LINEAR ALGEBRA

MATH 244

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Welcome to a Fascinating Linear Algebraic Journey

A picture generated using Dall- E in ChatGPT

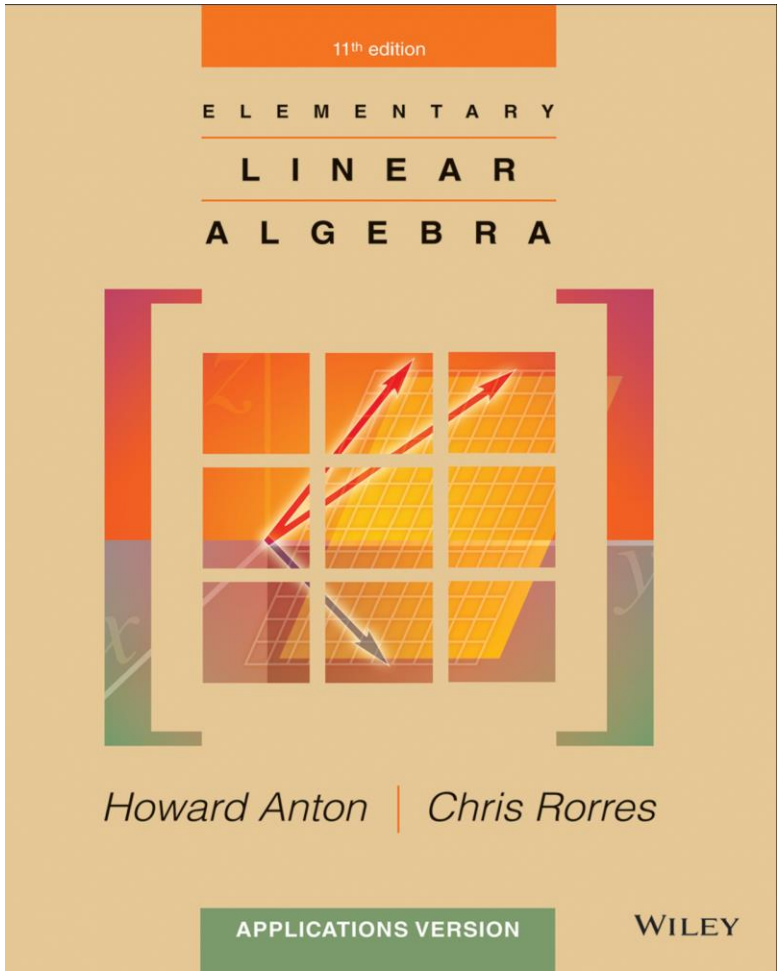
Course Outline and Organization

Course Outline

- **Systems of Linear Equations**; Gauss and Gauss – Jordan elimination; homogeneous systems; Cramer's Rule.
- **Matrices** and matrix operations; elementary row operations; inverse of a matrix; special matrices.
- **Determinants**; properties of determinants; adjoint and its properties.
- **Vector Spaces**; subspaces; combinations and span; dependence independence; basis and dimension; coordinates; change of basis; rank and nullity.
- **Inner Product Spaces**; orthogonal and normal sets of vectors; orthonormal basis; Gram-Schmidt orthonormalization process.
- **Linear Transformations**; basic properties; kernel and image spaces; matrix of linear transformation.
- **Eigenvalues, Eigenvectors and Diagonalization** of a matrix.

Recommended Book

Elementary Linear Algebra (Applications Version) by Howard Anton and Chris Rorres, 11th Edition, Wiley, USA, 2014.



Grading

A total of 100 points are eared as follows:

Midterm Exam 1	25 points
Midterm Exam 2	25 points
Quizzes	10 points
Final Exam	40 points

Course Organization

- Using **Backboard; ClassPoint; ChatGPT;** and **other A.I.s**
- **Interactive** Lectures and Exercise sessions**.**
- **Participation** is very important. Raise your hand at any point you have a question.
- **Attendance** is taken regularly in the beginning of class. More than 25% absence causes denial from final exam entry. **Being late more than 5 minutes twice counts as 1 absence.**
- **University Policies, Rules, and Regulations** are applied. Find them here, Read them, understand them, and live by them.
- Please inform me and/or the department if you have **Special needs** or require any **Accommodations.**
- **Finally**, I am really looking forward for an exciting semester together. So, sit back relax and let's enjoy the course.

What is Linear Algebra?

What is linear algebra?

Linear

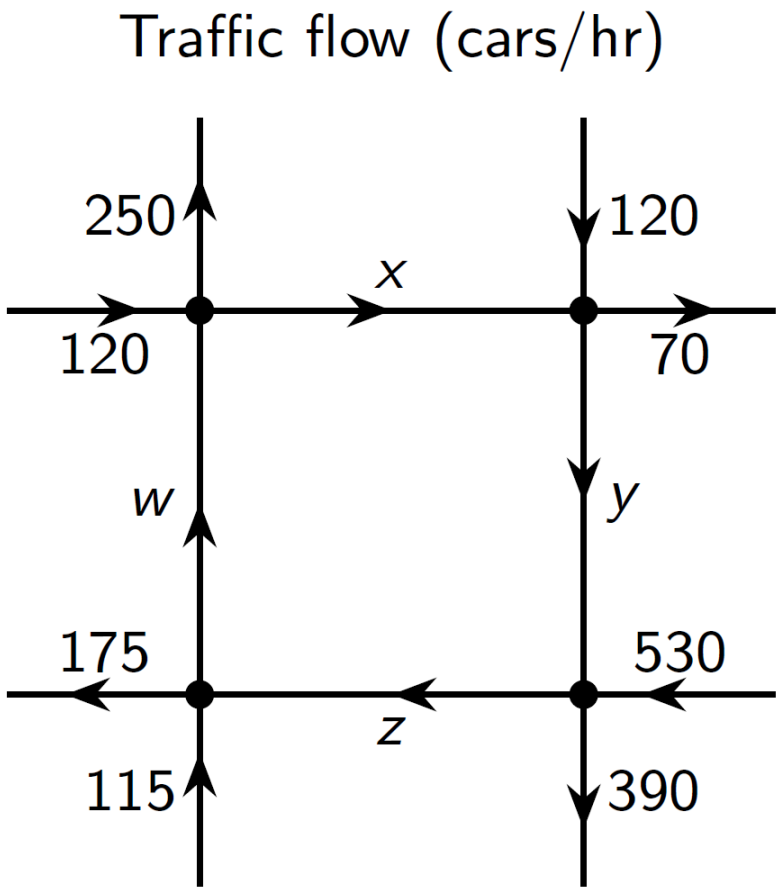
- having to do with lines/planes/etc.
- For example, $x + y + 3z = 7$, not $\sin, \log, x^2$, etc.

Algebra

- solving equations involving numbers and symbols
- from al-jebr (Arabic), meaning reunion of broken parts
- 9^{th} century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

Why is Linear Algebra?

Civil Engineering: How much traffic flows through the four labeled segments?



$\rightsquigarrow$ system of linear equations:

$$\begin{aligned} w + 120 &= x + 250 \\ x + 120 &= y + 70 \\ y + 530 &= z + 390 \\ z + 115 &= w + 175 \end{aligned}$$

Chemistry: Balancing reaction equations

$$\underline{\ x\ }\ C_2H_6 + \underline{\ y\ }\ O_2 \to \underline{\ z\ }\ CO_2 + \underline{\ w\ }\ H_2O$$

$\rightsquigarrow$ system of linear equations, one equation for each element.

$$2x = z$$
$$6x = 2w$$
$$2y = 2z + w$$

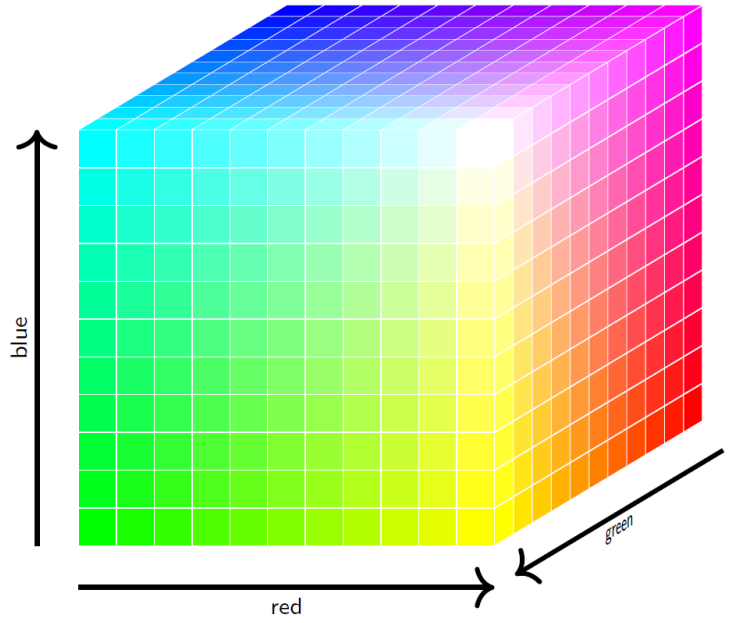
Geometry and Astronomy: Find the equation of a circle passing through 3 given points, say $(1,0)$, $(0,1)$, and $(1,1)$. The general form of a circle is $a(x^2+y^2)+bx+cy+d=0$.

$\rightsquigarrow$ system of linear equations:

$$\begin{aligned} a+b\quad\quad\quad+d&=0\\ a\quad\quad+c+d&=0\\ 2a+b+c+d&=0 \end{aligned}$$

Labeling with R^n

All colors you can see can be described by three quantities: the amount of red, green, and blue light in that color. Therefore, we can use the elements of $\mathbf{R}^3$ to *label* all colors: the point $(.2, .4, .9)$ labels the color with 20% red, 40% green, and 90% blue.



Data Science:

Table 1 Term-by-Document Matrix of the Four Most Venomous Animals

		Documents			
		Jellyfish	Cobra	Snail	Octopus
Terms	venom	32	44	1	18
	death	9	3	0	2
	danger	6	4	0	4
	survive	2	0	0	1
	Madagascar	0	0	2	0

$$A = \begin{pmatrix} 32 & 44 & 1 & 18 \\ 9 & 3 & 0 & 2 \\ 6 & 4 & 0 & 4 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

Economics:(Simple Production Model):

Suppose a firm produces cars and buses requiring steal and plastic, where:

1 car requires 2 tons of steal and 0.5 ton of plastic

1 bus requires 3 tons steal and 1 ton of plastic

Suppose that 14 tons of steal and 4 tons of plastic are available. Want to know: how many cars and buses can be produced? Is it possible to use all amounts?

Solution: If x_1 be the number of cars and x_2 be the number of buses, then

- the amount of steal needed is $2x_1 + 3x_2$,
- the amount of plastic needed is $0.5x_1 + x_2$.

These amounts should be equal to the amounts bought. Therefore,

$$2x_1 + 3x_2 = 14$$
$$0.5x_1 + x_2 = 4$$

Recourses and References

The following are freely available on the web which influenced the slides:

- *Interactive Linear Algebra* by Dan Margalit and Joseph Rabinoff.
- Courseware freely available at the university of Waterloo faculty of mathematics. https://open.math.uwaterloo.ca/.
- Georgia Tech Math 1554 linear algebra course page which has lots of available material including lecture videos. https://gatech.instructure.com/courses/114544/.
- Lecture notes of William Chen https://www.williamchen-mathematics.info/lnlafolder/lnla.html.