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الإجابة النموذجية للاختبار

111-MATH - second semester 1443

* الجزء الأول :

① ٣ درجات

$$f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$(c-2)^2=\frac{1}{4-1}\int_1^4 (x-2)^2\,dx$$

$$(c-2)^2=\frac{1}{3}\left[\frac{(x-2)^3}{3}\Big|_1^4\right]$$

$$(c-2)^2=\frac{1}{9}\left(2^3-(-1)^3\right)$$

$$(c-2)^2=\frac{1}{9}(8+1)$$

$$(c-2)^2=\frac{1}{\cancel{9}}(\cancel{9})=1$$

$$\Rightarrow \quad c-2=\pm 1$$

$$\Rightarrow C=1+2 \quad \text{or} \quad C=-1+2$$

$$C=3\in(1,4) \qquad C=1\notin(1,4)$$

$$\therefore \boxed{C=3}$$

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٢ درجتان :

$$F(x) = \int_{-3x}^{\sqrt{x}} \frac{t^2}{t^2+1}\, dt$$

$$F'(x) = \left[\frac{(\sqrt{x})^2}{(\sqrt{x})^2+1} \cdot \frac{1}{2\sqrt{x}}\right] - \left[\frac{(-3x)^2}{(-3x)^2+1} \cdot (-3)\right]$$

$$= \frac{x}{x+1} \cdot \frac{1}{2\sqrt{x}} + \frac{9x^2}{9x^2+1} \quad 27$$

$$= \frac{x}{2\sqrt{x}\,(x+1)} + \frac{9x^2}{9x^2+1}$$

٣ درجتان

$$f(x) = \tanh^{-1}\left(e^{3x^2}\right) + \log_2 \left|\sinh(2x) + 3^{x+1}\right|$$

$$f'(x) = \frac{1}{1-\left(e^{3x^2}\right)^2} \cdot e^{3x^2} \cdot 6x + \frac{1}{\ln 2} \frac{1}{\sinh(2x)+3^{x+1}} \left[\cosh 2x \cdot 2 + 3^{x+1} \cdot 1 \ln 3\right.$$

$$= \frac{6x\, e^{3x^2}}{1 - e^{6x^2}} + \frac{2\cosh 2x}{\ln 2\,(\sinh 2x + 3^{x+1})} + \frac{\ln 3 \cdot 3^{x+1}}{\ln 2\,(\sinh 2x + 3^{x+1})}$$

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الجزء ~~الثالث~~ الثاني :-

(١) ٣ درجات

$$\int \frac{3}{\sqrt{x^2-8x+25}}\,dx$$

$x^2-8x+25 = x^2-8x+16-16+25 = (x-4)^2+9$

$$3\int \frac{1}{\sqrt{(x-4)^2+9}}\,dx = 3\int \frac{1}{\sqrt{(x-4)^2+3^2}}\,dx$$

let $u = x-4$

$du = dx$

$$= 3\int \frac{1}{\sqrt{u^2+3^2}}\,du = 3\cdot \sinh^{-1}\frac{u}{3} + C$$

$$= 3\sinh^{-1}\frac{x-4}{3} + C$$

(٢) درجتان :-

$$\int x\sec^2 x\,dx$$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

$$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$$

$$= x\tan x - \int \frac{\sin x}{\cos x}\,dx$$

$$= x\tan x - (-\ln|\cos x|) + C$$

$$= x\tan x + \ln|\cos x| + C$$

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(٣) ٣ درجات

$$\int \frac{1}{(4+x^2)^{\frac{3}{2}}}\,dx$$

let $x = 2\tan\theta \Rightarrow x^2 = 4\tan^2\theta$

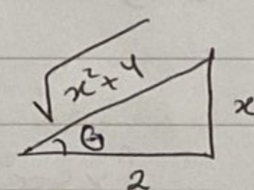
$dx = 2\sec^2\theta\, d\theta$

$$4+x^2 = 4+4\tan^2\theta = 4(1+\tan^2\theta) = 4\sec^2\theta$$

$$\int \frac{1}{(4+x^2)^{\frac{3}{2}}}\,dx = \int \frac{1}{(4\sec^2\theta)^{\frac{3}{2}}}\cdot 2\sec^2\theta\, d\theta$$

$$= \int \frac{2\sec^2\theta}{8\sec^3\theta}\,d\theta = \frac{1}{4}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{4}\int \cos\theta\, d\theta$$

$$= \frac{1}{4}\sin\theta + C$$



$$= \frac{1}{4}\,\frac{x}{\sqrt{4+x^2}} + C$$

(٤) ٣ درجات

$$\int \frac{x^2+1}{x^3-x^2}\,dx$$

$$\frac{x^2+1}{x^3-x^2} = \frac{x^2+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$x^2+1 = A(x-1)\cdot x + B(x-1) + Cx^2$$

if $x=0 \Rightarrow 1 = -B \Rightarrow \boxed{B=-1}$

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if $\boxed{x=1}$: $\boxed{2=C}$

Coefficient (x^2) : $1 = A + C \Rightarrow \boxed{A = -1}$

$$\therefore \int \frac{x^2+1}{x^3-x^2}\,dx = \int \frac{-1}{x}\,dx + \int \frac{-1}{x^2}\,dx + \int \frac{2}{x-1}\,dx$$

$$= -\int \frac{1}{x}\,dx - \int x^{-2}\,dx + 2\int \frac{1}{x-1}\,dx$$

$$= -\ln|x| - (-x^{-1}) + 2\ln|x-1| + C$$

$$= -\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$$

درجتان ٥

$$\int \sqrt{3+\sqrt{x}}\,dx$$

or $u=\sqrt{x}$

Let $u = 3+\sqrt{x} \Rightarrow \sqrt{x} = u-3$

$$du = \frac{1}{2\sqrt{x}}\,dx$$

$$\Rightarrow du = \frac{1}{2(u-3)}\,dx \Rightarrow dx = 2(u-3)\,du$$

$$\int \sqrt{3+\sqrt{x}}\,dx = \int \sqrt{u} \cdot 2(u-3)\,du = 2\int (u)^{\frac{1}{2}}(u-3)\,du$$

$$= 2\int \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) du$$

$$= 2\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - 3\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{4}{5}u^{\frac{5}{2}} - 4u^{\frac{3}{2}} + C$$

$$= \frac{4}{5}\left(3+\sqrt{x}\right)^{5/2} - 4\left(3+\sqrt{x}\right)^{\frac{3}{2}} + C$$

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الجزء الثالث:

(١) درجتان

$$\lim_{x\to\infty}\left(1+\frac{4}{x}\right)^x = 1^\infty$$

let $y=\left(1+\frac{4}{x}\right)^x \Rightarrow \ln y = \ln\left(1+\frac{4}{x}\right)^x = x\ln\left(1+\frac{4}{x}\right)$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\ln\left(1+\frac{4}{x}\right)$$

$$=\lim_{x\to\infty}\frac{\ln\left(1+\frac{4}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$$

Use L'Hoptial rule:

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\frac{1}{1+\frac{4}{x}}\cdot\frac{-4}{x^2}}{\frac{-1}{x^2}} = \lim_{x\to\infty}\frac{4}{1+\frac{4}{x}} = \frac{4}{1} = 4$$

$$\therefore \boxed{\lim_{x\to\infty} y = e^4}$$

(٢) ٣ درجات

$$\int_2^\infty \frac{9}{(1-3x)^4}\,dx$$

$$\int_2^\infty \frac{9}{(1-3x)^4}\,dx = \lim_{t\to\infty}\int_2^t \frac{9}{(1-3x)^4}\,dx$$

$$=\lim_{t\to\infty}\left.\frac{1}{(1-3x)^4}\right|_2^t$$

$$\int \frac{9}{(1-3x)^4}\,dx$$

$u=1-3x$

$du=-3\,dx \Rightarrow \frac{du}{-3}=dx$

$$\int \frac{9}{u^4}\frac{du}{-3} = -3\int u^{-4}\,du$$

$$=-3\frac{u^{-3}}{-3} = \frac{1}{u^3}+C$$

$$=\frac{1}{(1-3x)^3}+C$$

$$= \lim_{t\to\infty} \frac{1}{(1-3t)^4} - \frac{1}{-5}$$

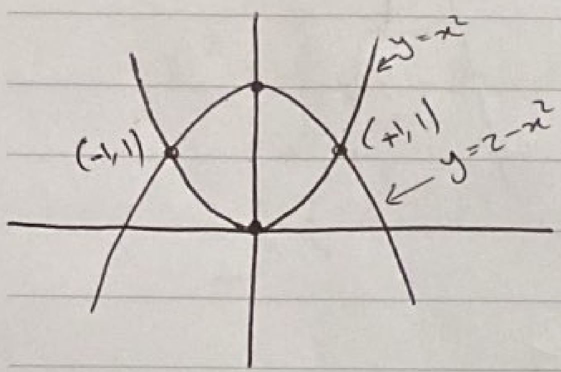
$$= 0 + \frac{1}{5} = \frac{1}{5}$$

$\therefore$ the improper integral converges

(ب) ۳ درجات

Area = ?? $y = x^2$ $y = 2 - x^2 \Rightarrow x^2 = 2 - y$

$x^2 = -(y-2)$



$\circ$ intersection points:

$y = x^2$ & $y = 2 - x^2$

$x^2 = 2 - x^2$

$2x^2 = 2 \Rightarrow x^2 = 1 \Rightarrow \boxed{x = \pm 1}$

$\Rightarrow$ if $x = 1 \Rightarrow y = 1$

$x = -1 \Rightarrow y = 1$

$\Rightarrow (1,1)$ & $(-1,1)$

$$A = \int_{-1}^{1} \left((2-x^2) - (x^2)\right) dx = \int_{-1}^{1} (2 - 2x^2)\, dx$$

$$= \left(2x - \frac{2}{3}x^3\right)\Big|_{-1}^{1}$$

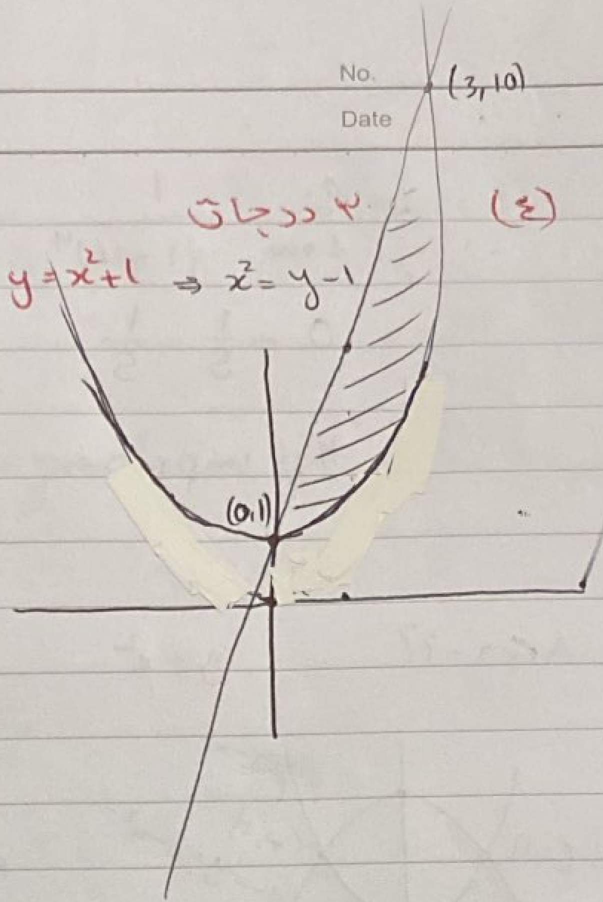
$$= \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = 4 - \frac{4}{3} = \frac{12-4}{3} = \boxed{\frac{8}{3}}$$

٣ درجات

(ع)

x

$y = 3x+1$, $y = x^2+1 \Rightarrow x^2 = y-1$



x	0	1
y	1	4

intersection points:

$y = 3x+1$ & $y = x^2+1$

$3x+1 = x^2+1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ or $x = 3$

$\Rightarrow y = 1$ or $y = 10$

$(0,1)$ & $(3,10)$

$$V = \pi \int_0^3 \left((3x+1)^2 - (x^2+1)^2\right) dx$$

$$= \pi \int_0^3 \left(9x^2 + 6x + \cancel{1} - x^4 - 2x^2 - \cancel{1}\right) dx$$

$$= \pi \int_0^3 \left(-x^4 + 7x^2 + 6x\right) dx$$

$$= \pi \left[-\frac{x^5}{5} + \frac{7}{3}x^3 + \frac{6}{2}x^2 \Big|_0^3 \right]$$

$$= \pi \left[-\frac{243}{5} + \frac{189}{3} + 27 - 0 \right] = \pi \left[\frac{-243}{5} + 63 + 27 \right]$$

$$= \pi \left[\frac{-243}{5} + 90 \right] = \pi \left[\frac{-243+450}{5} \right] = \boxed{\frac{207}{5}\pi}$$

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(٥) ٣ درجات

$y = \pi + \frac{2}{3}x^{\frac{3}{2}}$ من $x=0$ إلى $x=8$

$$L_0^8 = \int_0^8 \sqrt{1 + y'^2}\, dx$$

$$y' = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} x^{\frac{1}{2}} = x^{\frac{1}{2}}$$

$$y'^2 = (x^{\frac{1}{2}})^2 = x$$

$$L_0^8 = \int_0^8 \sqrt{1+x}\, dx = \int_0^8 (1+x)^{\frac{1}{2}} dx = \frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^8$$

$$= \frac{2}{3}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{2}{3}(27-1) = \frac{2}{3}(26) = \boxed{\frac{52}{3}}$$

(٦) درجتان

$$\frac{x}{\sqrt{x^2+y^2}} = 5y \qquad y \neq 0$$

$$\Rightarrow \frac{\cancel{r}\cos\theta}{\sqrt{r^2}} = 5\cancel{r}\sin\theta \Rightarrow \frac{\cos\theta}{\sin\theta} = 5r$$

$$\Rightarrow r = \frac{1}{5}\cot\theta$$

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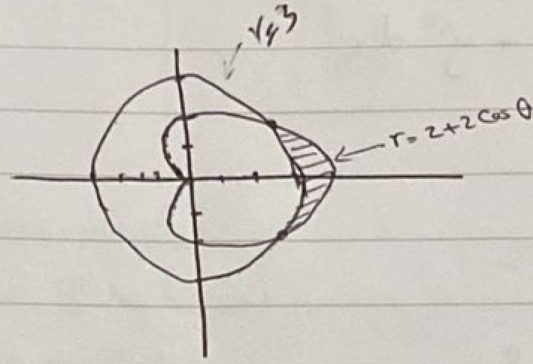
(٧)   ٤ درجات

Area = ?? $r = 3$ (outside) $r = 2+2\cos\theta$ (inside)



• intersection points

$r=3$ & $r = 2+2\cos\theta$

$3 = 2+2\cos\theta$

$1 = 2\cos\theta$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

$= \theta = 2n\pi - \frac{\pi}{3} = -\frac{\pi}{3}$ (n=0)

$$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left[\frac{1}{2}(2+2\cos\theta)^2 - \frac{1}{2}(3)^2\right] d\theta$$

$$= \cancel{2}\int_0^{\frac{\pi}{3}} \cancel{\frac{1}{2}}\left[(2+2\cos\theta)^2 - 9\right] d\theta$$

$$= \int_0^{\frac{\pi}{3}} (4 + 8\cos\theta + 4\cos^2\theta - 9)\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \left[8\cos\theta + 4\left(\frac{1}{2}(1+\cos 2\theta)\right) - 5\right] d\theta$$

$$= \int_0^{\frac{\pi}{3}} (8\cos\theta + 2 + 2\cos 2\theta - 5)\, d\theta$$

$$= 8\sin\theta + 2\theta + \frac{2}{2}\sin 2\theta - 5\theta \Big|_0^{\frac{\pi}{3}}$$

$$= \left(8\frac{\sqrt{3}}{2} - 3\left(\frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}\right) - 0 = \boxed{9\frac{\sqrt{3}}{2} - \pi}$$