

Chapter 2 – Part 1

Forces

STATICS, AGE-1330

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- ◆ Introduction
- ◆ Types of Vectors
- ◆ Vector Addition
- ◆ Force Systems
- ◆ Components of a Force
- ◆ Additional Exercises

1. Introduction



Definitions

Scalar - A quantity characterized by a positive or negative number is called a scalar. Examples of scalars used in Statics are mass, volume or length.

Definitions

Vector - A quantity that has both magnitude and a direction. Examples of vectors used in Statics are position, force, and moment.

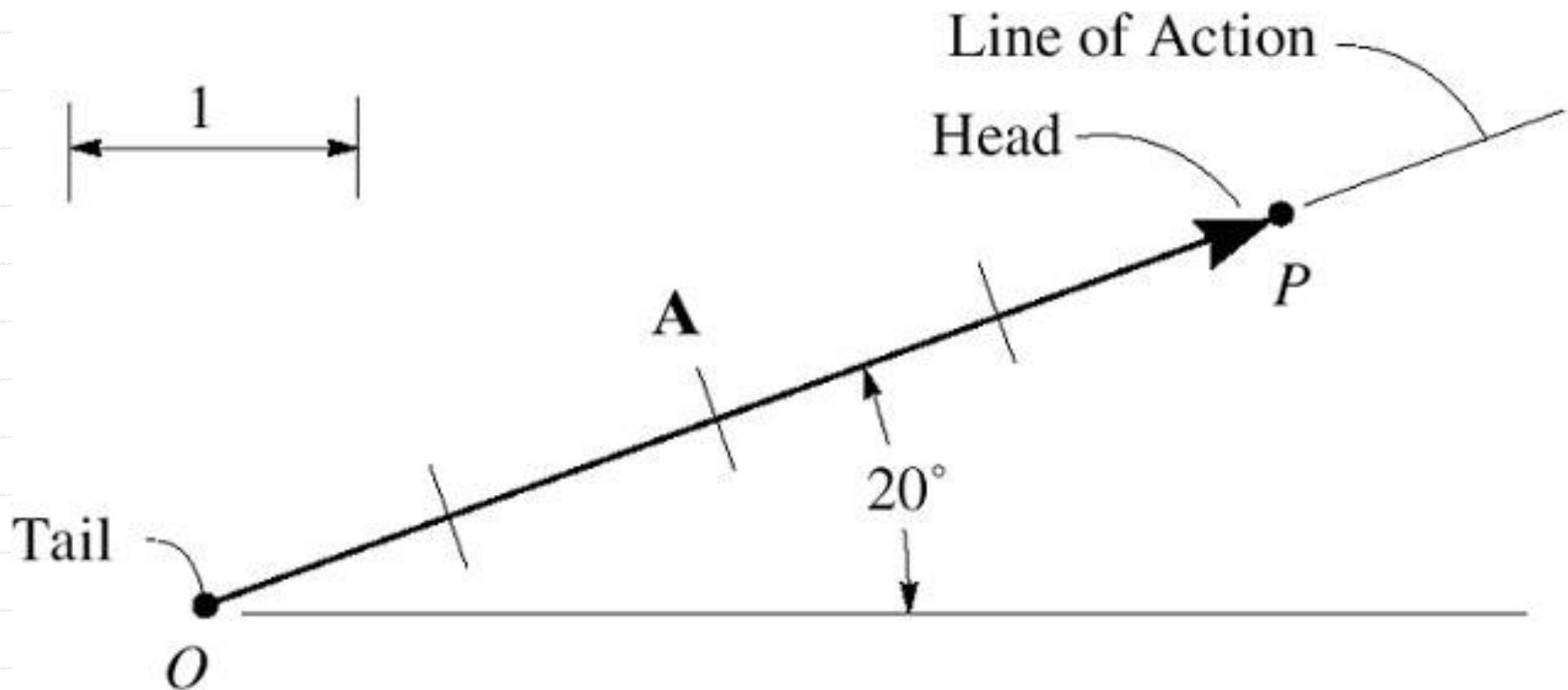
Mechanics: Scalars and Vectors

- Scalar
 - Only **magnitude** is associated with it
 - e.g., time, volume, density, speed, energy, mass etc.
- Vector
 - Possess **direction** as well as **magnitude**
 - Parallelogram law of addition (and the triangle law)
 - e.g., displacement, velocity, acceleration etc.

Symbols

Vectors are denoted by a letter with an arrow over it or a boldface letter such as **A**.

Vector Definitions



Mechanics: Scalars and Vectors

A Vector **$\mathbf{V}$** can be written as: **$\mathbf{V} = V\mathbf{n}$**

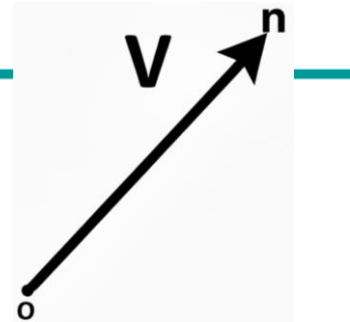
V = magnitude of **$\mathbf{V}$**

$\mathbf{n}$ = unit vector whose magnitude is one and whose direction coincides with that of **$\mathbf{V}$**

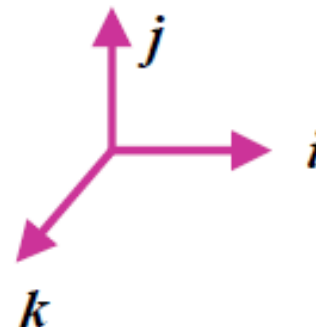
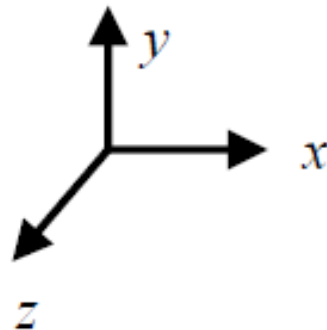
Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

Vectors represented by Bold and Non-Italic letters (**$\mathbf{V}$**)

Magnitude of vectors represented by Non-Bold, Italic letters (V)



$$\frac{\mathbf{V}}{V} = \mathbf{n}$$



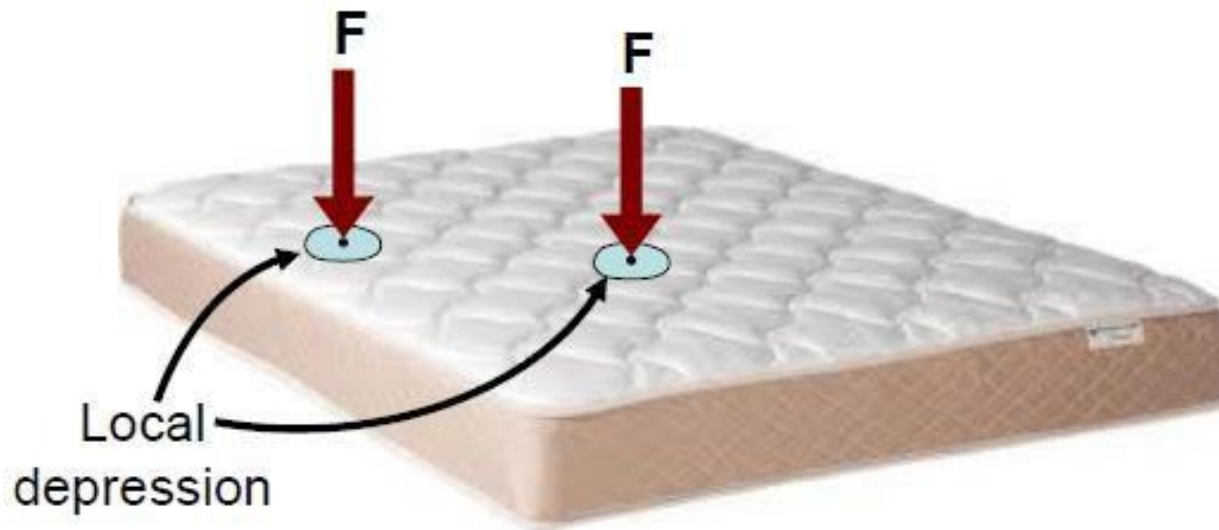
2. Types of Vectors



Types of Vectors: Fixed Vector

- **Fixed Vector**

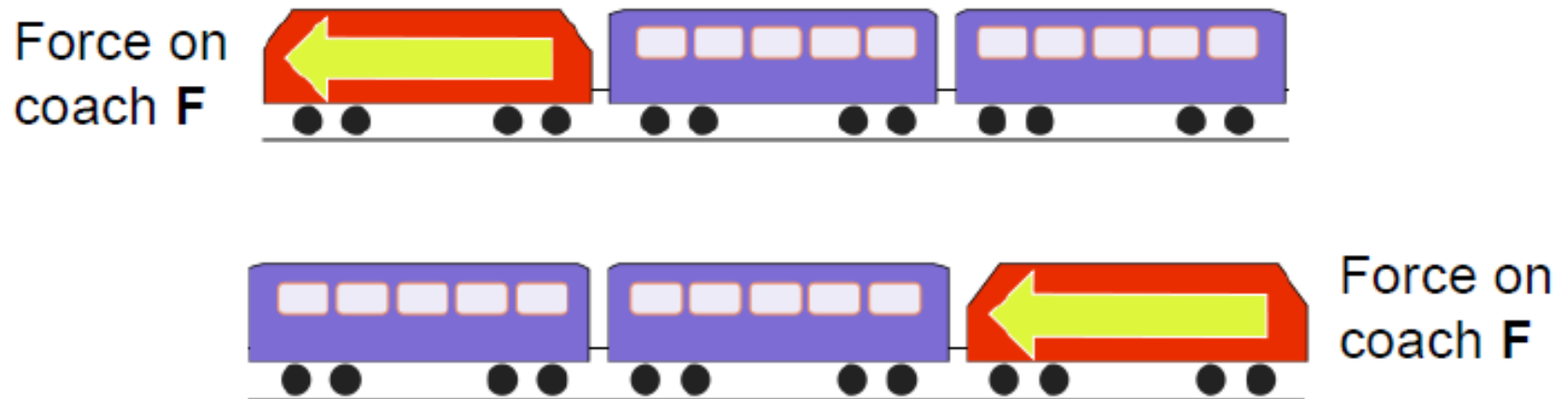
- Constant magnitude and direction
 - **Unique point of application**
- e.g., force on a deformable body



- e.g., force on a given particle

Types of Vectors: Sliding Vector

- **Sliding Vector**
 - Constant magnitude and direction
 - **Unique line of action**
 - “Slide” along the line of action
 - **No unique point of application**



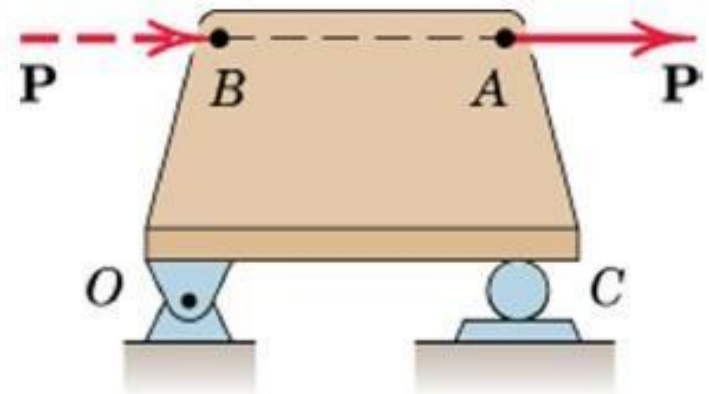
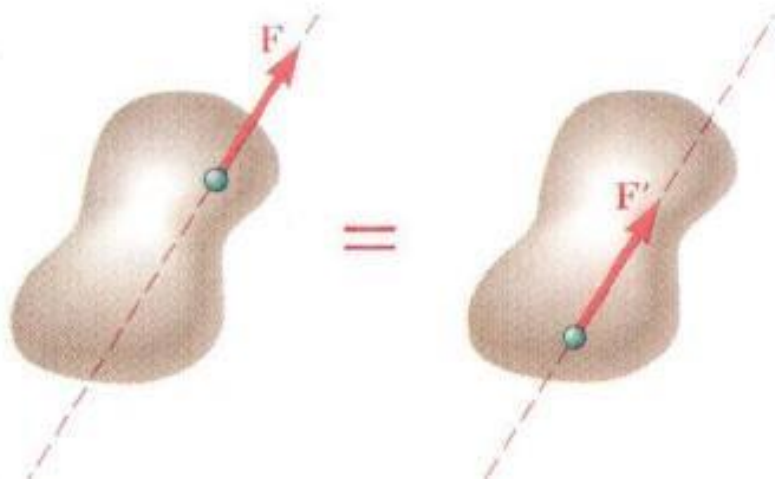
Types of Vectors: Sliding Vector

- **Sliding Vector**

- Principle of Transmissibility

- **Application of force at any point along a particular line of action**

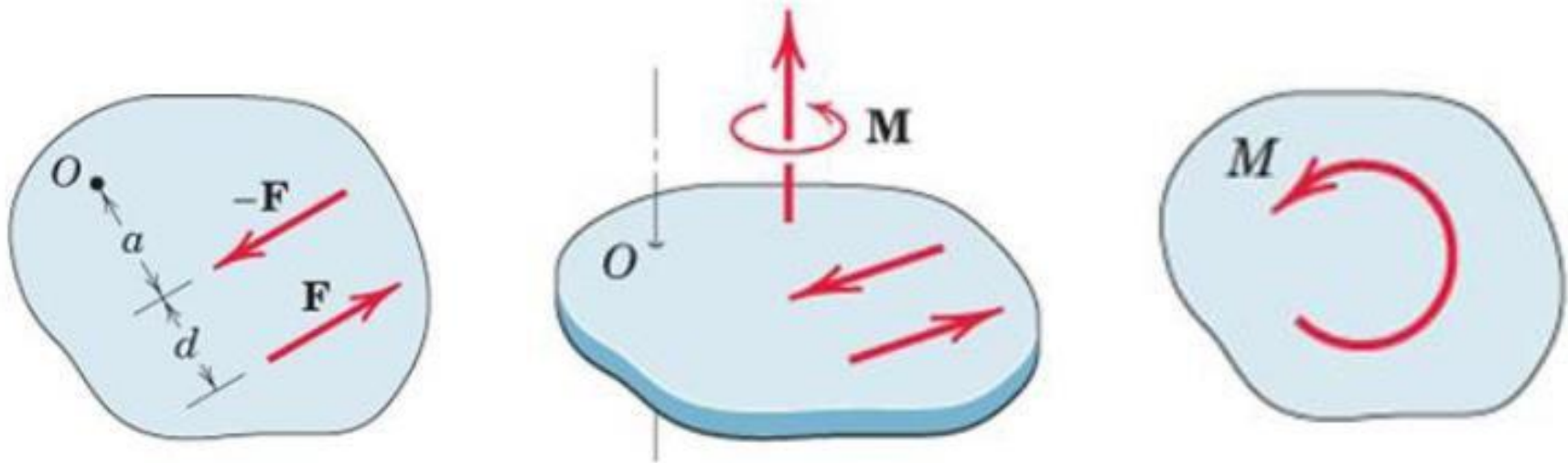
- No change in resultant external effects of the force



Types of Vectors: Free Vector

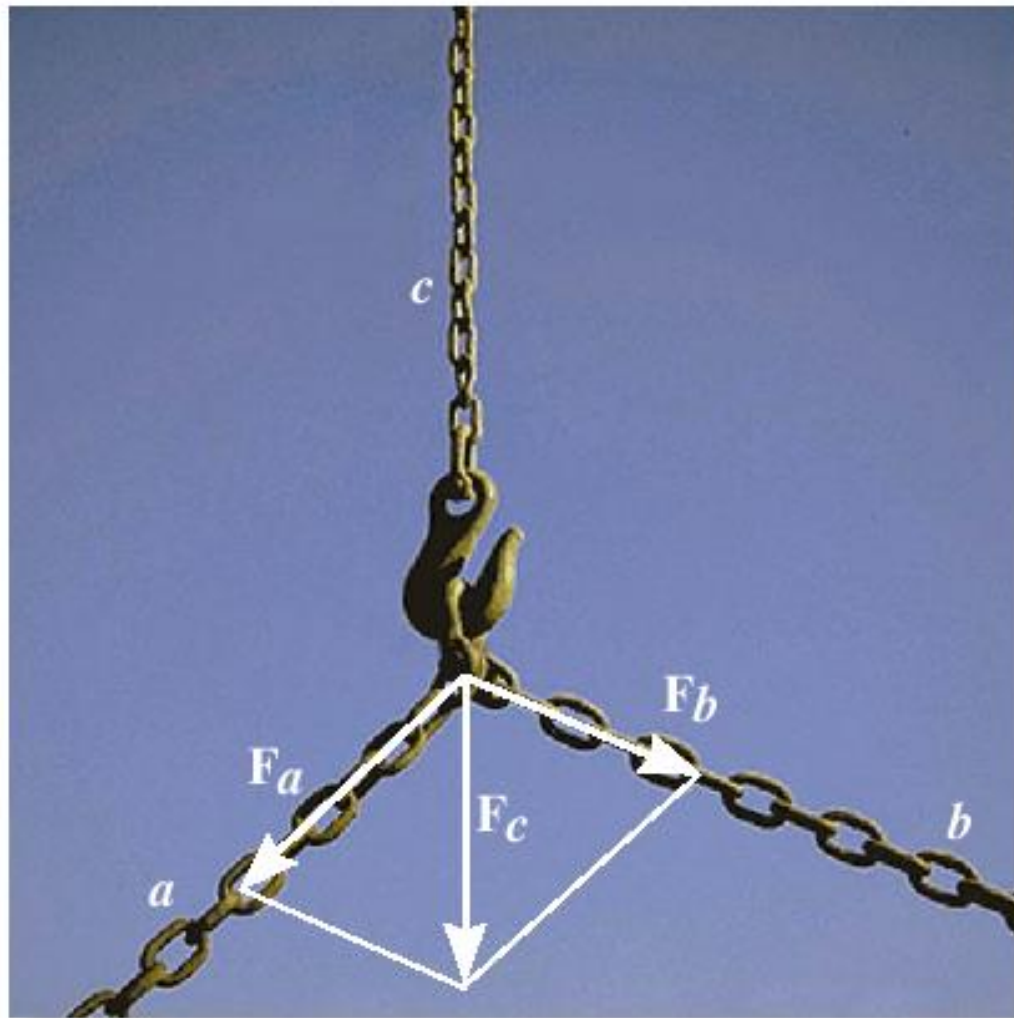
- **Free Vector**

- Freely movable in space
 - **No unique line of action**
 - **No unique point of application**
- e.g., moment of a couple



3. Vector Addition





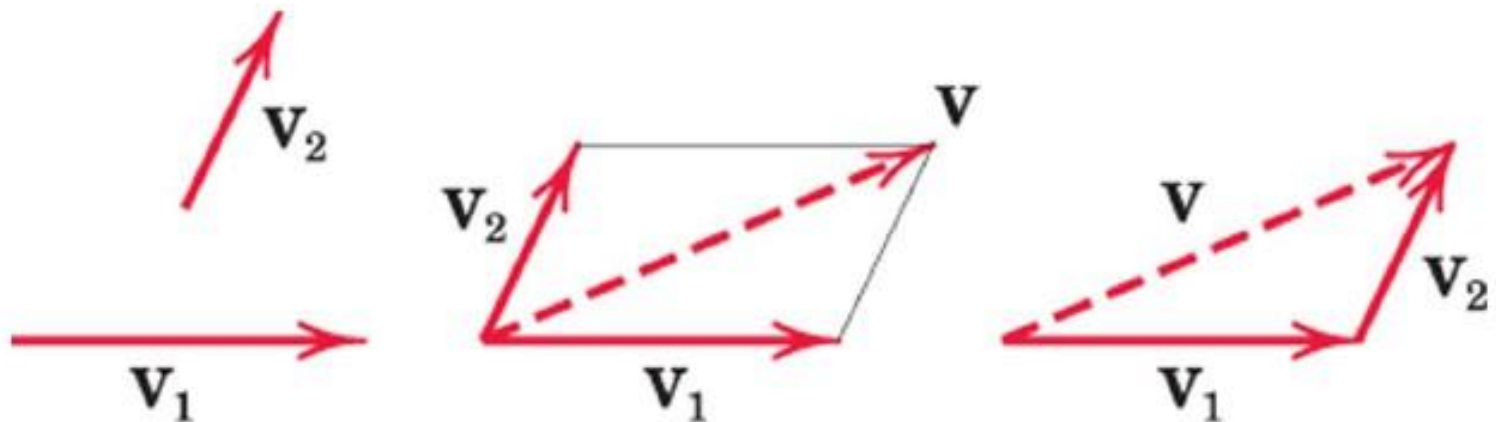
Vectors: Rules of addition

- **Parallelogram Law**

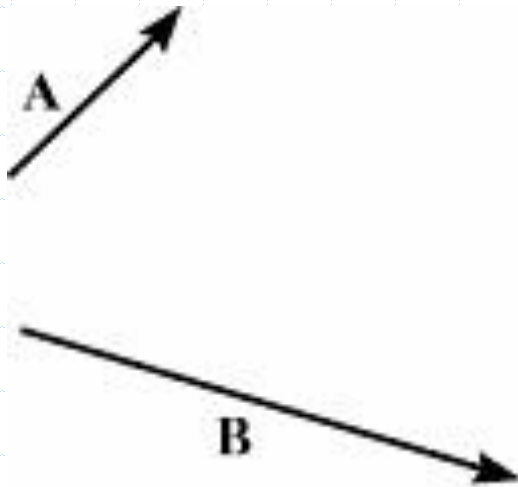
- Equivalent vector represented by the diagonal of a parallelogram

- $V = V_1 + V_2$ (*Vector Sum*)

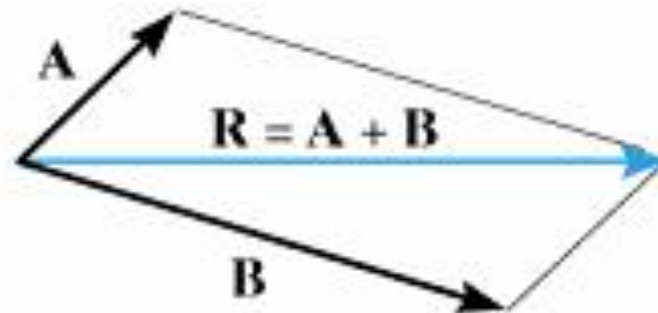
- $V \neq V_1 + V_2$ (*Scalar sum*)



Vector Addition



(a)



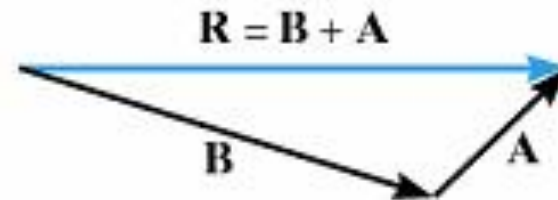
Parallelogram Law

(b)

Vector Addition

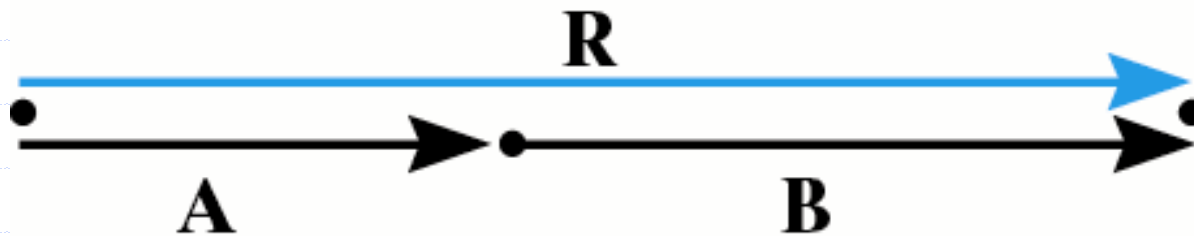


Triangle construction
(c)



Triangle construction
(d)

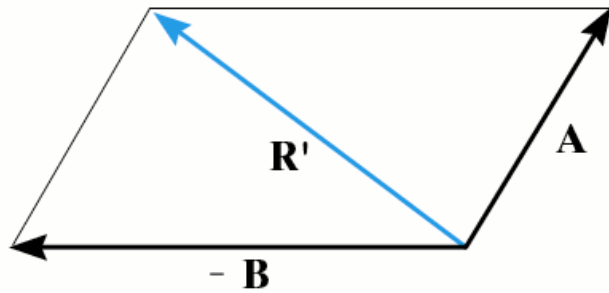
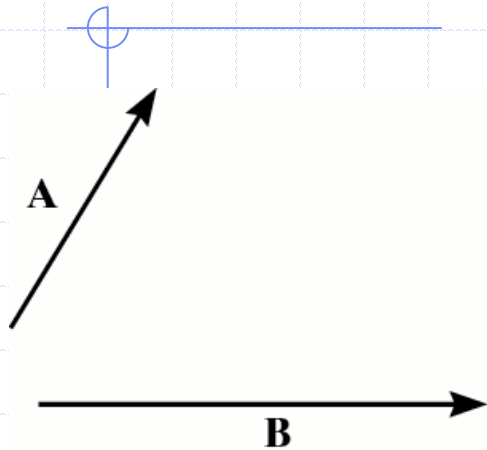
Vector Addition



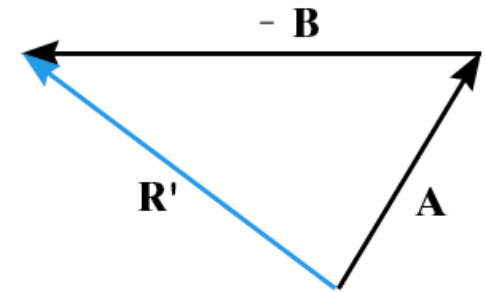
$$R = A + B$$

Addition of collinear vectors

Vector Subtraction



or



Parallelogram law

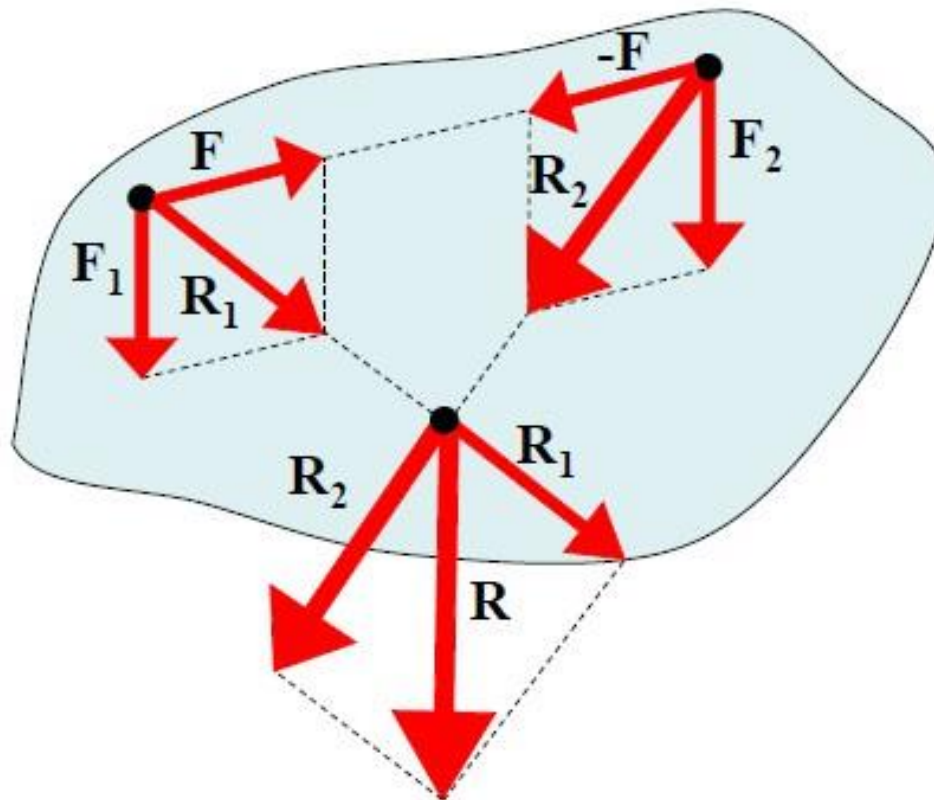
Triangle construction

Vector Subtraction

Vectors: Parallelogram law of addition

- Addition of two parallel vectors

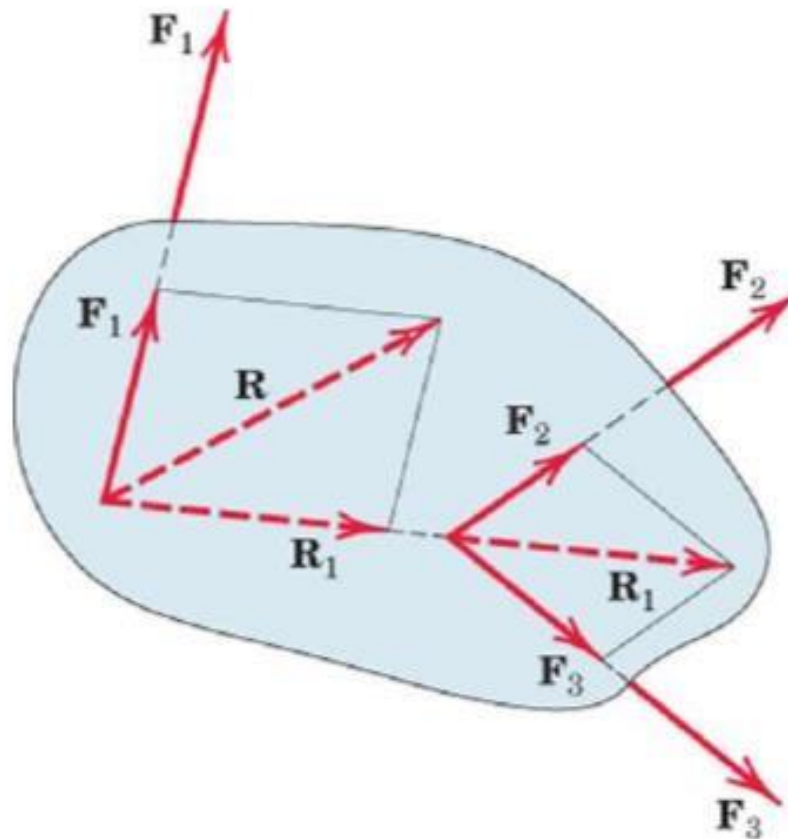
$$\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{R}$$

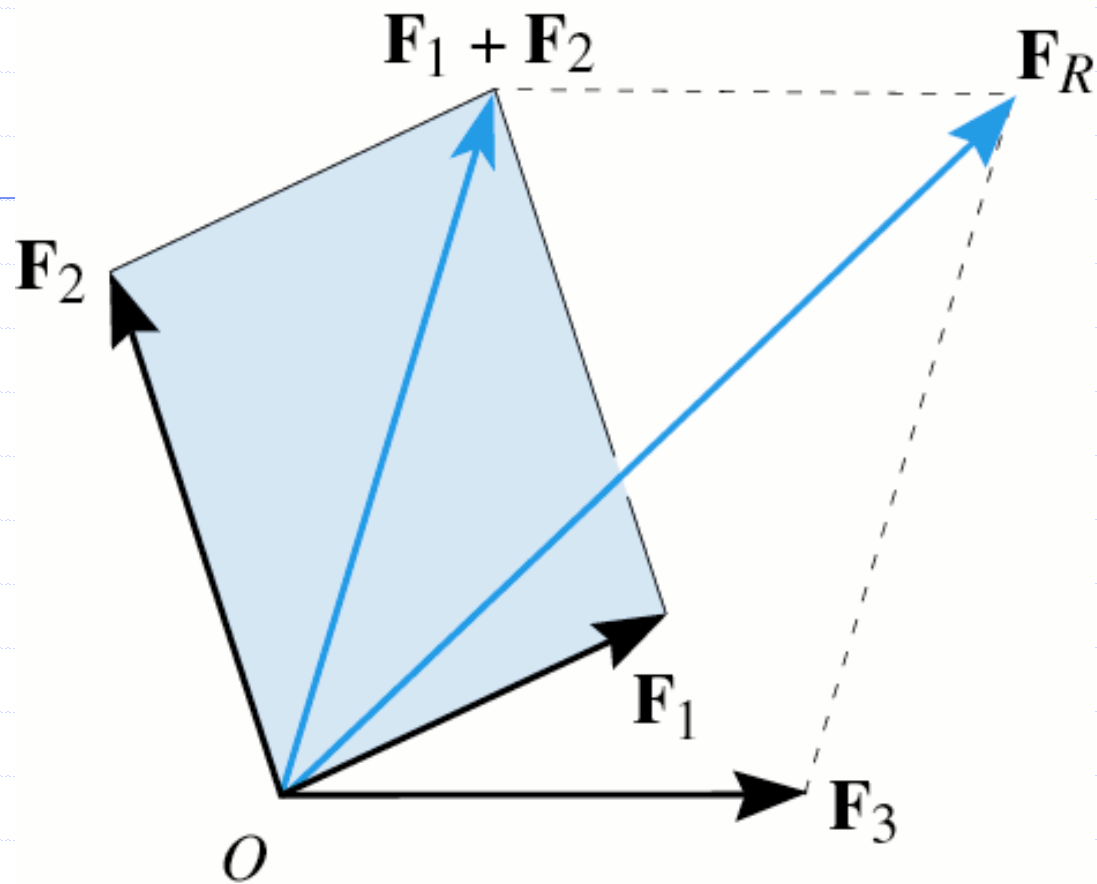


Vectors: Parallelogram law of addition

- Addition of 3 vectors

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{R}$$

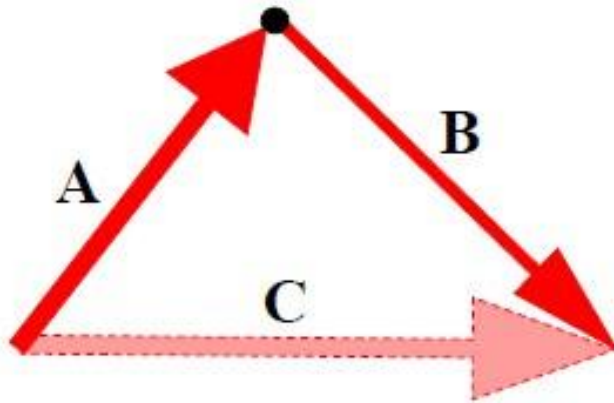




Vectors: Rules of addition

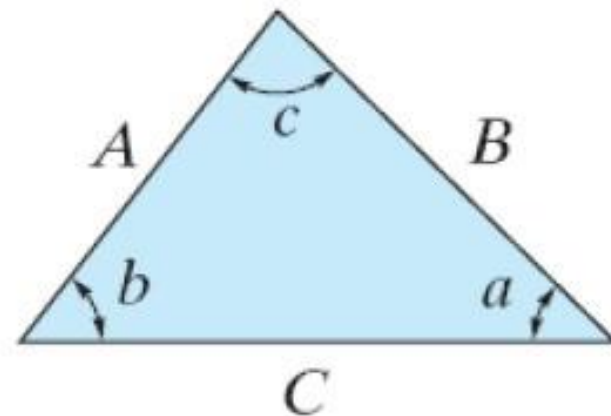
- **Trigonometric Rule**

- Law of Sines
- Law of Cosine



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

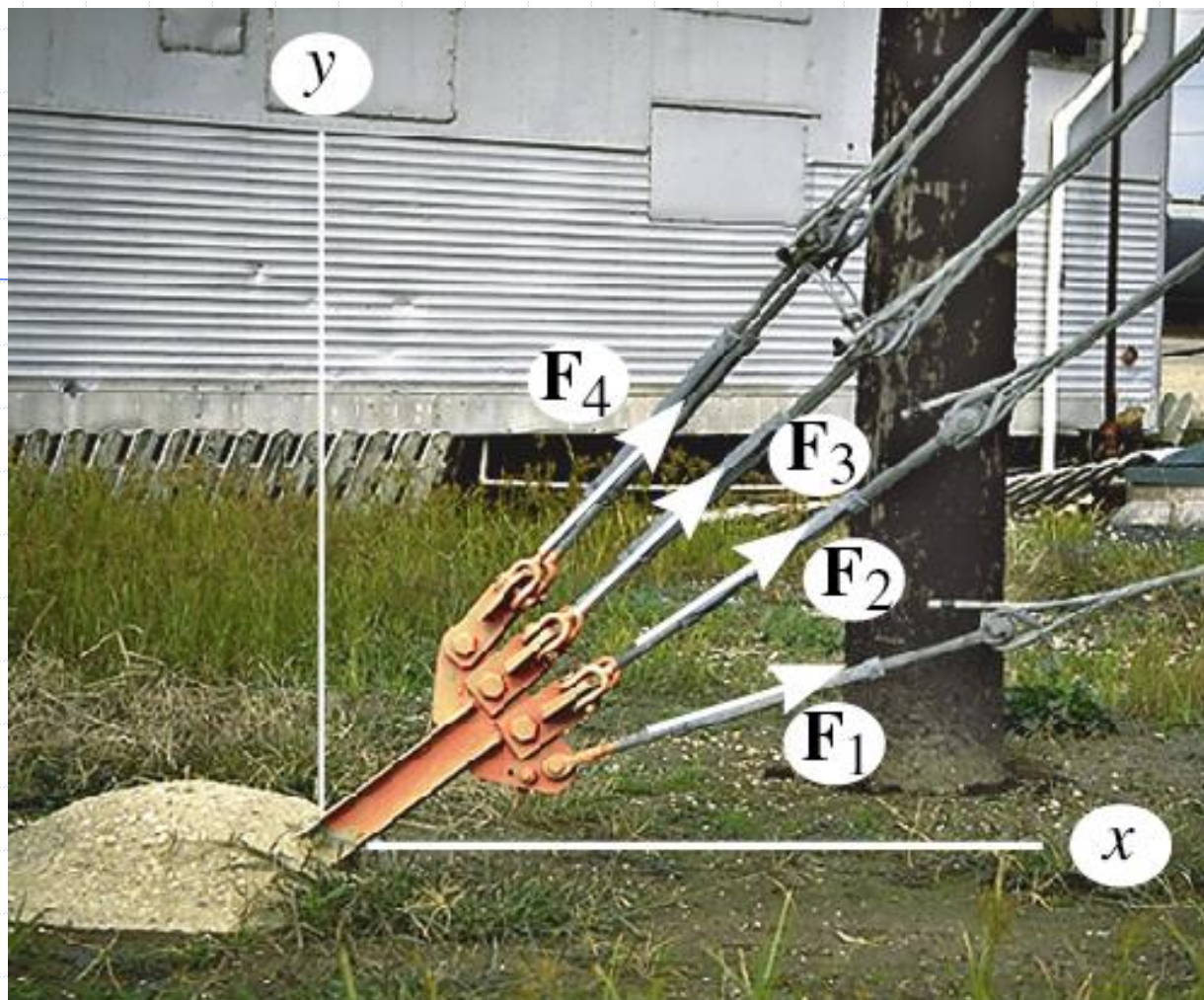


Cosine law:

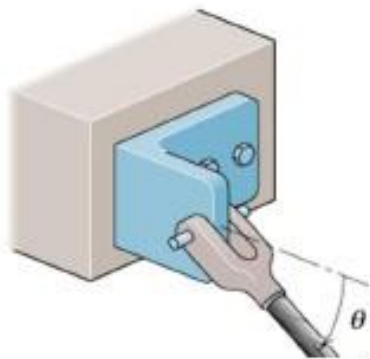
$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

4. Force Systems

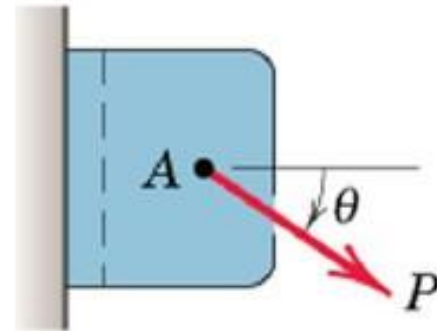




Force Systems



Cable Tension P



- **Force:** Represented by vector
 - Magnitude, direction, point of application
 - P : fixed vector (or sliding vector??)
 - External Effect
 - Applied force; Forces exerted by bracket, bolts, Foundation (reactive force)

Force Systems

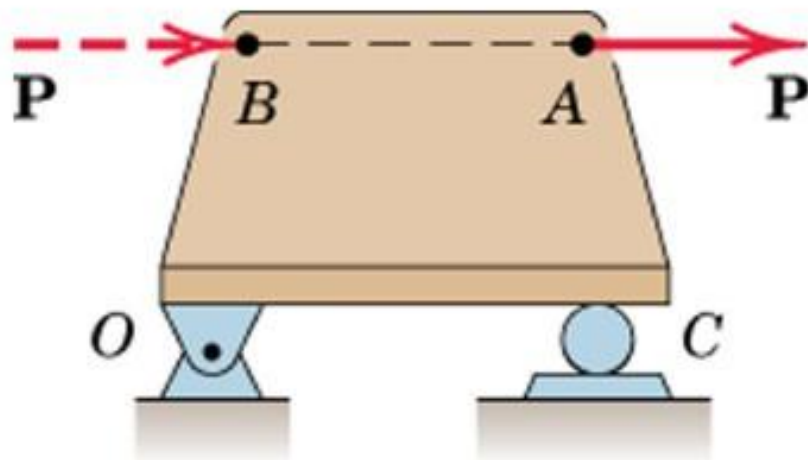
- **Rigid Bodies**

- External effects only

- **Line of action** of force is **important**

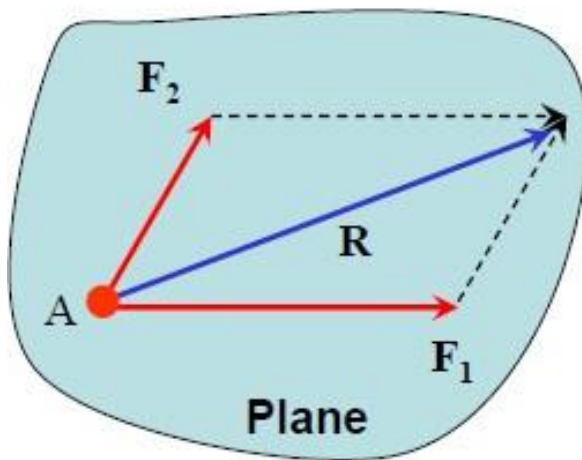
- Not its point of application

- Force as **sliding vector**

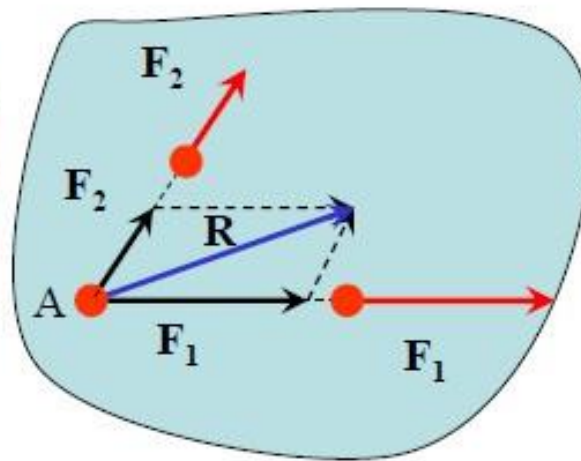


Force Systems

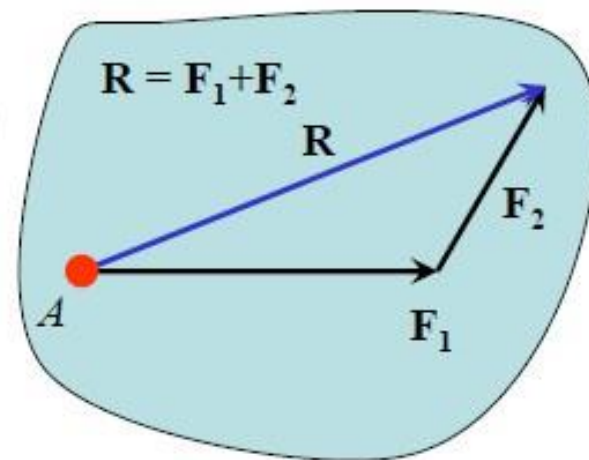
- **Concurrent forces**
 - Lines of action intersect at a point



Concurrent Forces
 F_1 and F_2



Principle of
Transmissibility

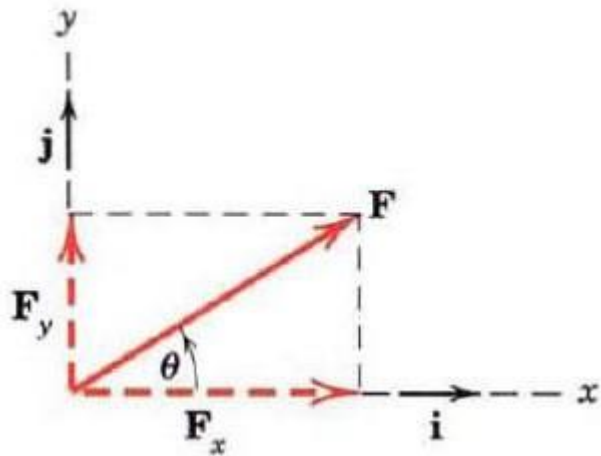


$$R = F_1 + F_2$$

5. Components of a Force



Rectangular components of 2-D Force System

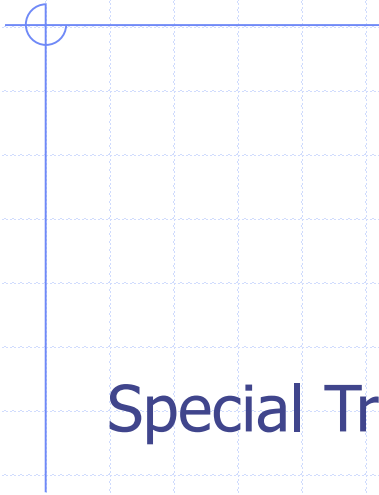


$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

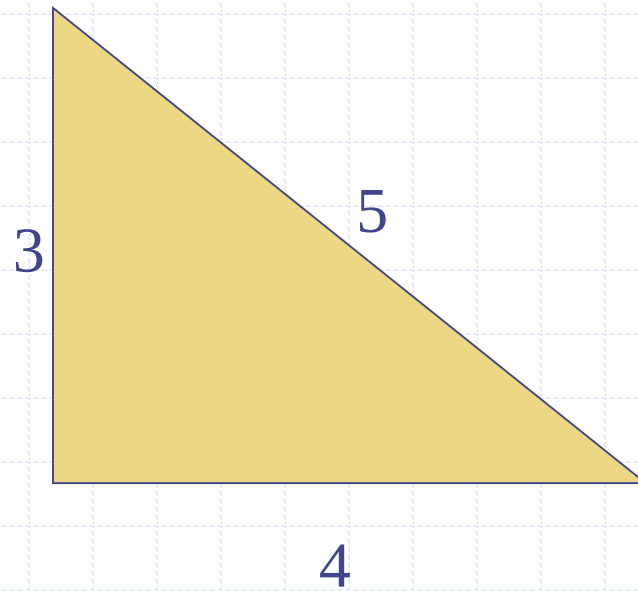
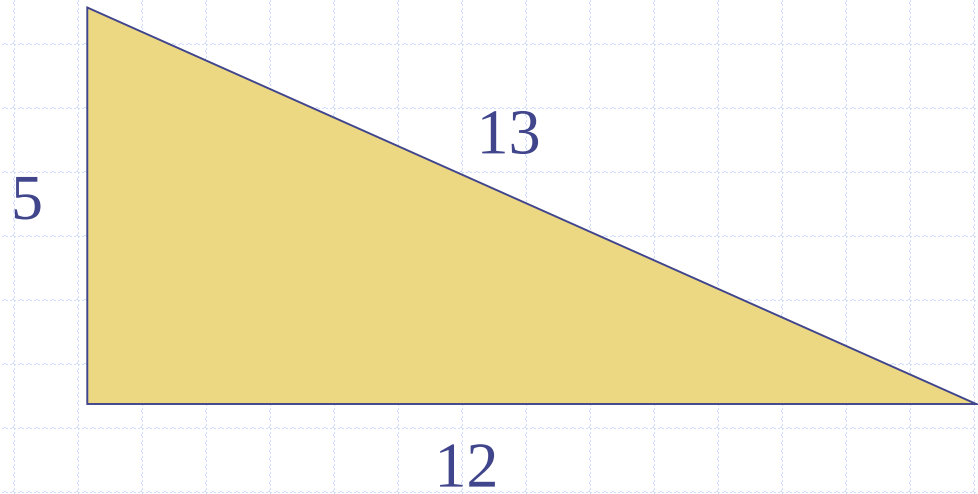
$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

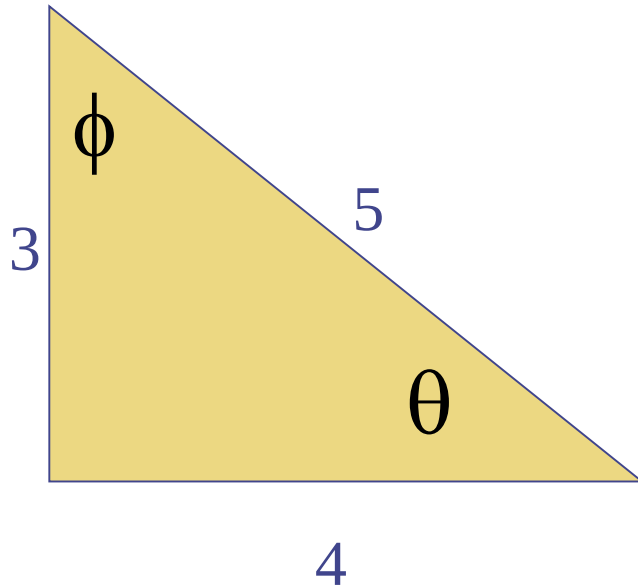
$$\mathbf{F} = F \angle \theta$$

$$F_x = F \cos \theta \quad F = \sqrt{F_x^2 + F_y^2}$$
$$F_y = F \sin \theta \quad \theta = \tan^{-1} \frac{F_y}{F_x}$$



Special Triangles



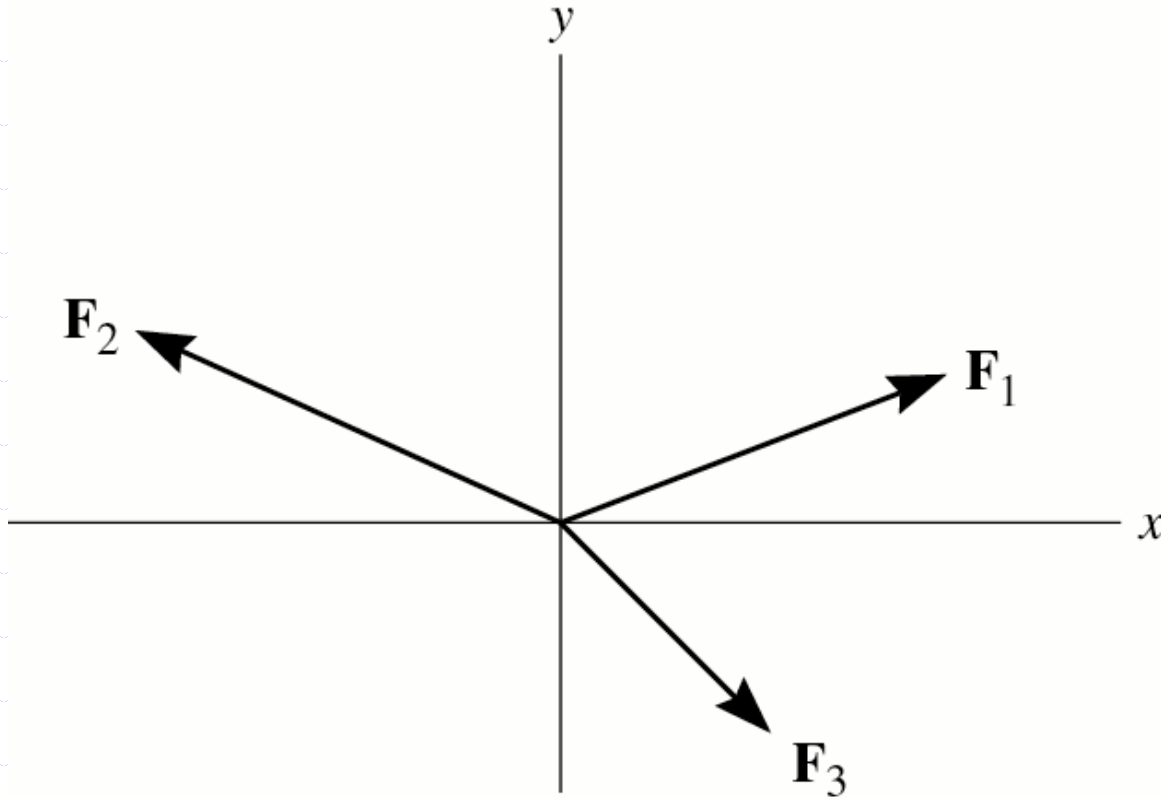


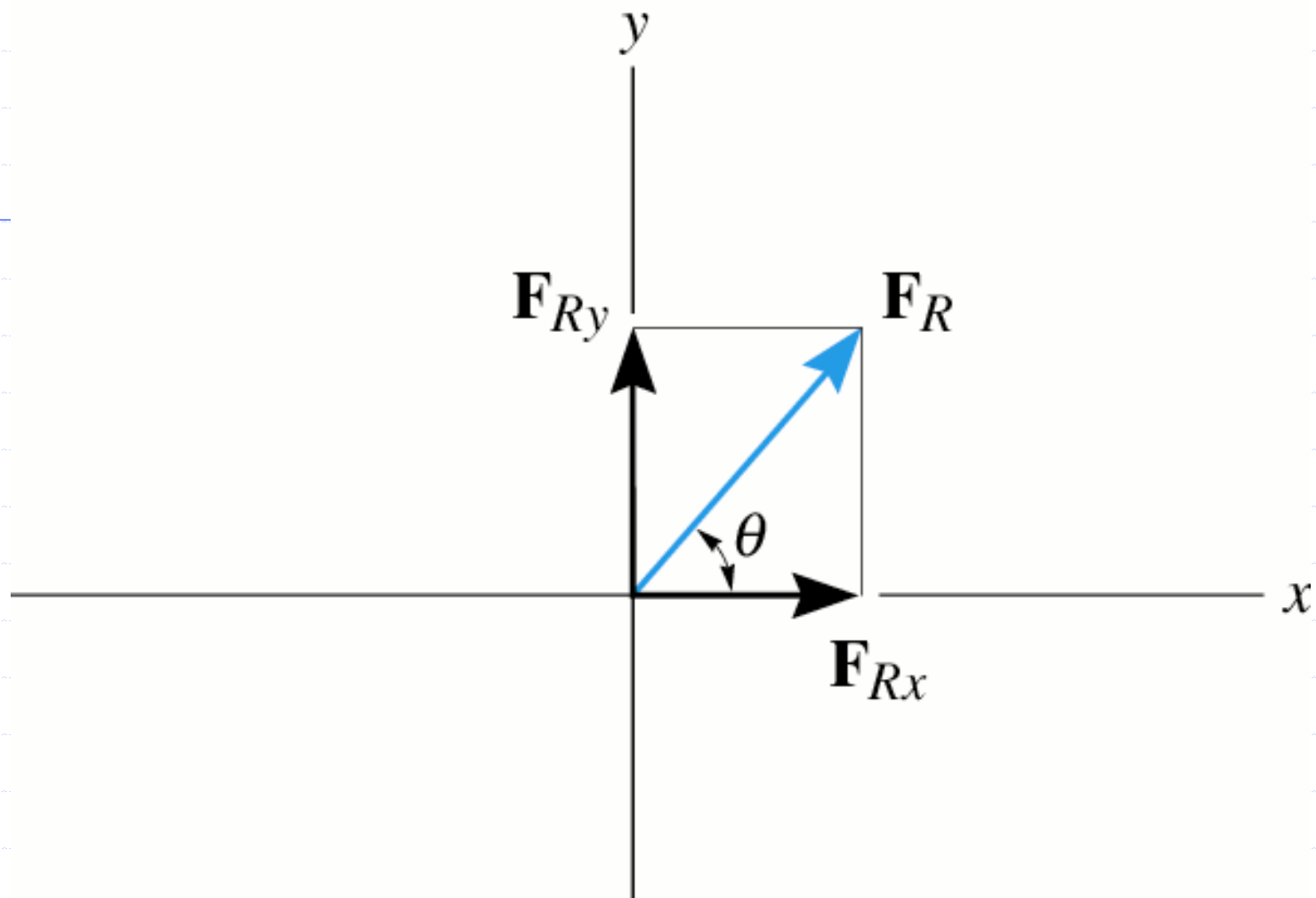
$$5 = \sqrt{3^2 + 4^2}$$

$$\cos \theta = \frac{4}{5} = 0.8 \quad \sin \theta = \frac{3}{5} = 0.6$$

$$\cos \phi = \frac{3}{5} = 0.6 \quad \sin \phi = \frac{4}{5} = 0.8$$

Coplanar Force Resultants





$$F_{Rx} = \sum F_x$$

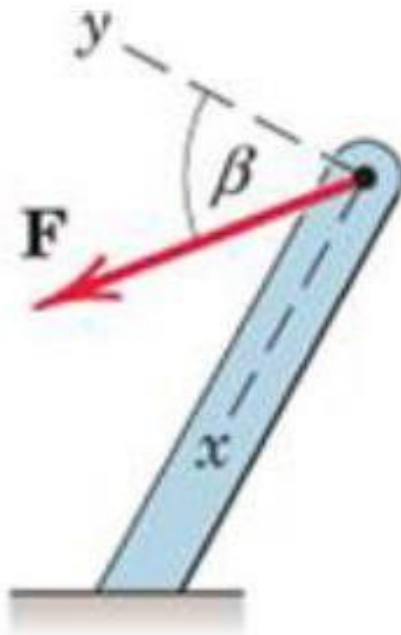
$$F_{Ry} = \sum F_y$$

$$|\vec{F}_R| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

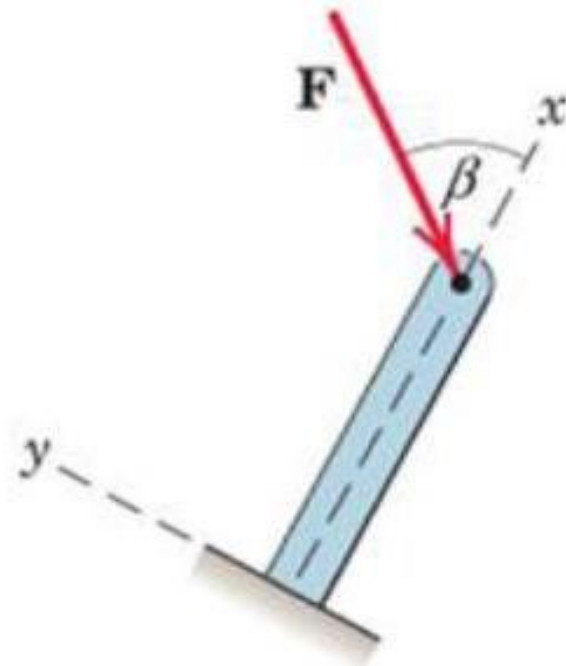
$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$

Components of a Force

- Examples



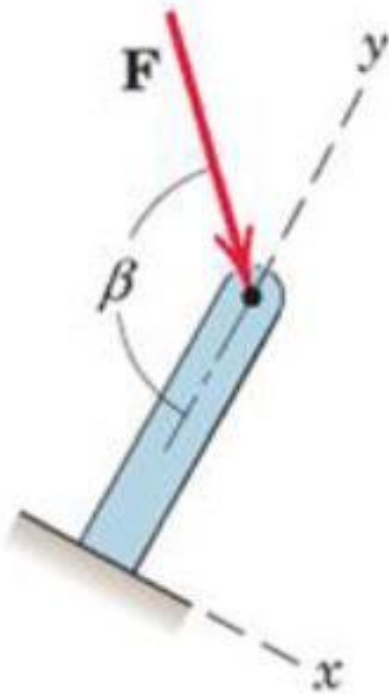
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



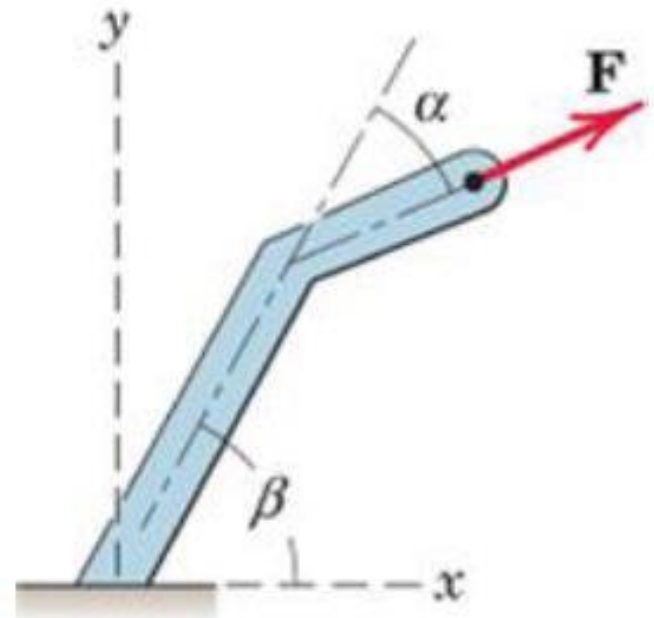
$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

Components of a Force

- Examples



$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$

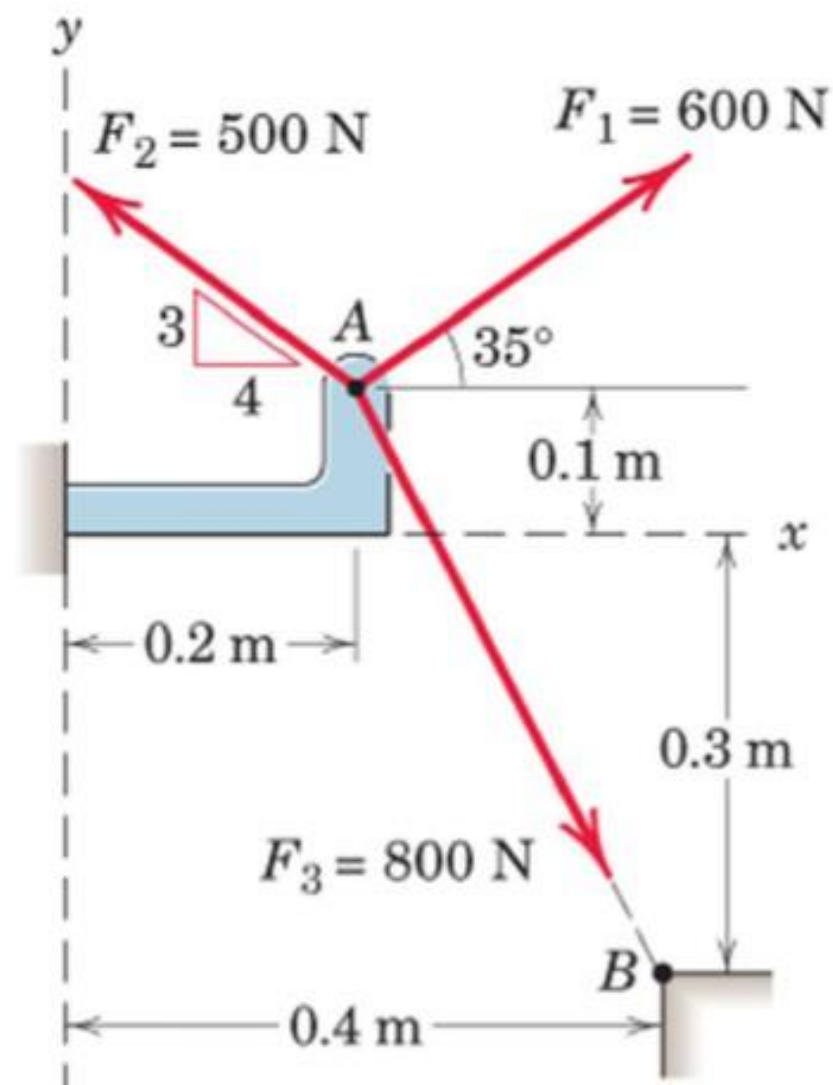


$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

Components of a Force

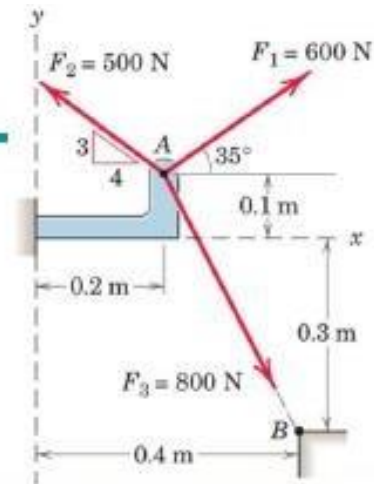
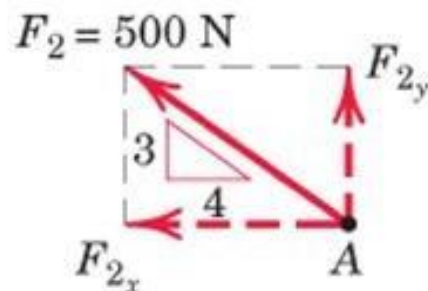
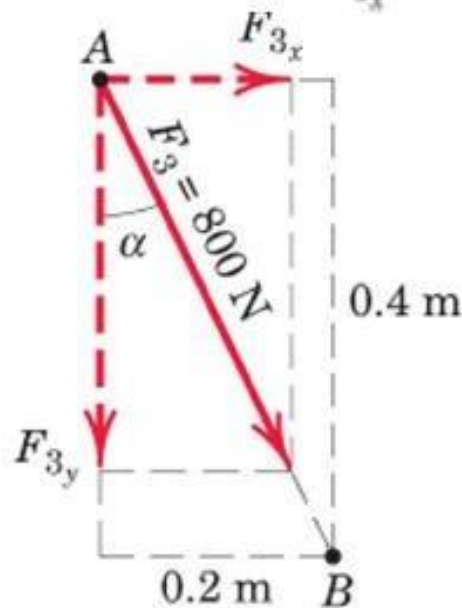
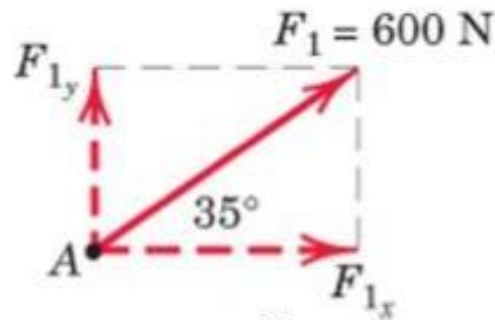
Example 1:

Determine the x and y scalar components of F_1 , F_2 , and F_3 acting at point A of the bracket



Components of Force

Solution:



$$F_{1x} = 600 \cos 35^\circ = 491\text{ N}$$

$$F_{1y} = 600 \sin 35^\circ = 344\text{ N}$$

$$F_{2x} = -500\left(\frac{4}{5}\right) = -400\text{ N}$$

$$F_{2y} = 500\left(\frac{3}{5}\right) = 300\text{ N}$$

$$\alpha = \tan^{-1} \left[\frac{0.2}{0.4} \right] = 26.6^\circ$$

$$F_{3x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358\text{ N}$$

$$F_{3y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716\text{ N}$$

Components of Force

Alternative Solution: Scalar components of $\mathbf{F}_3$ can be obtained by writing $\mathbf{F}_3$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment AB.

Unit vector can be formed by dividing any vector, such as the geometric position vector by its length or magnitude.

$$\mathbf{F}_3 = F_3 \mathbf{n}_{AB} = F_3 \frac{\overrightarrow{AB}}{AB} = 800 \left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}} \right]$$

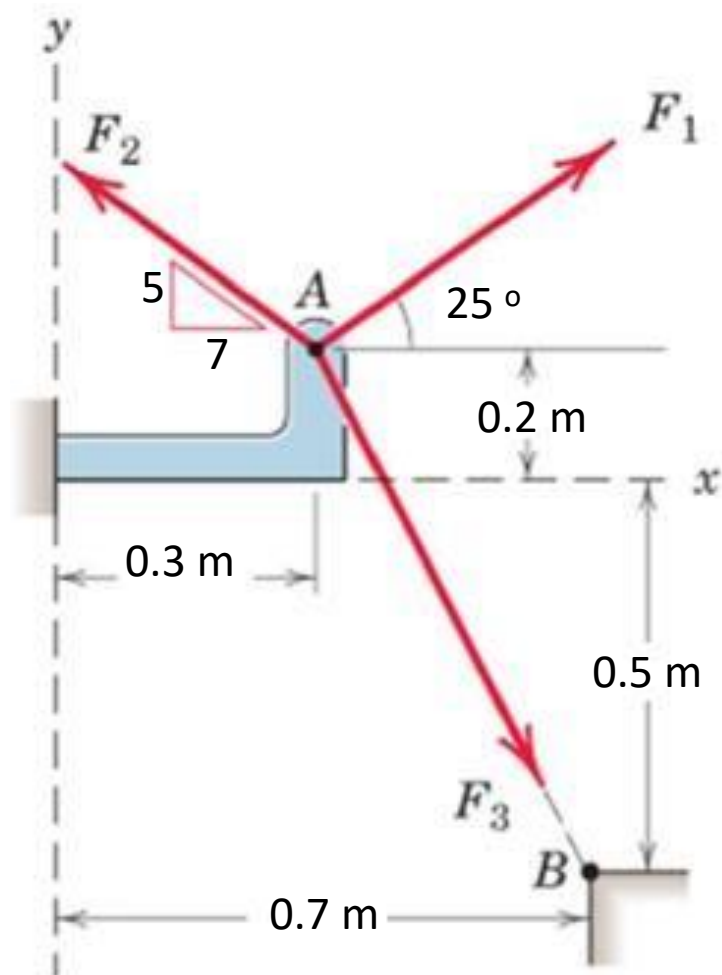
$$= 800[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j} \text{ N}$$

$$F_{3_x} = 358 \text{ N}$$

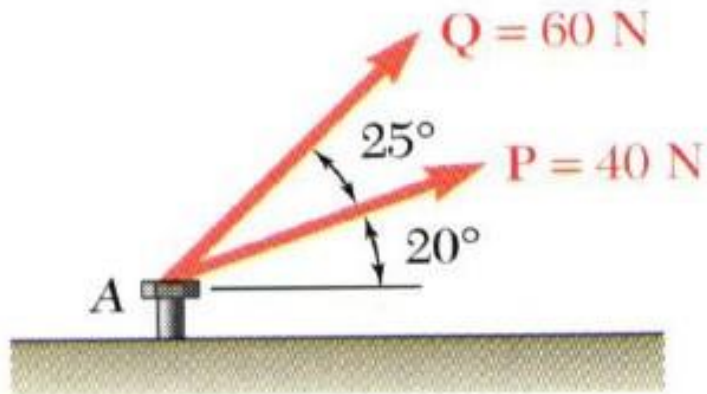
$$F_{3_y} = -716 \text{ N}$$

Determine the components of each force
Evaluate the resultant in x and y directions



Components of Force

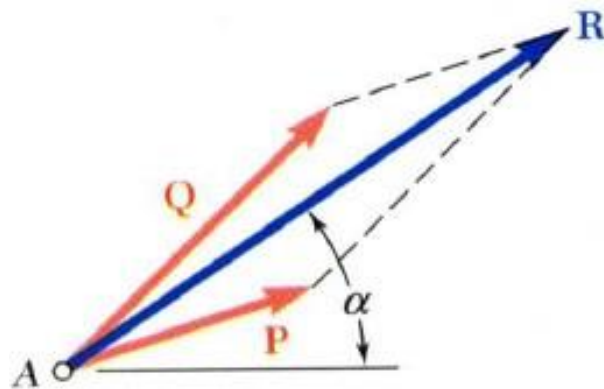
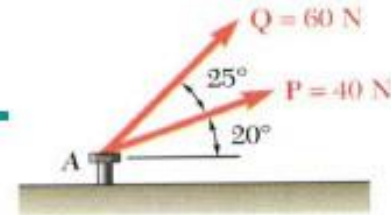
Example 2: The two forces act on a bolt at A. Determine their resultant.



- **Graphical solution –**
- Construct a parallelogram with sides in the same direction as P and Q and lengths in proportion.
- Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.
- **Trigonometric solution**
- Use the law of cosines and law of sines to find the resultant.

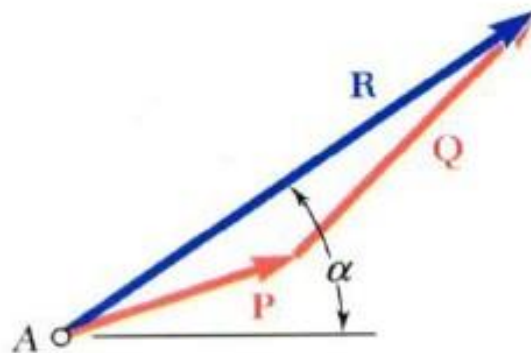
Components of Force

Solution:



- **Graphical solution** - A parallelogram with sides equal to P and Q is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$$\mathbf{R = 98\ N \quad \alpha = 35^\circ}$$

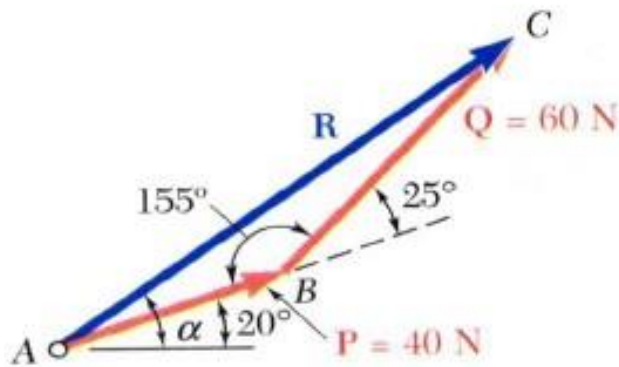
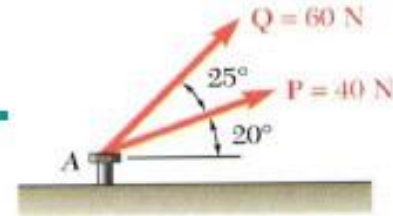


- **Graphical solution** - A triangle is drawn with P and Q head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\mathbf{R = 98\ N \quad \alpha = 35^\circ}$$

Components of Force

Trigonometric Solution:



$$\begin{aligned} R^2 &= P^2 + Q^2 - 2PQ \cos B \\ &= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155^\circ \end{aligned}$$

$$\boxed{R = 97.73\text{N}}$$

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$

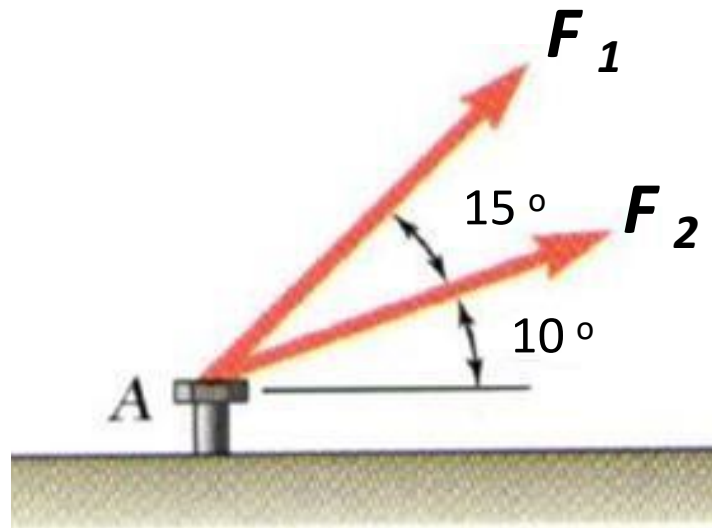
$$= \sin 155^\circ \frac{60\text{N}}{97.73\text{N}}$$

$$A = 15.04^\circ$$

$$\alpha = 20^\circ + A$$

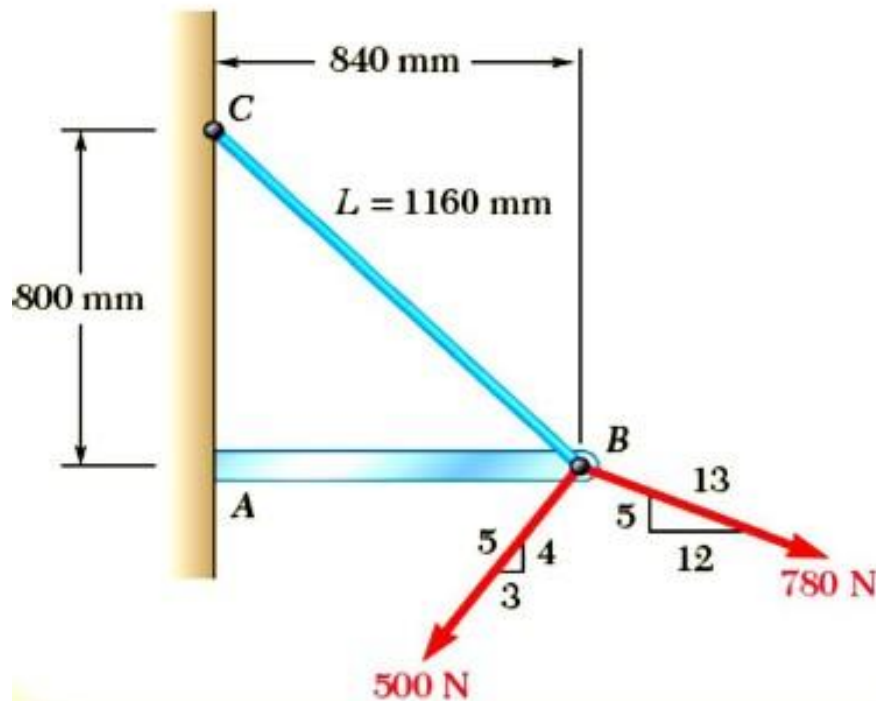
$$\boxed{\alpha = 35.04^\circ}$$

Determine the resultant force if $F_1 = F_2 = 200\text{ N}$.



Components of Force

Example 3: Tension in cable BC is 725 N; determine the resultant of the three forces exerted at point B of beam AB .

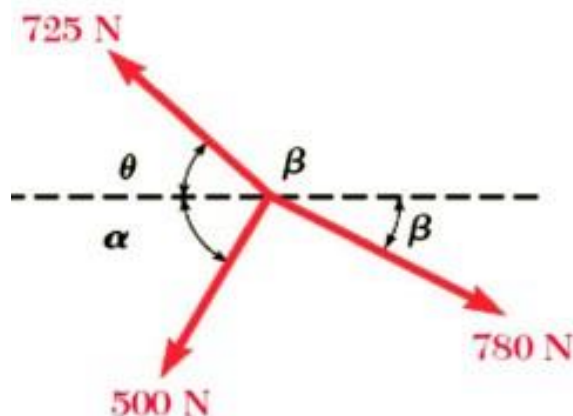


Solution:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force

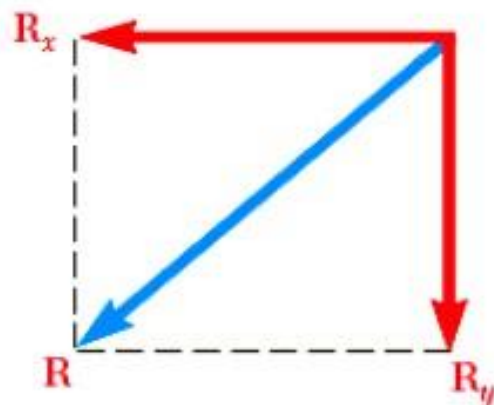
Solution



- Resolve each force into rectangular components.

Magnitude, N	x Component, N	y Component, N
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$

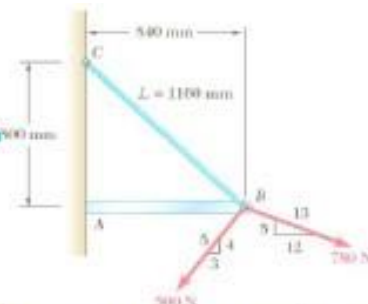
$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} \quad \mathbf{R} = (-105 \text{ N})\mathbf{i} + (-200 \text{ N})\mathbf{j}$$



- Calculate the magnitude and direction.

$$\tan \alpha = \frac{-R_y}{-R_x} = \frac{200 \text{ N}}{105 \text{ N}} \quad \alpha = 62.3^\circ$$

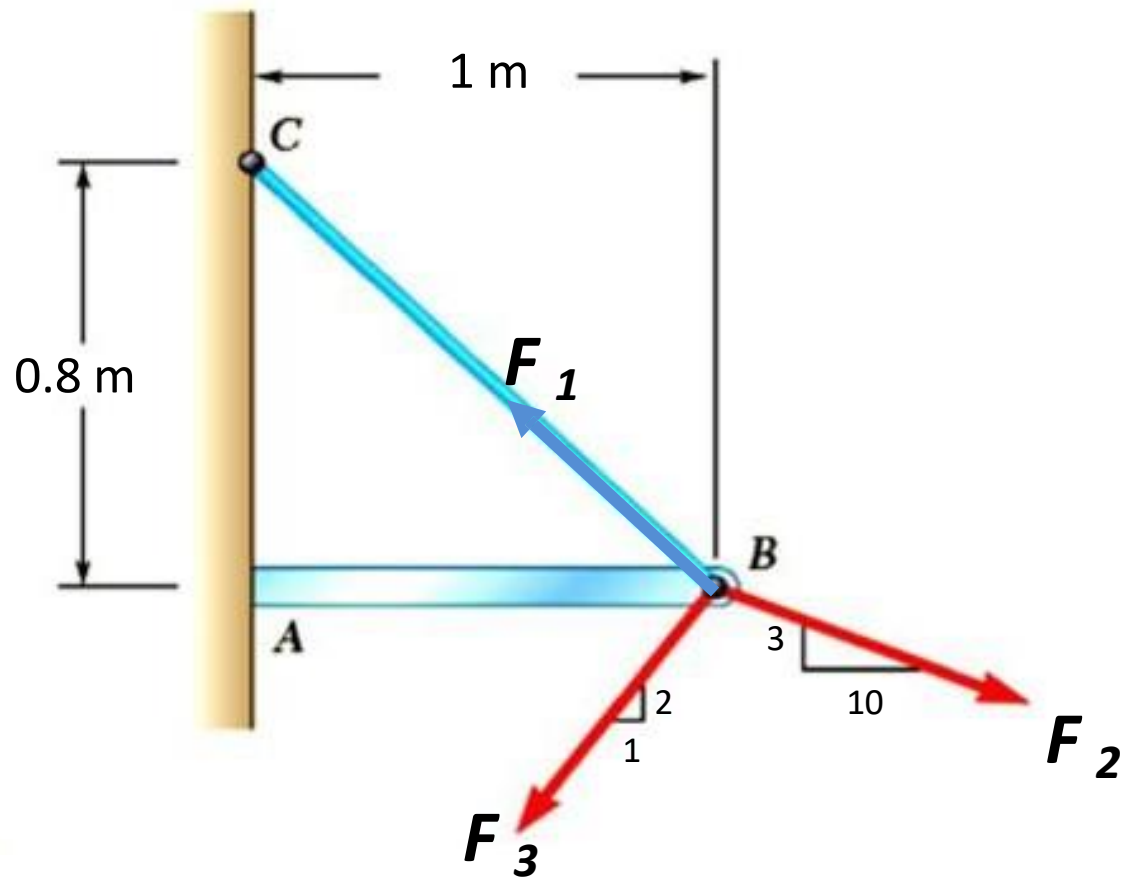
$$R = \sqrt{R_x^2 + R_y^2} = 225.9 \text{ N} \quad \angle 62.3^\circ$$



6. Additional Exercises

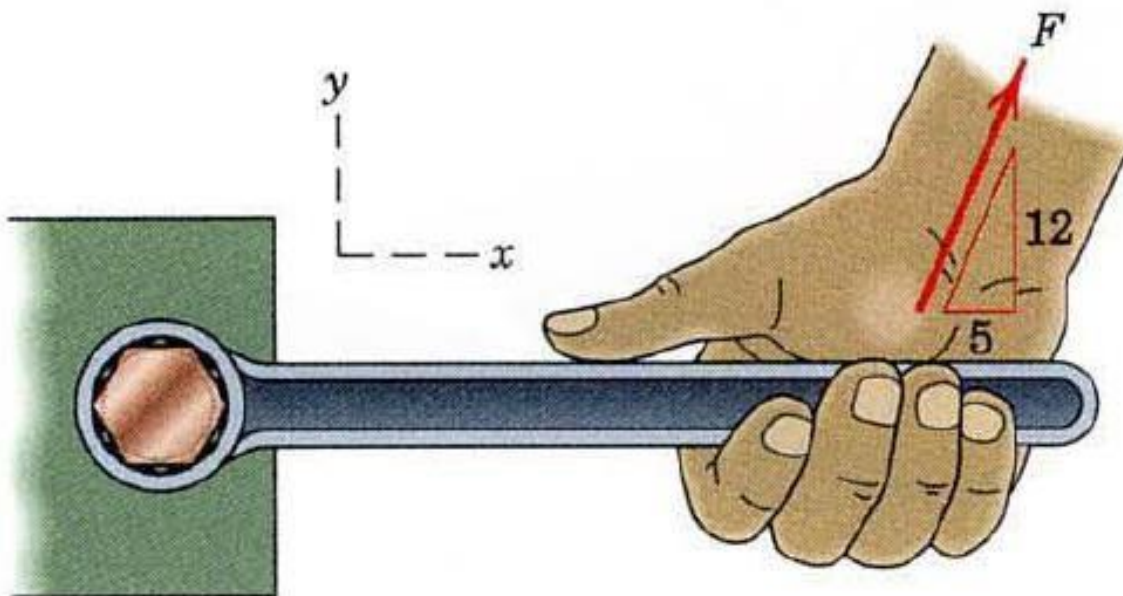


If $F_1 = 1 \text{ kN}$, $F_2 = 900 \text{ N}$, $F_3 = 1.2 \text{ kN}$, compute the resultant force acting on point B.

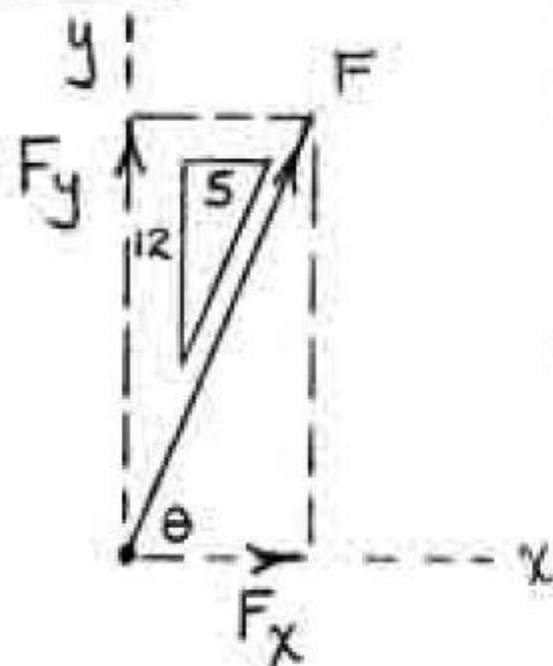


2/7 The y -component of the force $\mathbf{F}$ which a person exerts on the handle of the box wrench is known to be 70 lb. Determine the x -component and the magnitude of $\mathbf{F}$.

Ans. $F_x = 29.2$ lb, $F = 75.8$ lb



2/7



$$\cos \theta = \frac{5}{13}, \quad \sin \theta = \frac{12}{13}$$

$$F_y = F \sin \theta = F \frac{12}{13} = 70$$

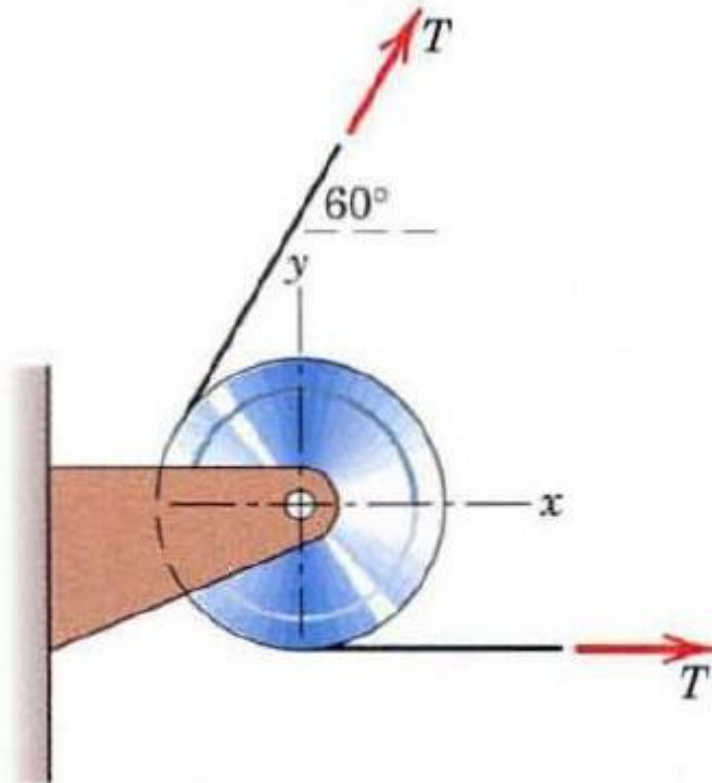
$$F = 75.8 \text{ lb}$$

$$F_x = F \cos \theta = 75.8 \left(\frac{5}{13} \right)$$

$$= \underline{29.2 \text{ lb}}$$

2/13 If the equal tensions T in the pulley cable are 400 N, express in vector notation the force $\mathbf{R}$ exerted on the pulley by the two tensions. Determine the magnitude of $\mathbf{R}$.

Ans. $\mathbf{R} = 600\mathbf{i} + 346\mathbf{j}$ N, $R = 693$ N



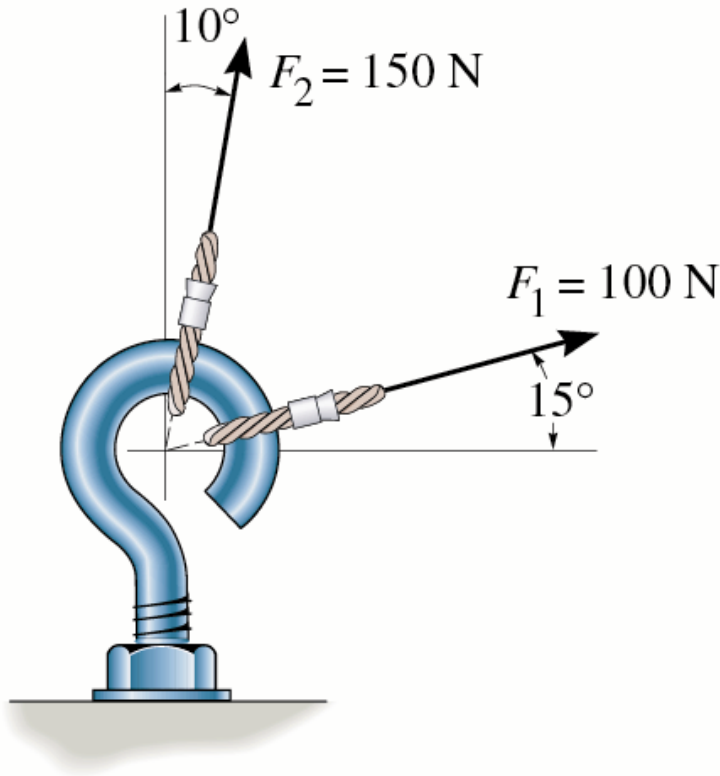
$$\underline{2/13} \quad R_x = \sum F_x = 400 + 400 \cos 60^\circ = 600 \text{ N}$$

$$R_y = \sum F_y = 400 \sin 60^\circ = 346 \text{ N}$$

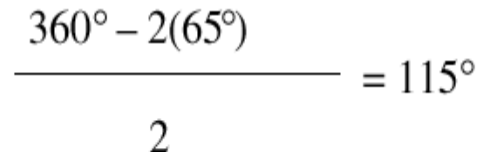
$$\Rightarrow \underline{R} = \underline{600\mathbf{i} + 346\mathbf{j} \text{ N}}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

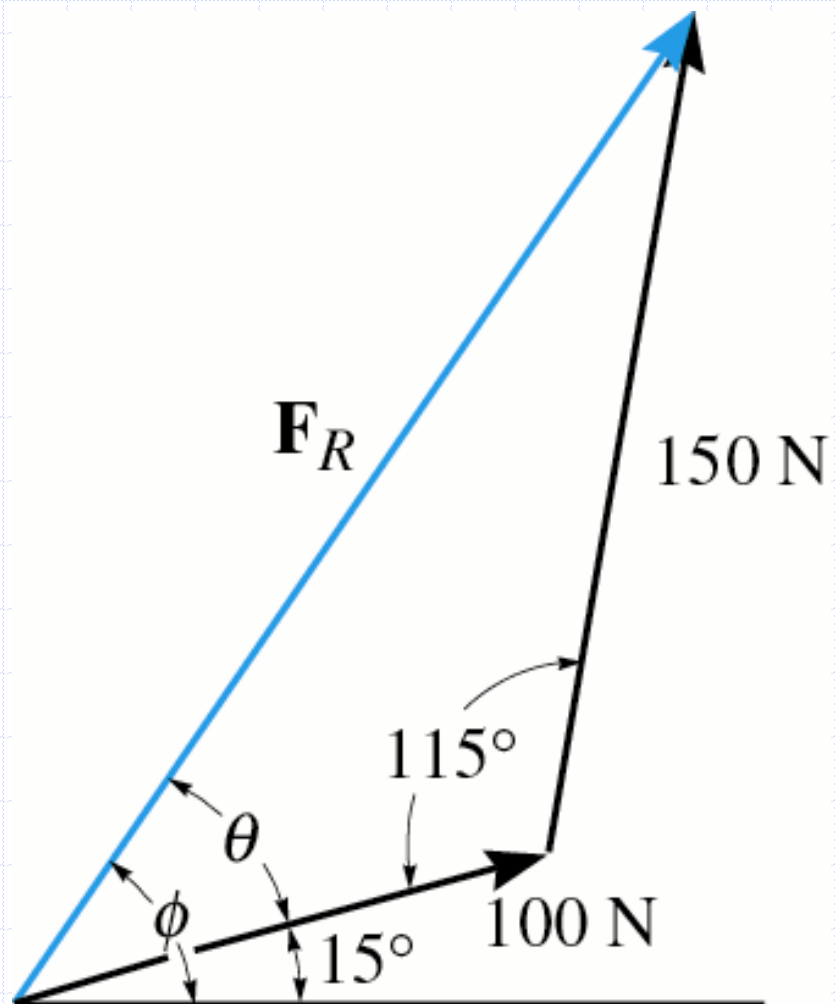
Example 1

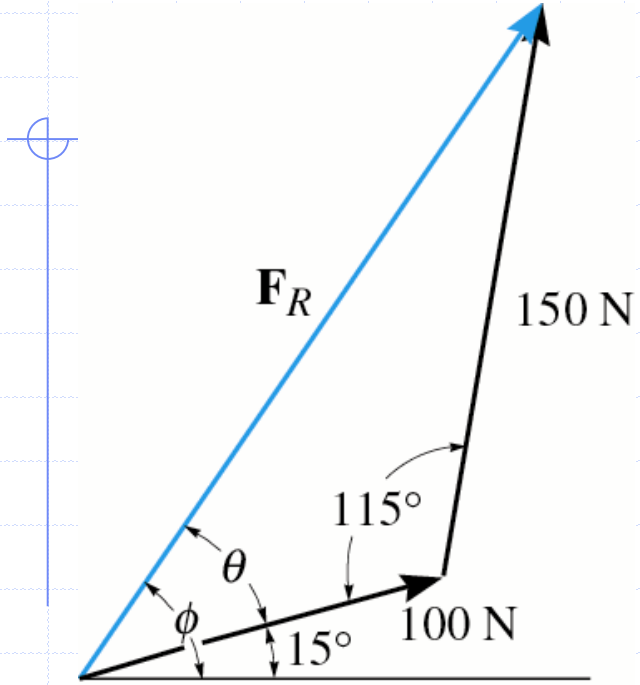


The screw eye in the figure at the left is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



Triangular Construction





Find F_R from law of cosines.

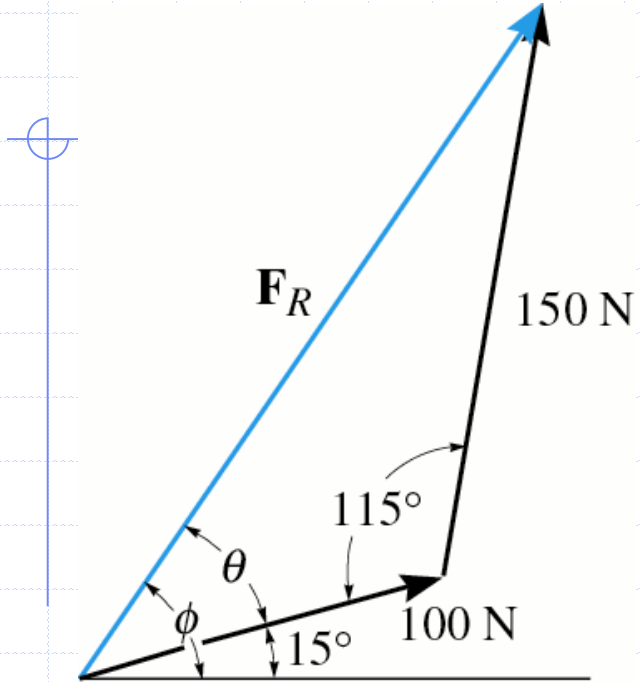
Find θ from law of sines.

Angle $\phi = \theta + 15^\circ$

$$F_R = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115^\circ}$$

$$F_R = \sqrt{10000 + 22500 - 30000(-0.4226)}$$

$$F_R = 212.6\text{N} = 213\text{N}$$



$$\frac{150}{\sin \theta} = \frac{212.6}{\sin 115^\circ}$$

$$\sin \theta = \frac{150}{212.6} (0.9063) = 0.6394$$

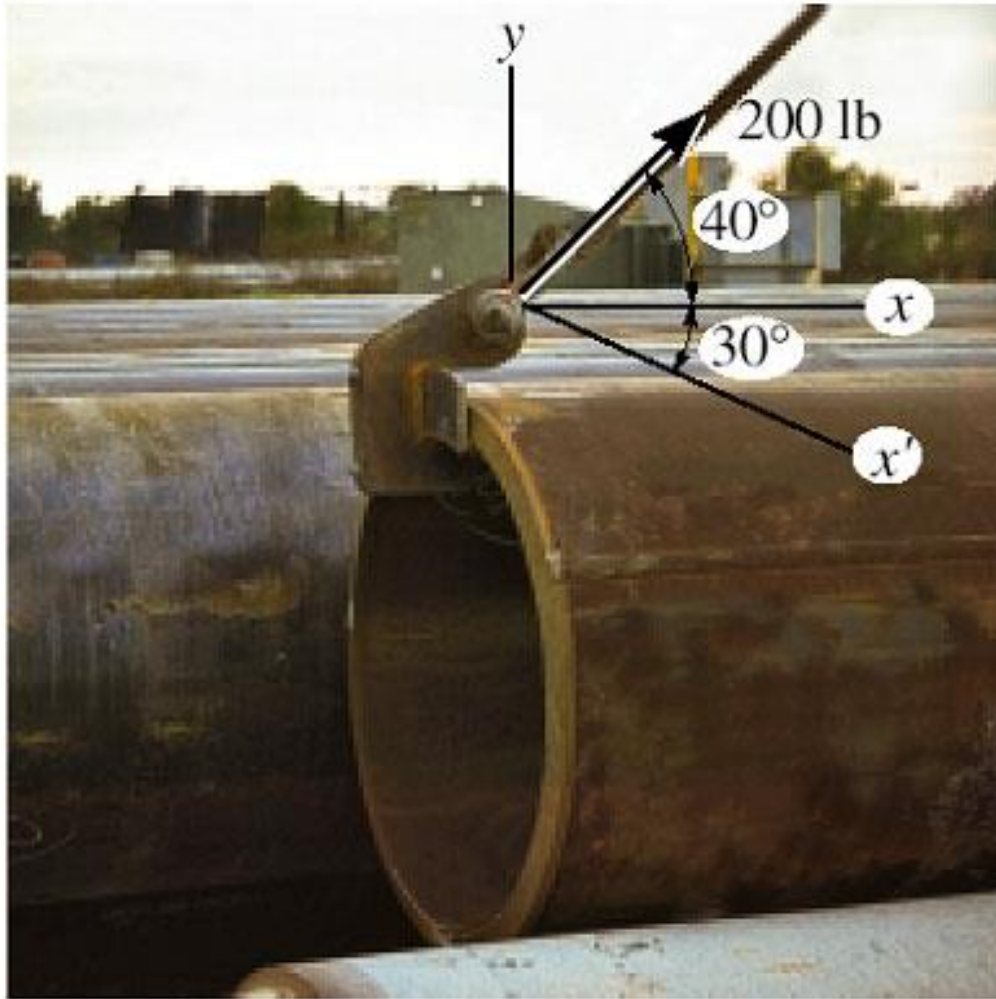
$$\theta = \sin^{-1}(0.6394) = 39.75^\circ = 39.8^\circ$$

$$\phi = \theta + 15^\circ$$

Answer

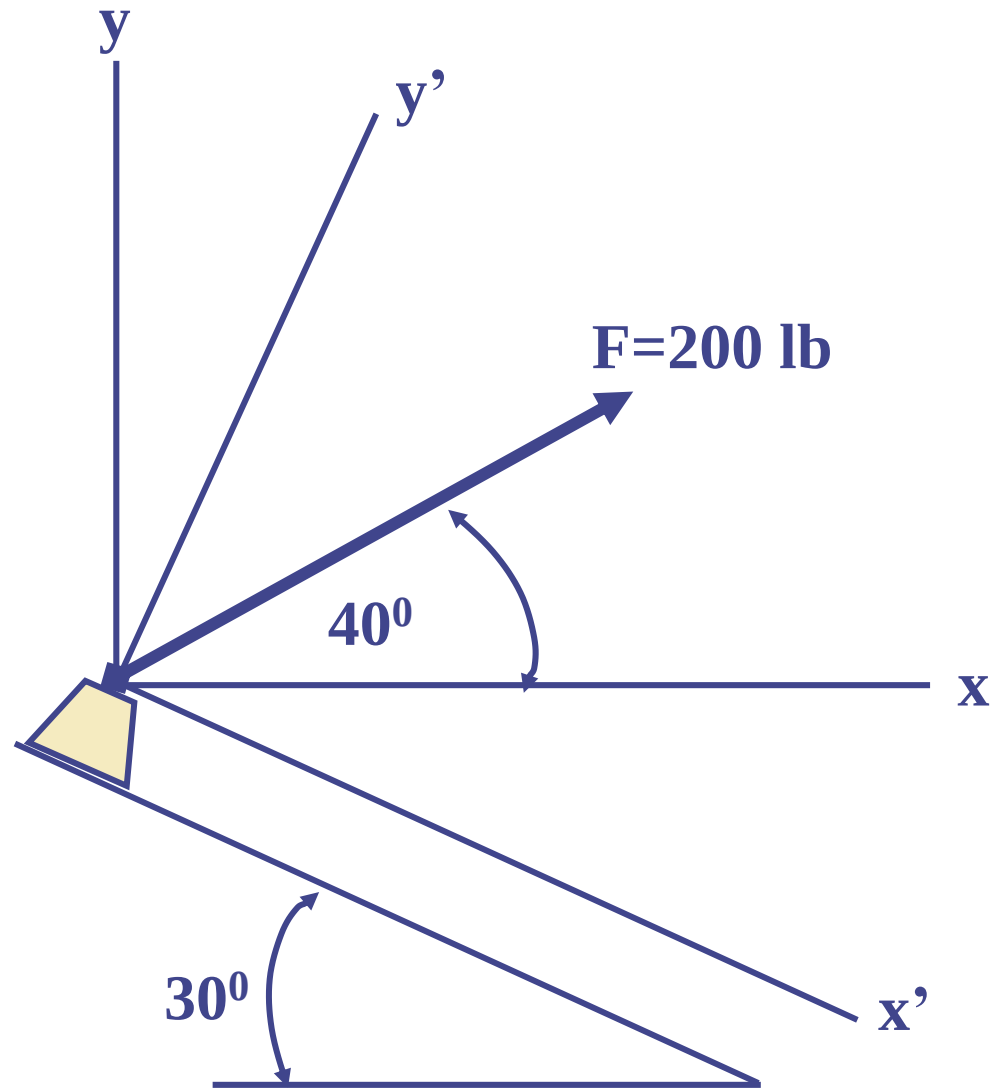
The resultant force has a magnitude of 213 N and is directed 54.8° from the horizontal.

Example 2

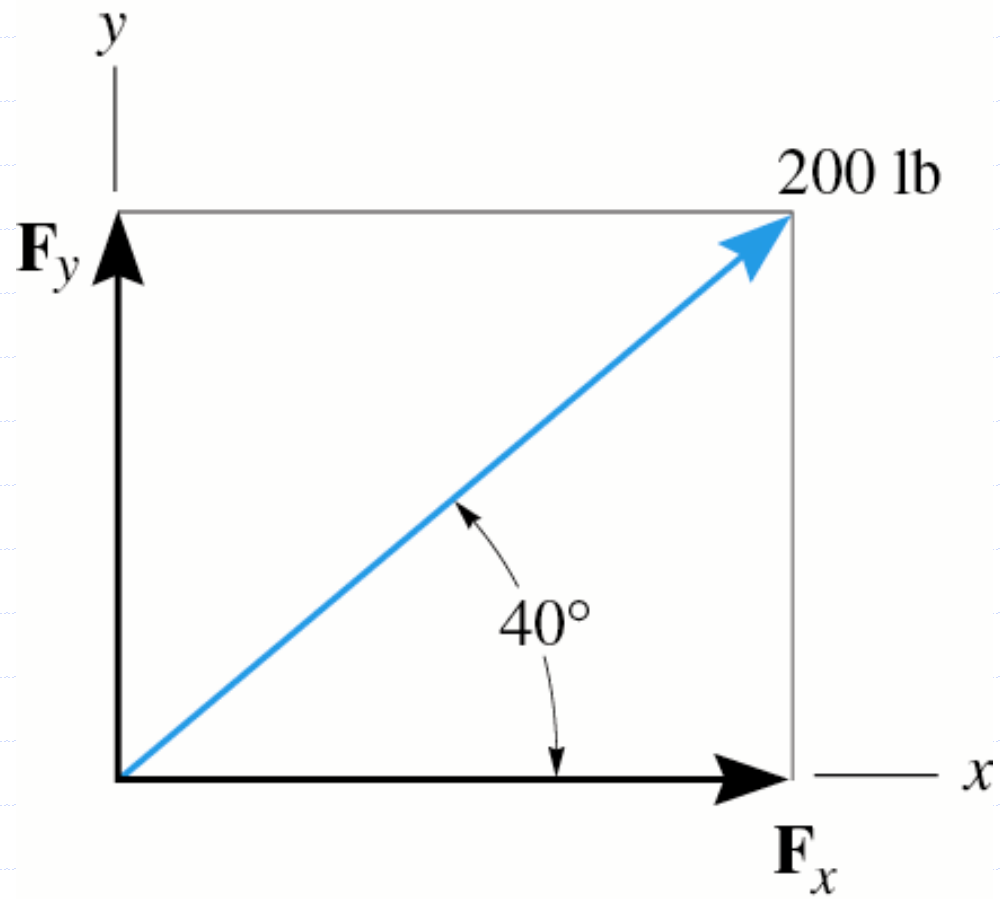


Resolve the 200 lb force into components in the x and y directions and in the x' and y directions

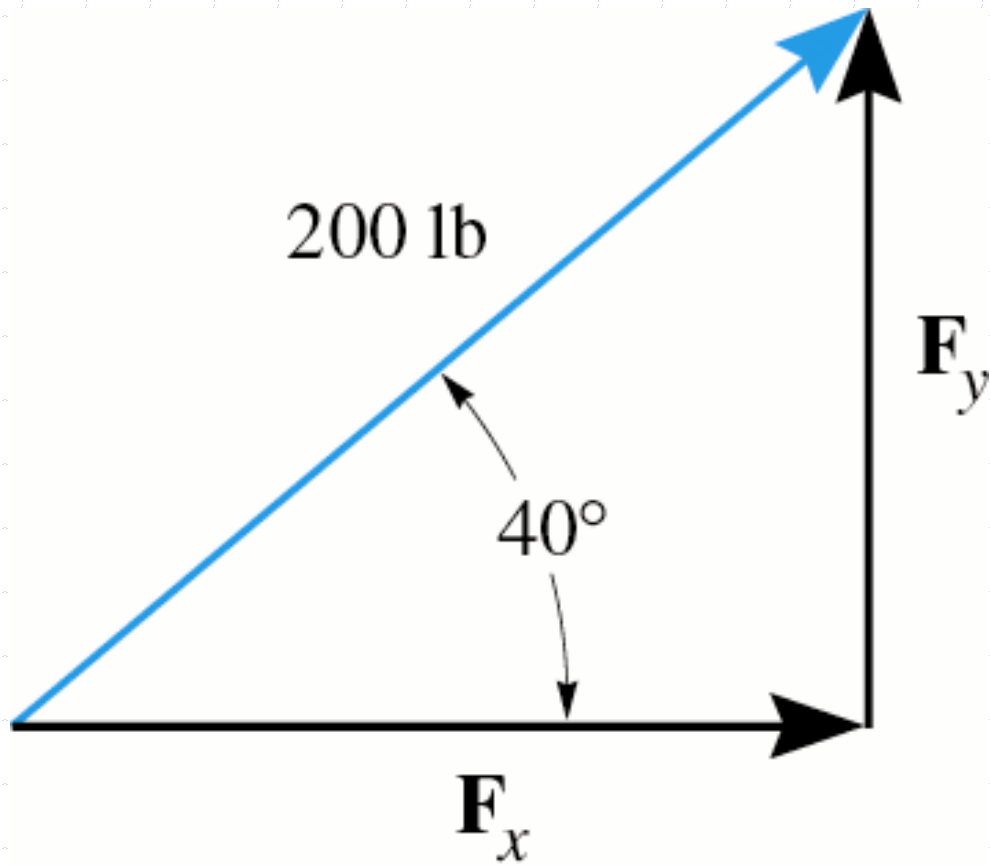
Resolve the 200 lb force into components in the x and y directions and in the x' and y' directions



Parallelogram Law



Triangular Construction



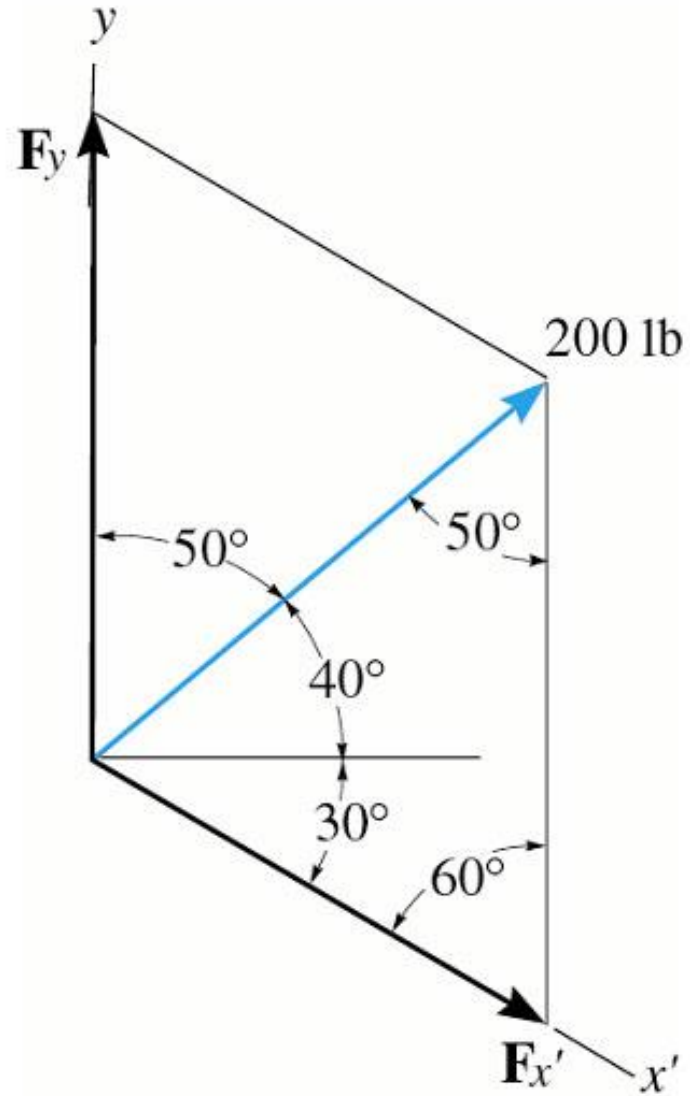
Solution – Part (a)

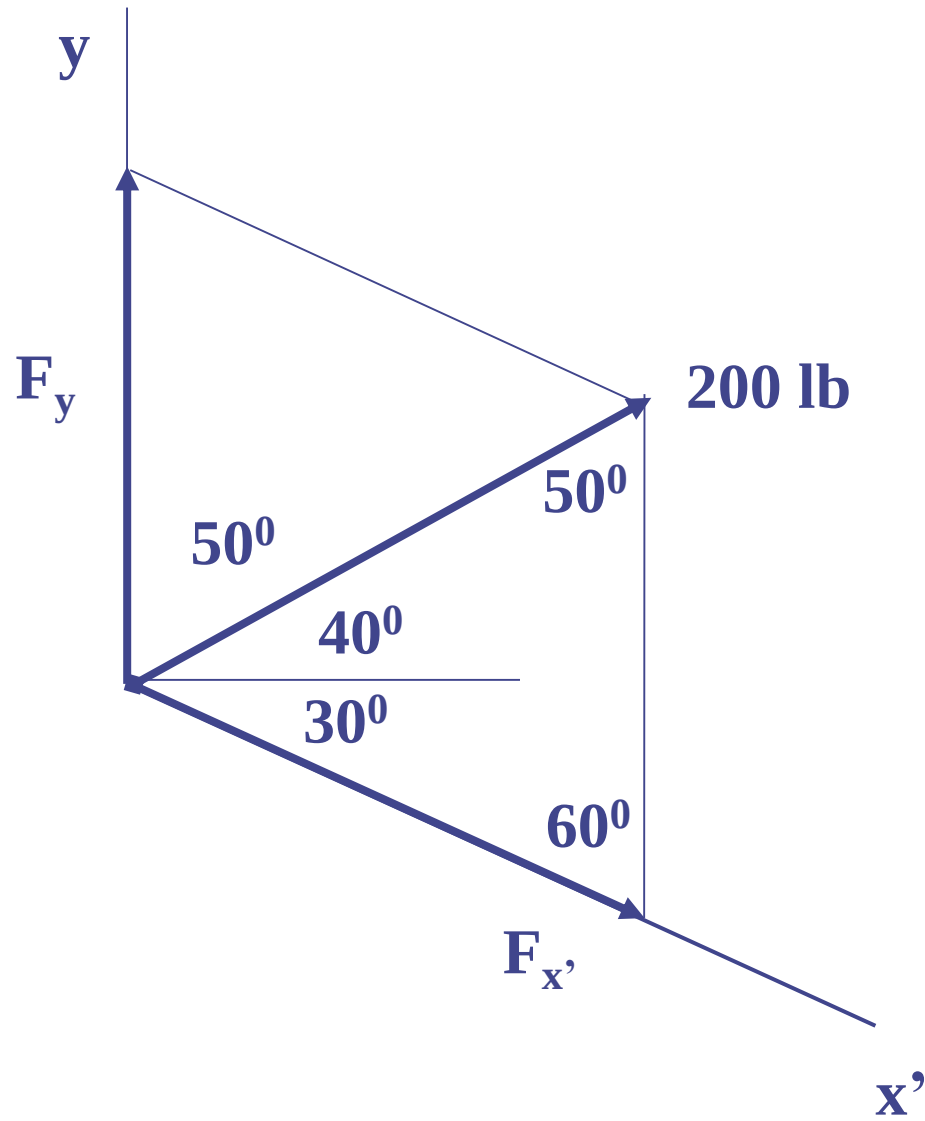
$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

$$\mathbf{F}_x = 200 \text{ lb} \cos 40^\circ = 153 \text{ lb}$$

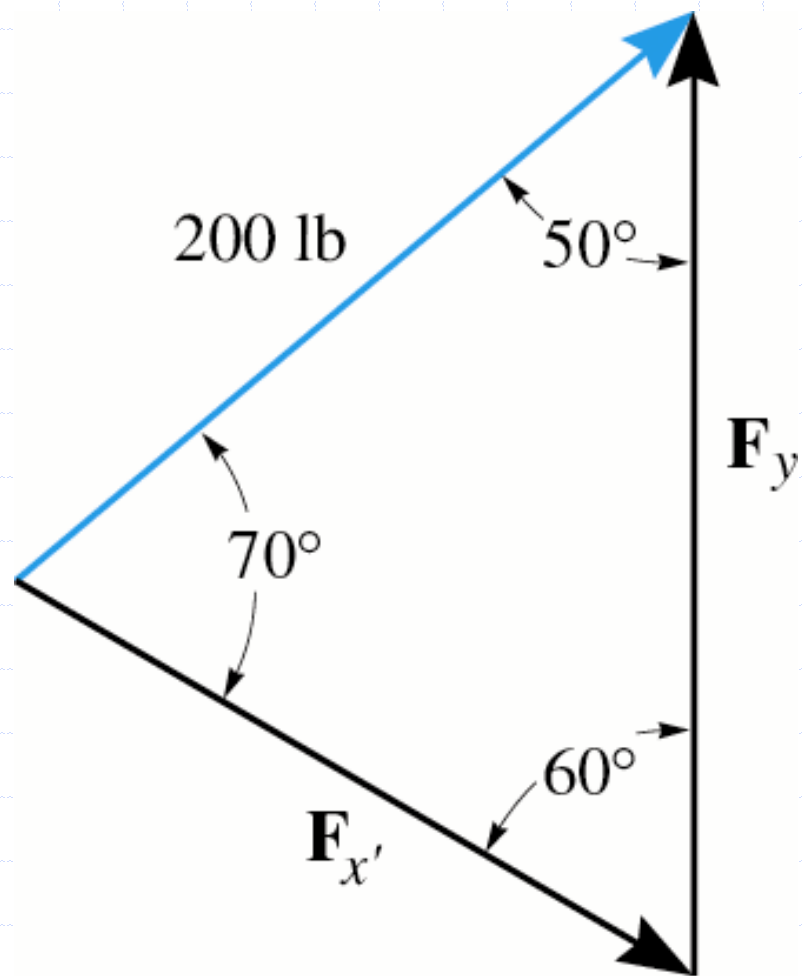
$$\mathbf{F}_y = 200 \text{ lb} \sin 40^\circ = 129 \text{ lb}$$

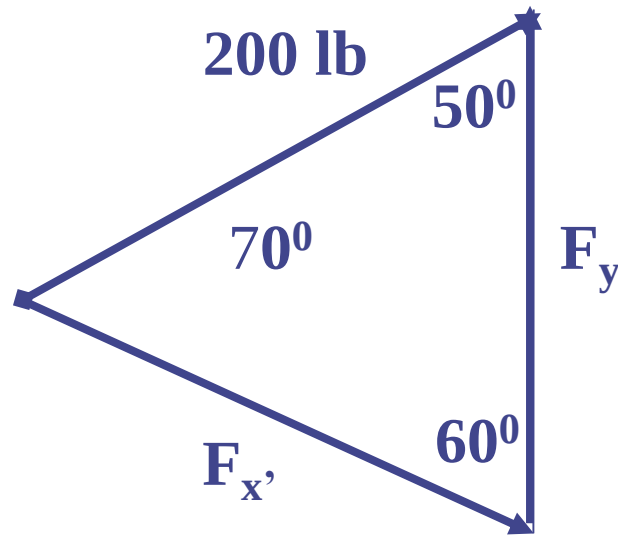
Parallelogram Law





Triangular Construction





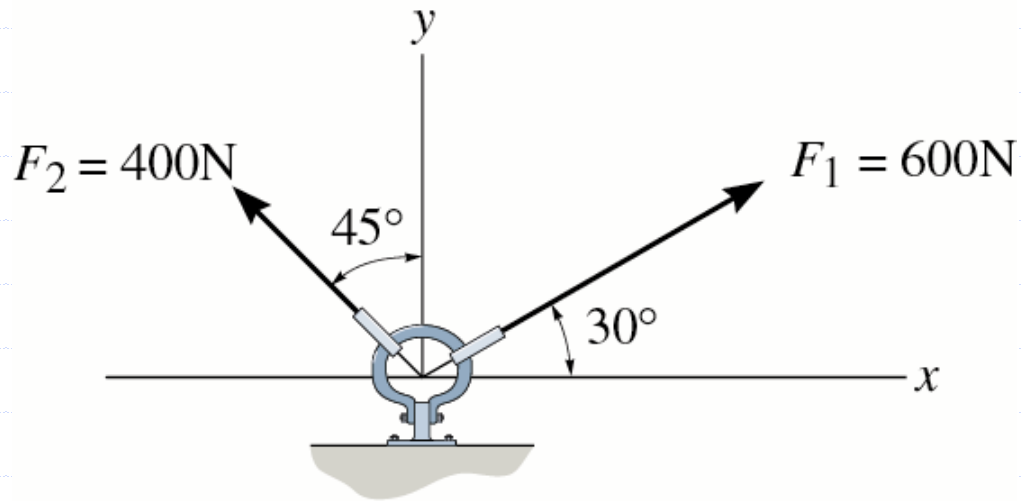
$$\mathbf{F} = \mathbf{F}_{x'} + \mathbf{F}_y$$

$$F_{x'} = 200 \frac{\sin 50^\circ}{\sin 60^\circ} = 177 \text{ lb}$$

$$\frac{F_{x'}}{\sin 50^\circ} = \frac{200}{\sin 60^\circ}$$
$$\frac{F_y}{\sin 70^\circ} = \frac{200}{\sin 60^\circ}$$

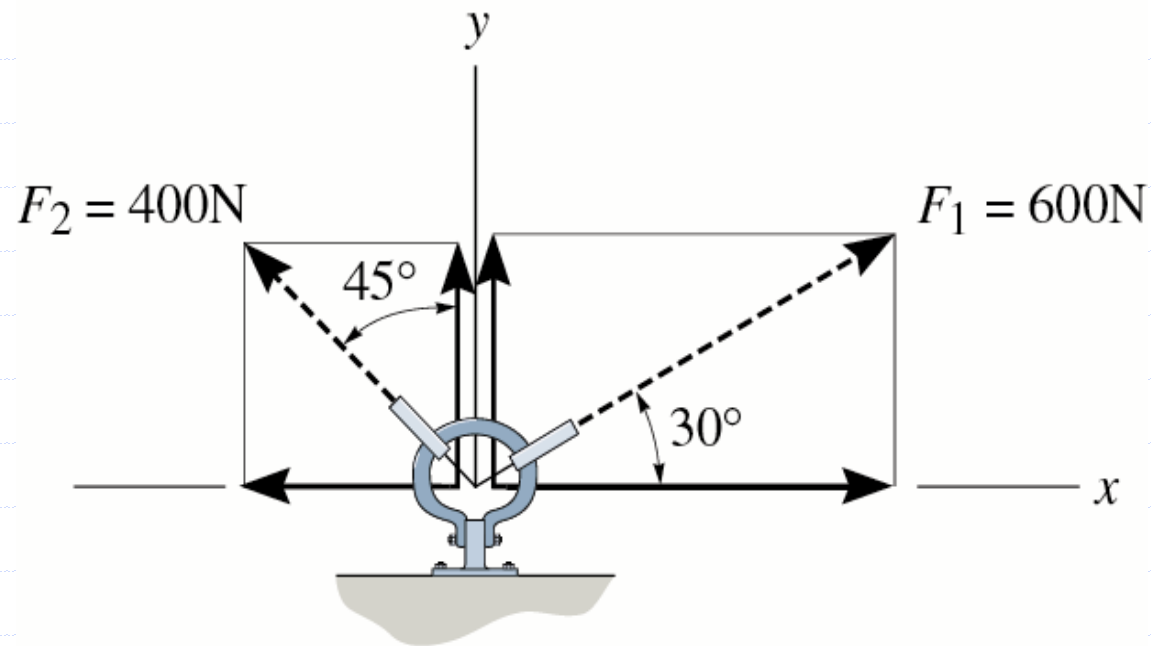
$$F_y = 200 \frac{\sin 70^\circ}{\sin 60^\circ} = 217 \text{ lb}$$

Example 3



The link in the figure is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the resultant magnitude and orientation of the resultant force.

Scalar Solution



Scalar Solution

$$\overset{+}{\rightarrow} F_{R_x} = \sum F_x$$

$$F_{R_x} = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N} = 236.8 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \sum F_y$$

$$F_{R_y} = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N} = 582.8 \text{ N} \uparrow$$

$$\theta = \tan^{-1} \left(\frac{582.8 \text{ N}}{236.8 \text{ N}} \right) = 67.9^\circ$$