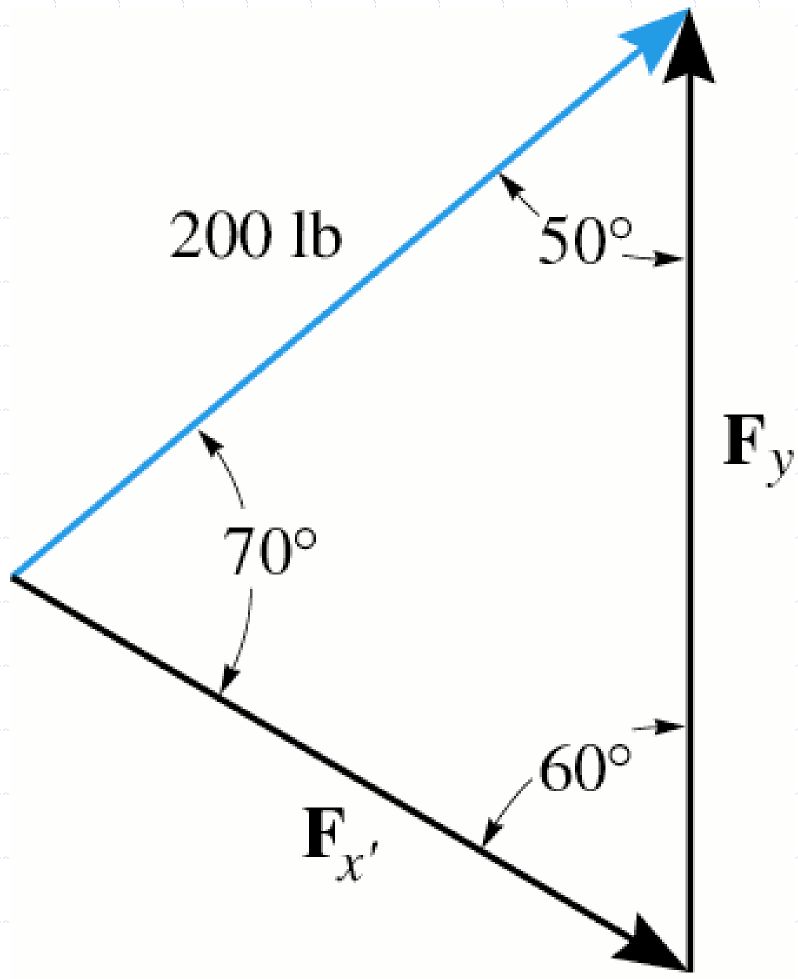
$$F_y = 200\,\text{lb} \sin 40^o = 129\,\text{lb}$$

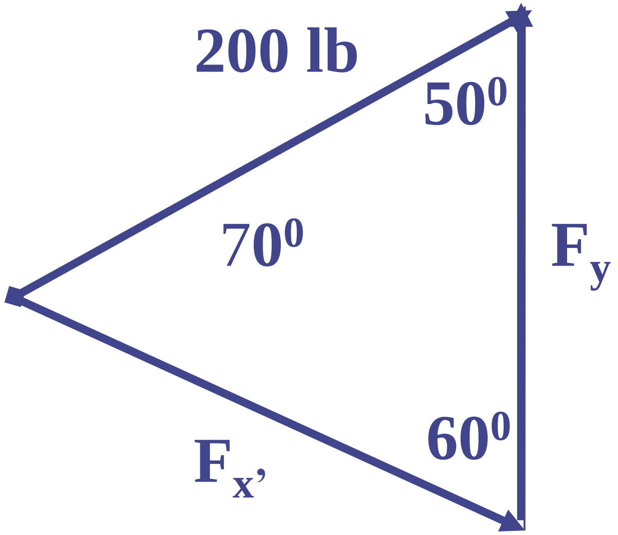
# Parallelogram Law




y
$F_y$
200 lb
$50^0$
$50^0$
$40^0$
$30^0$
$60^0$
$F_{x'}$
x'

# Triangular Construction






$$F = F_{x'} + F_y$$

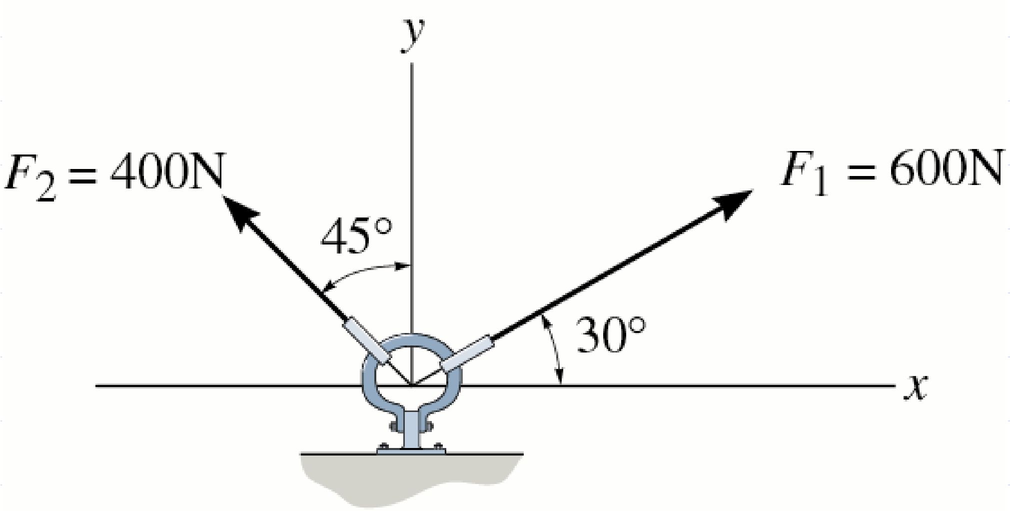
$$F_{x'} = 200\,\frac{\sin 50^0}{\sin 60^0} = 177\,lb$$

$$\frac{F_{x'}}{\sin 50^0} = \frac{200}{\sin 60^0}$$

$$\frac{F_y}{\sin 70^0} = \frac{200}{\sin 60^0}$$

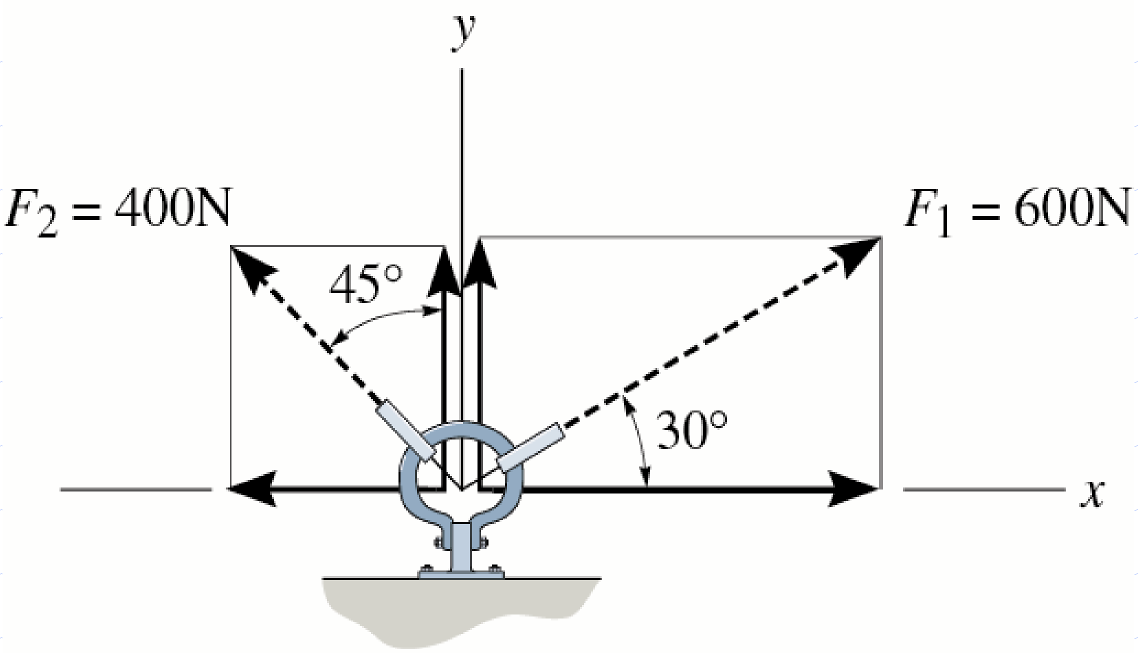
$$F_y = 200\,\frac{\sin 70^0}{\sin 60^0} = 217\,lb$$

# Example 3




The link in the figure is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the resultant magnitude and orientation of the resultant force.

# Scalar Solution



# Scalar Solution

$$\overset{+}{\rightarrow} \quad F_{R_x} = \sum F_x$$

$$F_{R_x} = 600\cos 30^{o}\,\mathrm{N} - 400\sin 45^{o}\,\mathrm{N} = 236.8\,\mathrm{N} \rightarrow$$

$$+\uparrow \quad F_{R_y} = \sum F_y$$

$$F_{R_y} = 600\sin 30^{o}\,\mathrm{N} + 400\cos 45^{o}\,\mathrm{N} = 582.8\,\mathrm{N} \uparrow$$

$$\theta = \tan^{-1}\left(\frac{582.8\,\mathrm{N}}{236.8\,\mathrm{N}}\right) = 67.9^{o}$$