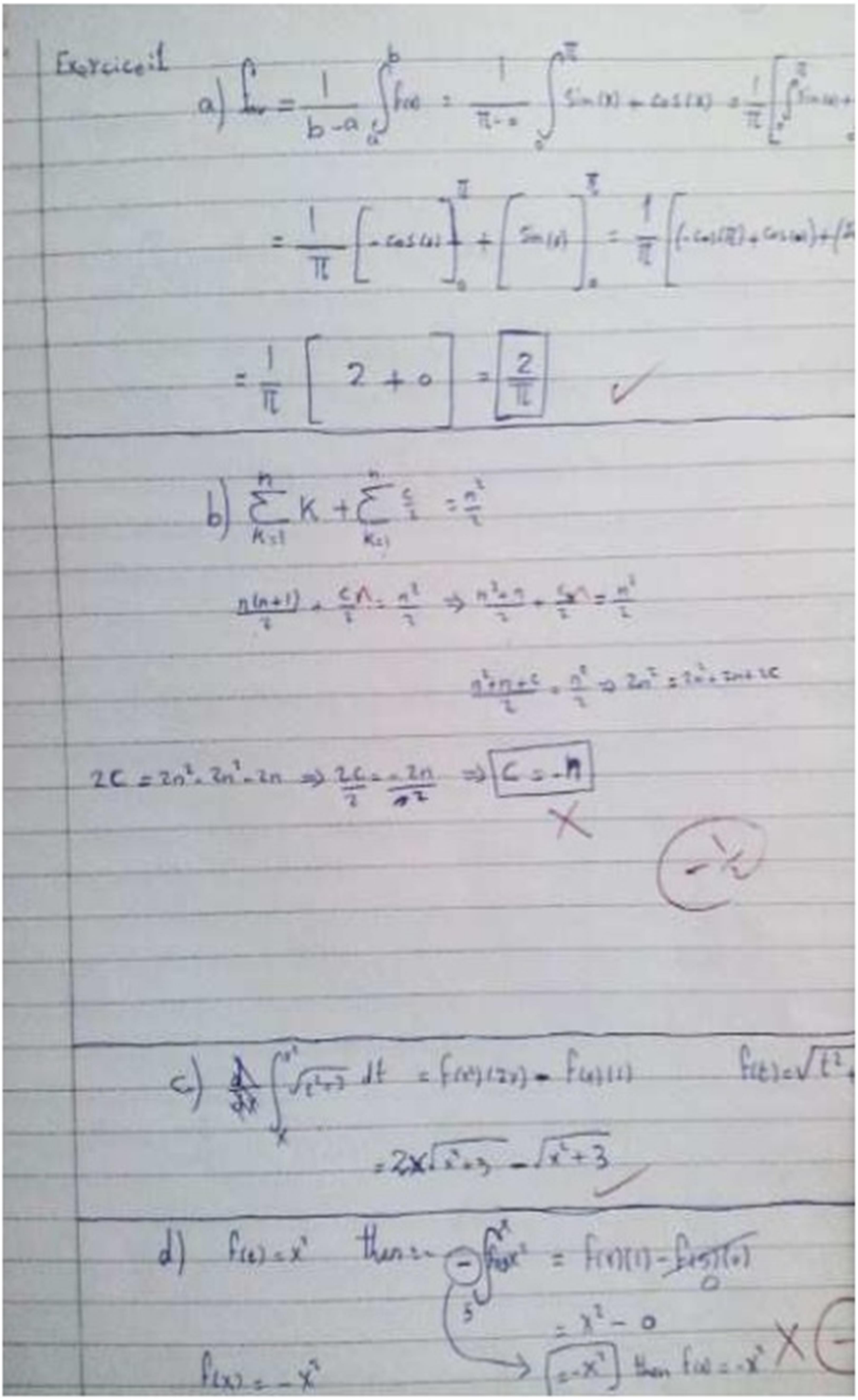
نموذج حل بعض الطلاب فى الاختبارات



Exercice 1

a) $f_{av} = \frac{1}{b-a}\int_a^b f(x) = \frac{1}{\pi-0}\int_0^{\pi} \sin(x) + \cos(x) = \frac{1}{\pi}\left[\int_0^{\pi}\sin(x) + \right.$

$= \frac{1}{\pi}\left[-\cos(x)\right]_0^{\pi} + \left[\sin(x)\right]_0^{\frac{\pi}{2}} = \frac{1}{\pi}\left[(-\cos(\pi) + \cos(0)) + (\right.$

$= \frac{1}{\pi}\left[2 + 0\right] = \boxed{\frac{2}{\pi}}$ ✓

b) $\sum_{k=1}^{n} k + \sum_{k=1}^{n} \frac{c}{2} = \frac{n^2}{2}$

$\frac{n(n+1)}{2} + \frac{cn}{2} = \frac{n^2}{2} \Rightarrow \frac{n^2+n}{2} + \frac{cn}{2} = \frac{n^2}{2}$

$\frac{n^2+n+c}{2} = \frac{n^2}{2} \Rightarrow 2n^2 = 2n^2 + 2n + 2c$

$2c = 2n^2 - 2n^2 - 2n \Rightarrow \frac{2c}{2} = \frac{-2n}{2} \Rightarrow \boxed{c = -n}$ ✗

-1

c) $\frac{d}{dx}\int_x^{x^2}\sqrt{t^2+3}\,dt = f(x^2)(2x) - f(x)(1)$ $\quad f(t) = \sqrt{t^2+}$

$= 2x\sqrt{x^2+3} - \sqrt{x^2+3}$ ✓

d) $f(t) = x^2$ then $= -\int_5^{x^2} f(t)\,dt^2 = f(x)(1) - \cancel{f(5)(0)}$ 0

$= x^2 - 0$

$\boxed{= -x^2}$ then $f(x) = -x^2$ ✗

$f(x) = -x^2$

a) $f(x) = \sin(x) + \cos(x) \quad [0, \pi]$

$f_{av} = \frac{1}{b-a} \int_0^{\pi} \sin(x) + \cos(x)$

$= \frac{1}{\pi} \left[-\cos(x) + \sin(x) \right]_0^{\pi}$

$= \frac{1}{\pi} \left[[1 + 0] - [-1 + 0] \right]$

$= \frac{1}{\pi} [1 + 1]$

$= 0.6366$

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b) find c $\quad \sum_{k=1}^{n} \left(k + \frac{c}{2}\right) = \frac{n^2}{2}$

$\sum_{k=1}^{n} k + \sum_{k=1}^{n} \frac{c}{2} = \frac{n^2}{2}$

$\frac{n(n+1)}{2} + \frac{c}{2} \cdot n = \frac{n^2}{2}$

$\frac{n^2+n}{2} + \frac{c}{2} \cdot n = \frac{n^2}{2}$

$\frac{n^2+n}{2} + \frac{cn}{2n} = \frac{n^2}{2}$

$\frac{n^2+n}{2} - \frac{n^2}{2} = -\frac{cn}{2n}$

$\frac{n}{2} = -\frac{c\cancel{n}}{2\cancel{n}}$

$\frac{n}{2} = -\frac{c}{2}$

1) a) $f(x) = \sin(x) + \cos(x) \quad [0, \pi]$

$$\text{average} = \frac{1}{b-a} \int_a^b f(x)$$

$$\text{average} = \frac{1}{\pi - 0} \int_0^\pi \sin(x) + \cos(x)$$

$$= \frac{1}{\pi} \left(-\cos(x) + \sin(x)\right)_0^\pi$$

$$= \frac{1}{\pi} \left[(-(-1)+0) - (-1+0\right]$$

$$\text{average} = \frac{1}{\pi} [2] = \frac{2}{\pi} \checkmark$$

1 b)

$$\sum_{k=1}^{n} k + \sum_{k=1}^{n} \frac{c}{2} = \frac{n^2}{2}$$

$$\frac{n(n+1)}{2} + \frac{cn}{2} = \frac{n^2}{2}$$

$$\frac{n^2+n}{2} + \frac{cn}{2} = \frac{n^2}{2}$$

$$\frac{cn}{2} = \frac{n^2}{2} - \frac{n^2+n}{2}$$

$$\cancel{\frac{cn}{2} = -\frac{n}{2}}$$

$$\cancel{\frac{-2cn}{-2n} = \frac{-2n}{-2n}}$$

$$\cancel{c = 1}$$

$$\frac{cn}{2} = \frac{-n}{2}$$

$$2cn = -2n$$

$$c = -1$$

or

$$\frac{cn}{2} = \frac{-n}{2}$$

$$\frac{2}{n} \cdot \frac{cn}{2} = \frac{-n}{2} \cdot \frac{2}{n}$$

$$c = -1 \checkmark$$

1 c)

$$\frac{d}{dx} \int_x^{x^2} \sqrt{t^2+3}\, dt = f(x^2) \cdot 2x - f(x)$$

$$= \cancel{f(x^2)} = \sqrt{(x^2)^2+3} \cdot 2x - \sqrt{x^2+3}$$