

Integral-differentiation rules

Type	$y = f(x)$	derivative $\frac{dy}{dx}$	$\int f(x)dx$
Basic rules	c	0	$\int c dx = cx + c'$
	1	0	$\int 1 dx = \int dx = x + c$
	x	1	$\int x dx = \frac{1}{2}x^2 + c$
	x^n	nx^{n-1}	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
	$c \cdot f(x)$	$c \cdot f'(x)$	$\int c f'(x)dx = cf(x) + c'$
Trigonometric function	$\sin x$	$\cos x$	$\int \cos x dx = \sin x + c$
	$\cos x$	$-\sin x$	$\int \sin x dx = -\cos x + c$
	$\tan x$	$\sec^2 x$	$\int \sec^2 x dx = \tan x + c$
	$\cot x$	$-\csc^2 x$	$\int \csc^2 x dx = -\cot x + c$
	$\sec x$	$\sec x \tan x$	$\int \sec x \tan x dx = \sec x + c$
	$\csc x$	$-\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + c$
	$\sin[f(x)]$	$\cos[f(x)]f'(x)$	$\int \cos[f(x)]f'(x)dx = \sin[f(x)] + c$
	$\cos[f(x)]$	$-\sin[f(x)]f'(x)$	$\int \sin[f(x)]f'(x) dx = -\cos[f(x)] + c$
	$\tan[f(x)]$	$\sec^2[f(x)]f'(x)$	$\int \sec^2[f(x)]f'(x) dx = \tan[f(x)] + c$
	$\cot[f(x)]$	$-\csc^2[f(x)]f'(x)$	$\int \csc^2[f(x)]f'(x)dx = -\cot[f(x)] + c$
	$\sec[f(x)]$	$\sec[f(x)] \tan[f(x)]f'(x)$	$\int \sec[f(x)]\tan[f(x)]f'(x)dx = \sec[f(x)] + c$
$\csc[f(x)]$	$-\csc[f(x)] \cot[f(x)]f'(x)$	$\int \csc[f(x)]\cot[f(x)]f'(x)dx = -\csc[f(x)] + c$	
Logarithmic functions	$\ln x$	$\frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + c$
	$\ln[f(x)]$	$\frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$
Exponential functions	e^x	e^x	$\int e^x dx = e^x + c$
	$e^{f(x)}$	$e^{f(x)}f'(x)$	$\int e^{f(x)}f'(x) = e^{f(x)} + c$
	a^x	$a^x \ln a$	$\int a^x dx = \frac{a^x}{\ln a} + c$
	$a^{f(x)}$	$a^{f(x)} \cdot f'(x) \cdot \ln a$	$\int a^{f(x)} \cdot f'(x) = \frac{a^{f(x)}}{\ln a} + c$
Radical functions	\sqrt{x}	$\frac{1}{2\sqrt{x}}$	$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$
	$\sqrt{f(x)}$	$\frac{f'(x)}{2\sqrt{f(x)}}$	$\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

Type	$y = f(x)$	derivative $\frac{dy}{dx}$	$\int f(x)dx$
Inverse trigonometric	$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + c$
	$\cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + c$
	$\tan^{-1}(x)$	$\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + c$
	$\cot^{-1}(x)$	$\frac{-1}{1+x^2}$	$\int \frac{-1}{1+x^2} dx = \cot^{-1}(x) + c$
	$\sec^{-1}(x)$	$\frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x) + c$
	$\csc^{-1}(x)$	$\frac{-1}{x\sqrt{x^2-1}}$	$\int \frac{-1}{x\sqrt{x^2-1}} dx = \csc^{-1}(x) + c$
	$\sin^{-1}[f(x)]$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}} dx = \sin^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\cos^{-1}[f(x)]$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	$\int \frac{-f'(x)}{\sqrt{a^2-[f(x)]^2}} dx = \cos^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\tan^{-1}[f(x)]$	$\frac{f'(x)}{1+[f(x)]^2}$	$\int \frac{f'(x)}{a^2+[f(x)]^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\cot^{-1}[f(x)]$	$\frac{-f'(x)}{1+[f(x)]^2}$	$\int \frac{-f'(x)}{a^2+[f(x)]^2} dx = \frac{1}{a} \cot^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\sec^{-1}[f(x)]$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	$\int \frac{f'(x)}{f(x)\sqrt{[f(x)]^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\csc^{-1}[f(x)]$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	$\int \frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-a^2}} dx = \frac{1}{a} \csc^{-1}\left(\frac{f(x)}{a}\right) + c$
Hyperbolic functions	$\sinh x$	$\cosh x$	$\int \cosh x dx = \sinh x + c$
	$\cosh x$	$\sinh x$	$\int \sinh x dx = \cosh x + c$
	$\tanh x$	$\operatorname{sech}^2 x$	$\int \operatorname{sech}^2 x dx = \tanh x + c$
	$\coth x$	$-\operatorname{csch}^2 x$	$\int \operatorname{csch}^2 x dx = -\coth x + c$
	$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$	$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + c$
	$\operatorname{csch} x$	$-\operatorname{csch} x \coth x$	$\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + c$
	$\sinh[f(x)]$	$\cosh[f(x)]f'(x)$	$\int \cosh[f(x)]f'(x)dx = \sinh[f(x)] + c$
	$\cosh[f(x)]$	$\sinh[f(x)]f'(x)$	$\int \sinh[f(x)]f'(x)dx = \cosh[f(x)] + c$
	$\tanh[f(x)]$	$\operatorname{sech}^2[f(x)]f'(x)$	$\int \operatorname{sech}^2[f(x)]f'(x)dx = \tanh[f(x)] + c$
	$\coth[f(x)]$	$-\operatorname{csch}^2[f(x)]f'(x)$	$\int \operatorname{csch}^2[f(x)]f'(x)dx = -\coth[f(x)] + c$
	$\operatorname{sech}[f(x)]$	$-\operatorname{sech}[f(x)] \tanh[f(x)]f'(x)$	$\int \operatorname{sech}[f(x)] \tanh[f(x)]f'(x)dx = -\operatorname{sech}[f(x)] + c$
	$\operatorname{csch}[f(x)]$	$-\operatorname{csch}[f(x)] \coth[f(x)]f'(x)$	$\int \operatorname{csch}[f(x)] \coth[f(x)]f'(x)dx = -\operatorname{csch}[f(x)] + c$

prepared by Mr. Ahmed Ebrahim

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Type	$y = f(x)$	$\frac{dy}{dx}$	$\int f(x)dx$
Inverse Hyperbolic functions	$\sinh^{-1}x$	$\frac{1}{\sqrt{1+x^2}}$	$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}x + c$
	$\cosh^{-1}x$	$\frac{1}{\sqrt{x^2-1}}$	$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1}x + c$
	$\tanh^{-1}x$	$\frac{1}{1-x^2}$	$\int \frac{1}{1-x^2} dx = \tanh^{-1}x + c; x < 1$
	$\coth^{-1}x$	$\frac{1}{1-x^2}$	$\int \frac{1}{1-x^2} dx = \tanh^{-1}x + c; x > 1$
	$\operatorname{sech}^{-1}x$	$\frac{1}{x\sqrt{1-x^2}}$	$\int \frac{1}{x\sqrt{1-x^2}} dx = \operatorname{sech}^{-1}x + c$
	$\operatorname{csch}^{-1}x$	$\frac{1}{ x \sqrt{1+x^2}}$	$\int \frac{1}{ x \sqrt{1+x^2}} dx = \operatorname{csch}^{-1}x + c$
	$\sinh^{-1}[f(x)]$	$\frac{1}{\sqrt{1+[f(x)]^2}}$	$\int \frac{f'(x)}{\sqrt{a^2+[f(x)]^2}} dx = \sinh^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\cosh^{-1}[f(x)]$	$\frac{1}{\sqrt{[f(x)]^2-1}}$	$\int \frac{f'(x)}{\sqrt{[f(x)]^2-a^2}} dx = \cosh^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\tanh^{-1}[f(x)]$	$\frac{1}{1-[f(x)]^2}$	$\int \frac{f'(x)}{a^2-[f(x)]^2} dx = \frac{1}{a} \tanh^{-1}\left(\frac{f(x)}{a}\right) + c; f(x) < a$
	$\coth^{-1}[f(x)]$	$\frac{1}{1-[f(x)]^2}$	$\int \frac{f'(x)}{a^2-[f(x)]^2} dx = \frac{1}{a} \coth^{-1}\left(\frac{f(x)}{a}\right) + c; f(x) > a$
	$\operatorname{sech}^{-1}[f(x)]$	$\frac{1}{[f(x)]\sqrt{1-[f(x)]^2}}$	$\int \frac{f'(x)}{[f(x)]\sqrt{a^2-[f(x)]^2}} dx = \frac{1}{a} \operatorname{sech}^{-1}\left(\frac{f(x)}{a}\right) + c$
	$\operatorname{csch}^{-1}[f(x)]$	$\frac{1}{ [f(x)] \sqrt{1+[f(x)]^2}}$	$\int \frac{f'(x)}{ [f(x)] \sqrt{a^2+[f(x)]^2}} dx = \frac{1}{a} \operatorname{csch}^{-1}\left(\frac{f(x)}{a}\right) + c$

Additional formulas

$\int \tan x dx = -\ln \cos x + c$
$\int \cot x dx = \ln \sin x + c$
$\int \sec x dx = \ln \sec x + \tan x + c$
$\int \csc x dx = \ln \csc x - \cot x + c$
$\int \sin ax \cos bx dx = \frac{1}{2} \int \sin(ax+bx) dx + \frac{1}{2} \int \sin(ax-bx) dx$
$\int \sin ax \sin bx dx = \frac{1}{2} \int \cos(ax-bx) dx - \frac{1}{2} \int \cos(ax+bx) dx$
$\int \cos ax \cos bx dx = \frac{1}{2} \int \cos(ax+bx) dx + \frac{1}{2} \int \cos(ax-bx) dx$
$\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$
$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$
$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$
$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$

Integration by parts

التكامل بالتجزئة

$$\int u dv = uv - \int v du$$

how to choose u??

Logarithmic,
inverse trigonometric, Algebraic,
trigonometric
Exponential

Rational Integration

تكامّل الدوال الكسرية (النسبية)

- partial fractions
- completing the square of denominator
- long division (OR factoring)

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Product of power sine and cosine

$$\int \sin^m x \cos^n x$$

n odd split $\cos x$ use $\sin^2 x = 1 - \cos^2 x$
take $u = \sin x$

m odd split $\sin x$ use $\cos^2 x = 1 - \sin^2 x$
take $u = \cos x$

$$\begin{cases} m \text{ even} \\ n \text{ even} \end{cases} \text{ use } \begin{cases} \cos^2(x) = \frac{1}{2}(1 + \cos 2x) \\ \sin^2(x) = \frac{1}{2}(1 - \cos 2x) \end{cases}$$

Product of power tan and sec

$$\int \tan^m x \sec^n x$$

n even split $\sec^2 x$ use $\sec^2 x = \tan^2 x + 1$
take $u = \tan x$

m odd split $\sec x \tan x$ use $\tan^2 x = \sec^2 x - 1$
take $u = \sec x$

$$\begin{cases} m \text{ even} \\ n \text{ odd} \end{cases} \text{ secx alone use } \tan^2 x = \sec^2 x - 1$$

Fundamental theorem of calculus

Part 1 $\int_a^b \frac{d}{dx} G(x) dx = G(b) - G(a)$

Part 2 $\frac{d}{dx} \int_a^x f(t) dt = f(x)$

Part 3 $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt = f(h(x))h'(x) - f(g(x))g'(x)$

Trigonometric substitutions

$$\sqrt{a^2 - x^2} \rightarrow x = a \sin \theta$$

$$dx = a \cos \theta d\theta$$

$$\sqrt{a^2 + x^2} \rightarrow x = a \tan \theta$$

$$dx = a \sec^2 \theta d\theta$$

$$\sqrt{x^2 - a^2} \rightarrow x = a \sec \theta$$

$$dx = a \sec \theta \tan \theta d\theta$$

Trigonometric Identities

$$\sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1, \quad \sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\tan^2 x + 1 = \sec^2 x, \quad \tan^2 x = \sec^2 x - 1$$

$$1 + \cot^2 x = \csc^2 x, \quad \cot^2 x = \csc^2 x - 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Hyperbolic Identities

$$\operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\sinh x}, \quad \tanh x = \frac{\sinh x}{\cosh x}$$

$$\cosh x + \sinh x = e^x$$

$$\cosh x - \sinh x = e^{-x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x = 1 - \operatorname{sech}^2 x$$

$$\coth^2 x = 1 + \operatorname{csch}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 2\cosh^2 x - 1$$

$$= 2\sinh^2 x + 1$$

$$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$$

$$\cosh^2 x = \frac{1}{2}\cosh 2x + 1$$

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Hyperbolic Derivatives

$\sinh x = \frac{e^x - e^{-x}}{2}$	$\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$
$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	$\operatorname{coth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

Summation and Riemann sum

$\sum_{i=1}^n c = cn$	$\sum_{i=1}^n i = \frac{n(n+1)}{2}$
$\sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$	$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^n ca_i = cn$	$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$
$\sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$	<p style="text-align: center; color: red;">Riemann sum</p> $R_n = \sum_{i=1}^n f(c_i)\Delta x$

area under curve

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i)\Delta x \quad , \quad \Delta x = \frac{b-a}{n} \quad , \quad c_i = a + i\Delta x$$

Average value function

f continuous on $[a, b]$

$$f_{av} = \frac{\int_a^b f(x)dx}{b-a}$$

Mean value theorem

f continuous on $[a, b]$ there exists number *c* on (a, b)

$$f(c) = \frac{\int_a^b f(x)dx}{b-a}$$

Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$$

Simpson's rule

$$\int_a^b f(x)dx \approx \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

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التكامل المعطل Imperator integrals

$$\int_a^{\infty} f(x)dx = \lim_{t \rightarrow \infty} \int_a^t f(x)dx$$

$$\int_{-\infty}^a f(x)dx = \lim_{t \rightarrow -\infty} \int_t^a f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{t \rightarrow -\infty} \int_t^a f(x)dx + \lim_{t \rightarrow \infty} \int_a^t f(x)dx$$

Area between two curves

With respect to X-Axis

$$A = \int_a^b [f(x) - g(x)]dx \quad , f(x) \geq g(x)$$

With respect to Y-Axis

$$A = \int_c^d [w(y) - v(y)] dy \quad , w(y) \geq v(y)$$

Volume of Revolution

Rotation about X-Axis

$$V = \pi \int_a^b ([f(x)]^2 - [g(x)]^2)dx \quad , f(x) \geq g(x)$$

Rotation about Y-Axis

$$V = \pi \int_c^d ([w(y)]^2 - [v(y)]^2)dy \quad , w(y) \geq v(y)$$

Arc length

With respect to X-Axis

$$y = f(x) , a \leq x \leq b$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

With respect to Y-Axis

$$x = g(y) , c \leq y \leq d$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Surface Area

Rotation about X-Axis

$$y = f(x) , a \leq x \leq b$$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

With respect to Y-Axis

$$x = g(y) , c \leq y \leq d$$

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

The slope of tangent line of parametric curve $y = y(t)$ at t_0

$$m = \frac{dy}{dx} \Big|_{t=t_0} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \Big|_{t=t_0}$$

Arc length of parametric curve

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Surface area of parametric curve

$$SA = 2\pi \int_a^b |y(t)| \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$SA = 2\pi \int_a^b |x(t)| \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

To convert from polar to rectangular coordinates

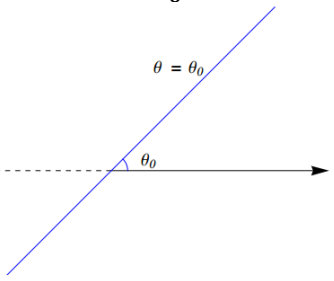
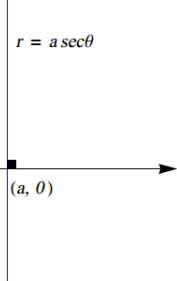
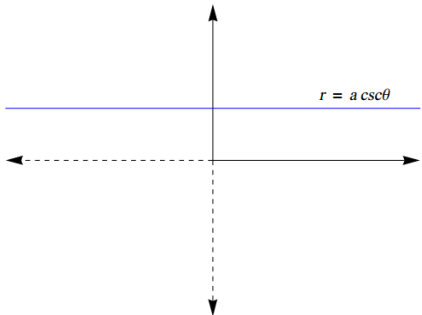
$$x = r \cos \theta \quad , \quad y = r \sin \theta$$

To convert from rectangular to polar coordinates

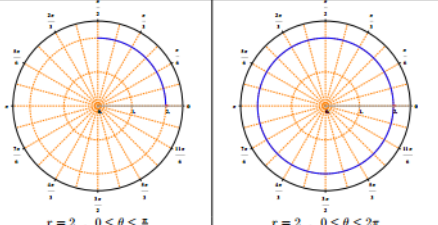
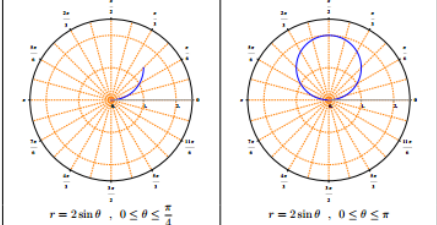
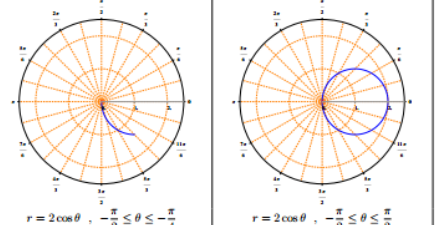
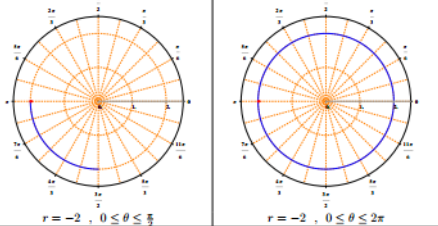
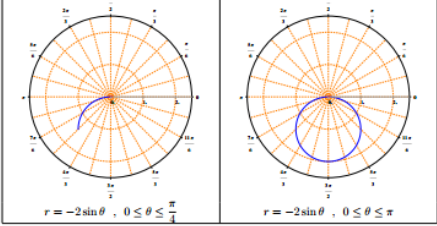
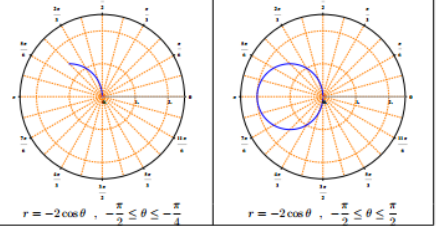
$$r^2 = x^2 + y^2 \quad , \quad \tan \theta = \frac{y}{x}$$

Polar curves

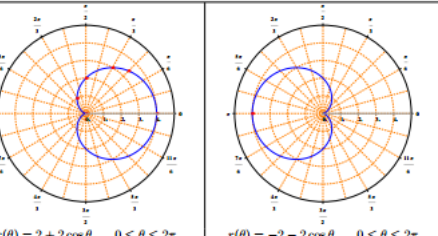
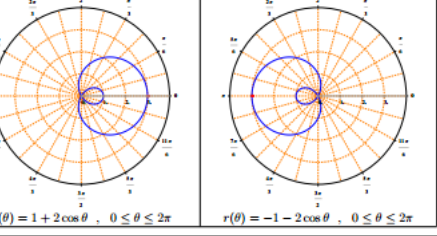
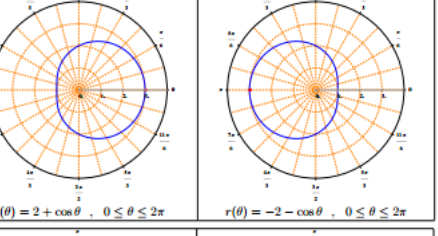
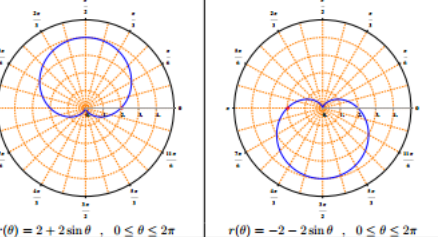
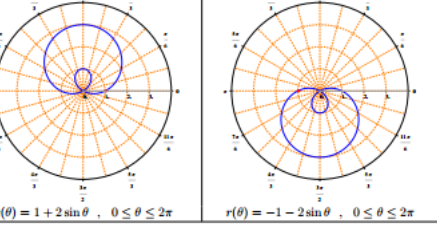
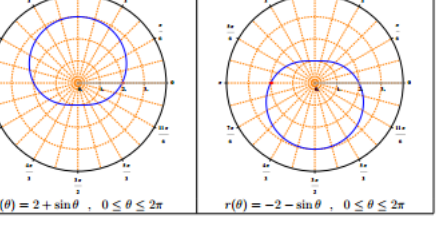
① lines

Lines passing through the pole	Lines perpendicular to the polar axis	Lines parallel to the polar axis
$\theta = \theta_0$ 	$r = a \sec \theta$ 	$r = a \csc \theta$ 

② circles

$r = a$	$r = a \sin \theta$	$r = a \cos \theta$
		
		

③ Lima con curves

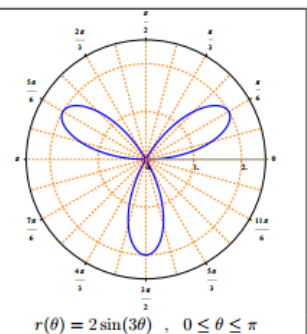
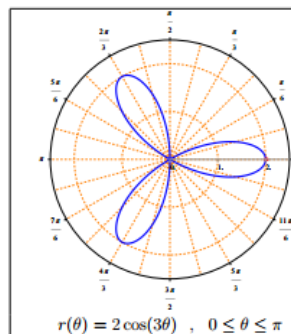
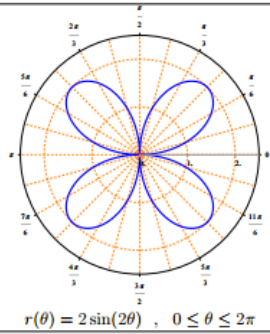
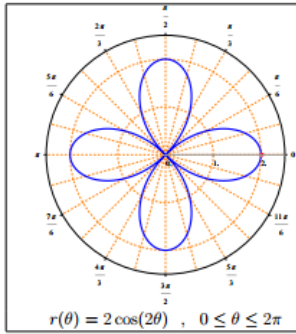
Cardioid (heart shape)	Lima con with inner loop	Dimpled Lima con
$r(\theta) = a + a \sin \theta$ $r(\theta) = a + a \cos \theta$	$r(\theta) = a + b \sin \theta, a < b $ $r(\theta) = a + b \cos \theta, a < b $	$r(\theta) = a + b \sin \theta, a > b $ $r(\theta) = a + b \cos \theta, a > b $
		
		

4 Rose curves

$$r(\theta) = a \cos(n\theta) \quad , \quad r(\theta) = a \sin(n\theta)$$

n even

n odd



Test of symmetry

With respect to the
polar axis

With respect to the line
 $\theta = \frac{\pi}{2}$

With respect to the pole

$$r(\theta) = r(-\theta)$$

$$r(\theta) = -r(-\theta)$$

$$r(\theta) = r(\pi - \theta)$$

$$r(\theta) = r(\pi + \theta)$$

Area inside-between polar Axis

Arc length of a polar Axis

bounded by Curves $r = r(\theta)$, $\theta = \theta_1$,

Curve $r = r(\theta)$, from θ_1 to θ_2

$$A = \int_{\theta_1}^{\theta_2} [r(\theta)]^2 d\theta$$

$$A = \int_{\theta_1}^{\theta_2} \sqrt{[r(\theta)]^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Surface area generated by Revolving a polar curve

$r = r(\theta)$, $\theta_1 \leq \theta \leq \theta_2$

Around the polar axis

Around the line $\theta = \frac{\pi}{2}$

$$A = 2\pi \int_{\theta_1}^{\theta_2} r(\theta) \sin \theta \sqrt{[r(\theta)]^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

$$A = 2\pi \int_{\theta_1}^{\theta_2} r(\theta) \cos \theta \sqrt{[r(\theta)]^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

prepared by Mr. Ahmed Ebrahim

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