LINEAR ALGEBRA

MATH 244
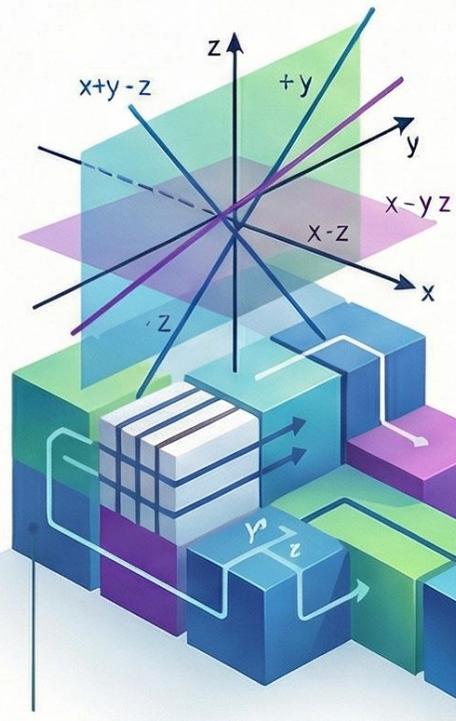
FAHD M ALSHAMMARI

Welcome to a Fascinating Linear Algebraic Journey

MATH 244 - 14472 - Fahd M Alshammari

Course Outline and Organization

A Visual Journey Through Linear Algebra

ABSTRACT STRUCTURES: SPACES & TRANSFORMATIONS

LINEAR TRANSFORMATIONS
Functions that map vectors from one space to another while preserving structure.

MATRICES
Rectangular arrays of numbers used to represent and manipulate linear systems efficiently.

VECTOR SPACES
The fundamental context for linear algebra, defining rules for vectors and scalars.
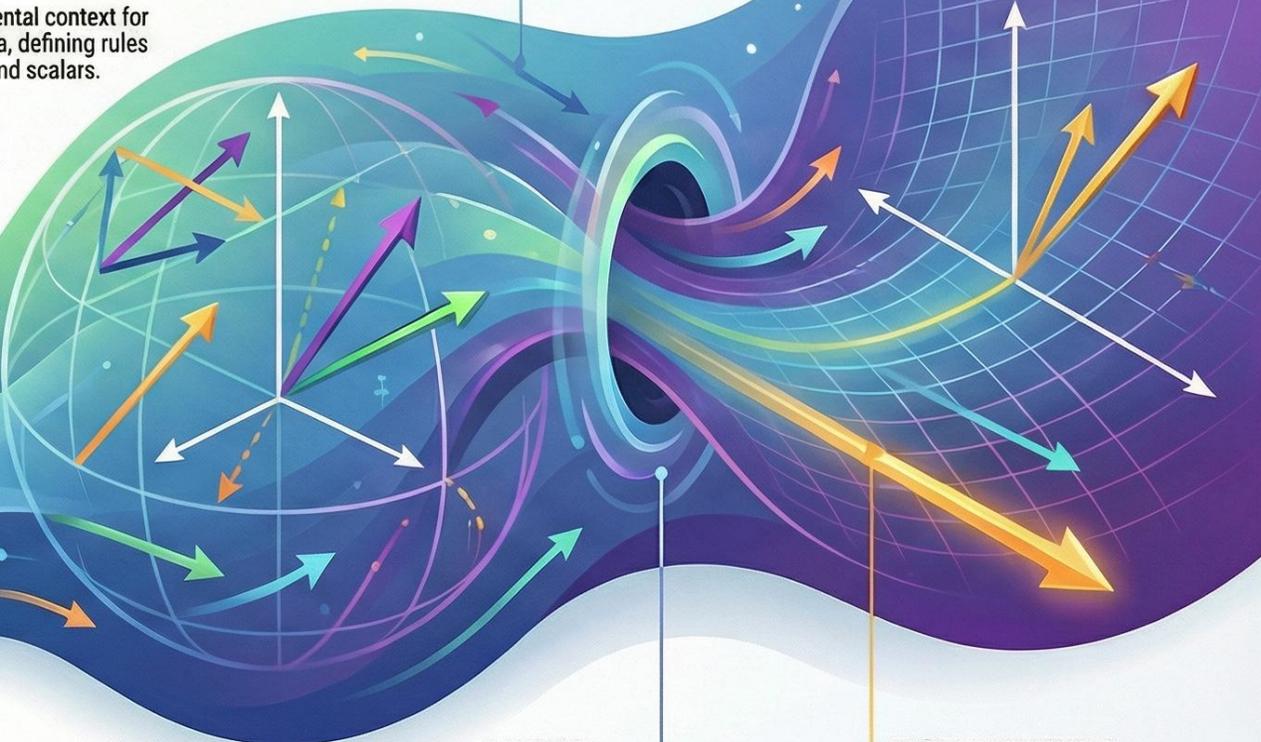
THE BUILDING BLOCKS: EQUATIONS & MATRICES

Systems of Linear Equations
The starting point of linear algebra, solved using methods like Gaussian elimination.

MATRICES
Rectangular arrays of numbers used to represent and manipulate linear systems efficiently.

DETERMINANTS
A unique scalar value calculated from a square matrix, revealing its properties.

LINEAR TRANSFORMATIONS
Functions that map vectors from one space to another while preserving structure.

EIGENVALUES & EIGENVECTORS
The special vectors that are only scaled, not changed in direction, by a transformation.

NotebookLM

Course Outline

- **Systems of Linear Equations**; Gauss and Gauss – Jordan elimination; homogeneous systems; Cramer's Rule.

- **Matrices** and matrix operations; elementary row operations; inverse of a matrix; special matrices.

- **Determinants**; properties of determinants; adjoint and its properties.

- **Vector Spaces**; subspaces; combinations and span; dependence independence; basis and dimension; coordinates; change of basis; rank and nullity.

- **Inner Product Spaces**; orthogonal and normal sets of vectors; orthonormal basis; Gram-Schmidt orthonormalization process.

- **Linear Transformations**; basic properties; kernel and image spaces; matrix of linear transformation.

- **Eigenvalues, Eigenvectors and Diagonalization** of a matrix.

Recommended Book

Elementary Linear Algebra (Applications Version) by Howard Anton and Chris Rorres, 11th Edition, Wiley, USA, 2014.
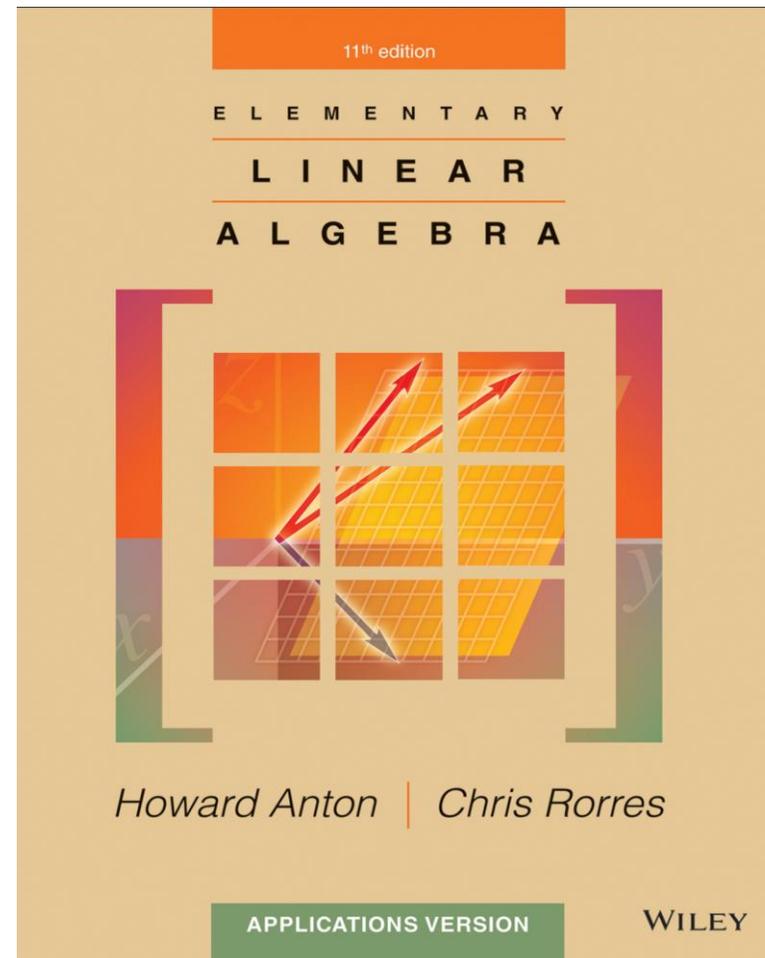
Grading

A total of 100 points are eared as follows:

Midterm Exam 1	25 points
Midterm Exam 2	25 points
Quizzes	10 points
Final Exam	40 points

Course Organization

- Using **Backboard; ClassPoint; ChatGPT;** and **other A.I.s**
- **Interactive** Lectures and Exercise sessions.
- **Participation** is very important. Raise your hand at any point you have a question.
- **Attendance** is taken regularly in the beginning of class. More than 25% absence causes denial from final exam entry. **Being late more than 5 minutes twice counts as 1 absence.**
- **University Policies, Rules, and Regulations** are applied. Find them here, Read them, understand them, and live by them.
- Please inform me and/or the department if you have **Special needs** or require any **Accommodations.**
- **Finally**, I am really looking forward for an exciting semester together. So, sit back relax and let's enjoy the course.

What is Linear Algebra?

What is linear algebra?

Linear

- ▶ having to do with lines/planes/etc.
- ▶ For example, $x + y + 3z = 7$, not $\sin, \log, x^2$, etc.

Algebra

- ▶ solving equations involving numbers and symbols
- ▶ from al-jebr (Arabic), meaning reunion of broken parts
- ▶ 9th century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

Why is Linear Algebra?

Civil Engineering: How much traffic flows through the four labeled segments?
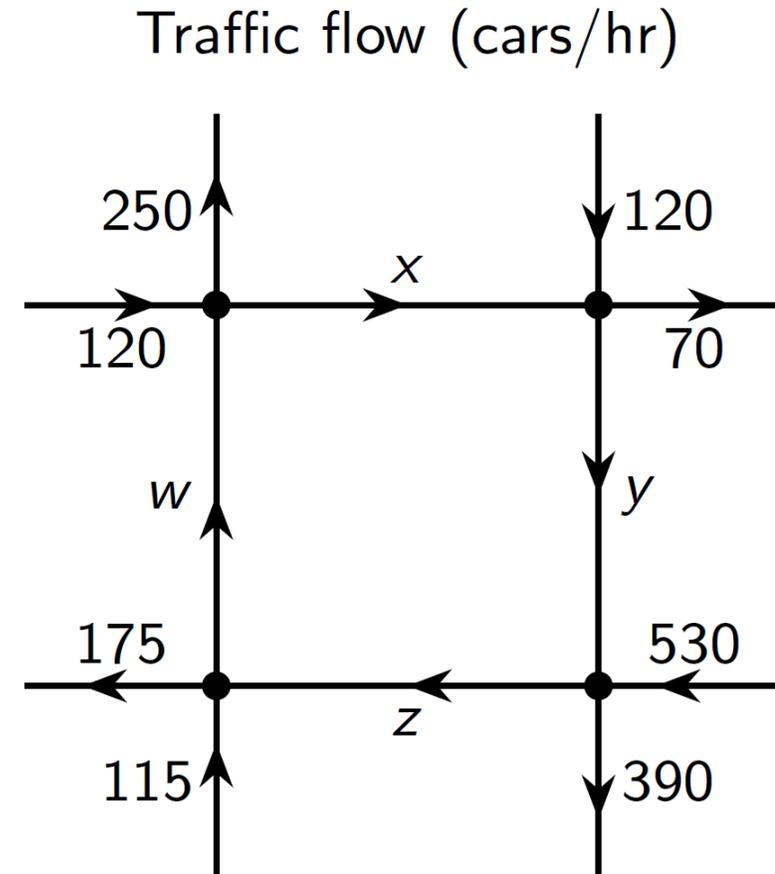
Traffic flow (cars/hr)

$\rightsquigarrow$ system of linear equations:

$$w + 120 = x + 250$$
$$x + 120 = y + 70$$
$$y + 530 = z + 390$$
$$z + 115 = w + 175$$

Chemistry: Balancing reaction equations

$$\underline{\ \ x\ \ }\ C_2H_6 + \underline{\ \ y\ \ }\ O_2 \to \underline{\ \ z\ \ }\ CO_2 + \underline{\ \ w\ \ }\ H_2O$$
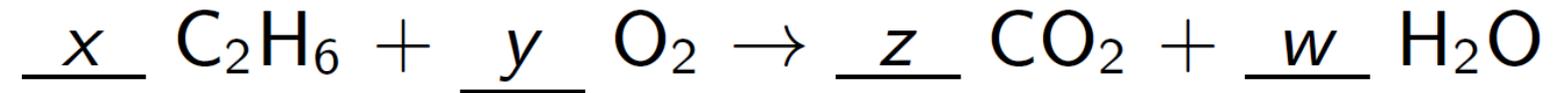
⤳ system of linear equations, one equation for each element.

$$2x = z$$
$$6x = 2w$$
$$2y = 2z + w$$

Geometry and Astronomy: Find the equation of a circle passing through 3 given points, say $(1,0)$, $(0,1)$, and $(1,1)$. The general form of a circle is $a(x^2 + y^2) + bx + cy + d = 0$.

$\rightsquigarrow$ system of linear equations:

$$
\begin{aligned}
a + b \qquad\ \ + d &= 0 \\
a \qquad + c + d &= 0 \\
2a + b + c + d &= 0
\end{aligned}
$$

Labeling with R^n

All colors you can see can be described by three quantities: the amount of red, green, and blue light in that color. Therefore, we can use the elements of $\mathbf{R}^3$ to *label* all colors: the point $(.2, .4, .9)$ labels the color with 20% red, 40% green, and 90% blue.
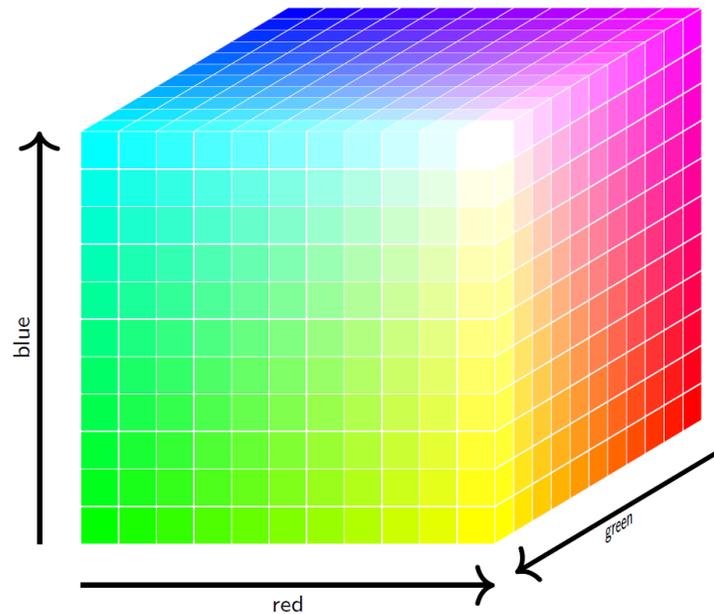
Data Science:

Table 1 Term-by-Document Matrix of the Four Most Venomous Animals

		Documents			
		Jellyfish	Cobra	Snail	Octopus
Terms	venom	32	44	1	18
	death	9	3	0	2
	danger	6	4	0	4
	survive	2	0	0	1
	Madagascar	0	0	2	0

$$A = \begin{pmatrix} 32 & 44 & 1 & 18 \\ 9 & 3 & 0 & 2 \\ 6 & 4 & 0 & 4 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

Economics:(Simple Production Model):

Suppose a firm produces cars and buses requiring steal and plastic, where:

      1 car requires 2 tons of steal and 0.5 ton of plastic

      1 bus requires 3 tons steal and 1 ton of plastic

Suppose that 14 tons of steal and 4 tons of plastic are available. Want to know: how many cars and buses can be produced? Is it possible to use all amounts?

Solution: If x_1 be the number of cars and x_2 be the number of buses, then

- the amount of steal needed is $2x_1 + 3x_2$,

- the amount of plastic needed is $0.5x_1 + x_2$.

These amounts should be equal to the amounts bought. Therefore,

$$2x_1 + 3x_2 = 14$$
$$0.5x_1 + x_2 = 4$$

Recourses and References

The following are freely available on the web which influenced the slides:

- *Interactive Linear Algebra* by Dan Margalit and Joseph Rabinoff.

- Courseware freely available at the university of Waterloo faculty of mathematics. https://open.math.uwaterloo.ca/.

- Georgia Tech Math 1554 linear algebra course page which has lots of available material including lecture videos. https://gatech.instructure.com/courses/114544/.

- Lecture notes of William Chen https://www.williamchen-mathematics.info/lnlafolder/lnla.html.