King Saud University | College of Sciences | Department of Mathematics
Midterm Examination | Math 132 | 1444(II) | Time:2H

Question 1: (7 marks)

1. Decide whether the following propositions is a tautology or a contradiction or a contingency? **(3 marks)**
$$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r.$$
2. Without using truth tables, prove that the following statement is a contradiction: **(2 marks)**
$$p \wedge (p \to q) \wedge (p \to \neg q)$$
3. Without using truth tables, prove the following logical equivalence: **(2 marks)**
$$\neg [p \vee (\neg p \wedge q)] \equiv (\neg p \wedge \neg q)$$

Question 2: (8 marks)

1. Let n be an integer. Show that if $n^2 + 5$ is an odd integer, then n is even. **(2 marks)**
2. Use mathematical induction to prove the following statement: **(3 marks)**
$$1 + 3 + 5 + \cdots + (2n-1) = n^2 \quad \forall n \geq 1.$$
3. Consider the sequence $\{u_n\}_{n=0}^{\infty}$ defined as follows: $\begin{cases} u_0 = 6 \\ u_1 = 9 \\ u_{n+1} = 2u_n - u_{n-1}; \quad n \geq 1 \end{cases}$

   Use mathematical induction to prove the following statement:**(~~4~~3 marks)**
$$u_n = 3n + 6, \quad \text{for each integer } n, \text{ with } n \geq 0.$$

Question 3: (6 marks)

1. Consider the set $A := \{1, 2, \{\emptyset\}, \{2, \{\emptyset\}\}, \{1, \{\emptyset\}\}, \{1, 2\}, \{1\}\}$.
   Determine whether each of the following statements is true or false. (Justify your answer). **(2 marks)**
   (a) S_1: "$\{\{1\}, \{\emptyset\}\} \in A$". (b) S_2: "$\{\{1\}, \{\emptyset\}\} \subseteq A$".
   (c) S_3: "$\{1, \{2\}\} \subseteq A$". (d) S_4: $\{1, 2, \{\{1\}, \{\emptyset\}\}\} - A = \{1, \{\emptyset\}\}$.
2. Consider the following two sets $B := \{a, 1, \{1\}\}$ and $C := \{1, 2, a, \{1\}\}$.
   Find the following two sets: **(4 marks)**
   (i) $(B \cap C) \times C$. **(ii)** $(B - C) \times (C - B)$. **(iii)** $\emptyset \times C$. **(iv)** $B \times \{\emptyset\}$.

Question 4: (9 marks)

Let $A := \{1, 3, 5, 7\}$ and $B := \{0, 1, 2, 3, 4\}$ be two sets.
Let R be the relation from the set A to the set B defined as follows:
for $a \in A$ and $b \in B$, $[(aRb) \Leftrightarrow (a \leq b)]$.
Let S be the relation from B to A defined by $S := \{(0, 1), (1, 1), (2, 1), (3, 5)\}$.

1. List all the ordered pairs in the relation R. **(1 marks)**
2. Find the domain and the image (range) of R. **(1 marks)**
3. Find the relations R^{-1} and S^{-1}. **(2 marks)**
4. Find the relations $R \cup S^{-1}$ and $S \cap R^{-1}$. **(2 marks)**
5. Find the relations : $S \circ R$ and $R \circ S$. **(3 marks)**

Answer mid Math 132

(442)

⑦

Q_1) 1). $\Big[\overbrace{\overbrace{(p\vee q)\wedge(p\to r)}^{A}\wedge(q\to r)}^{B}\Big]\to r.$

p	q	r	$p\vee q$	$p\to r$	$q\to r$	A	B	R.
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

is a tautology.

③

2) $p\wedge(p\to q)\wedge(p\to\neg q)\equiv p\wedge[p\to(q\wedge\neg q)]$

$\equiv p\wedge(p\to F)$

$\equiv p\wedge(\neg p\vee F)$ ②

$\equiv p\wedge\neg p$

$\equiv F.$

3).

$\neg[p\vee(\neg p\wedge q)]\equiv\neg((p\vee\neg q)\wedge(p\vee q))$

$\equiv\neg(T\wedge(p\vee q))$ ②

$\equiv\neg(p\vee q)$

$\equiv(\neg p\wedge\neg q).$

2

Q2) 8

1) we prove by contraposition that: if n is odd, then n^2+5 is even.

n is odd $\Rightarrow n = 2k+1,\ k \in \mathbb{Z},$

$$n^2+5 = (2k+1)^2+5 = 4k^2+4k+1+5$$
$$= 4k^2+4k+6$$
$$= 2(2k^2+2k+3)$$
$$= 2t;\ t = 2k^2+2k+3 \in \mathbb{Z}.$$

$\Rightarrow n^2+5$ is even. 2

2) $P(n): 1+3+5+\dots+(2n-1) = n^2;\ n \geq 1.$

B.S: $P(1)$: L.H.S $= 1$, R.HS $= 1^2 = 1$.

$\Rightarrow$ L.H.S = R.H.S $\Rightarrow P(1)$ is true.

I.S: Let $k \geq 1$; we prove that $P(k) \longrightarrow P(k+1)$.

We assume that $P(k)$ is true $\Rightarrow 1+3+5+\dots+(2k-1) = k^2$.

and we prove that $P(k+1)$ is true. $(1+3+5+\dots+(2k+1)) = (k+1)^2$.

$$1+3+5+\dots+(2k+1) = 1+3+5+\dots+(2k-1)+(2k+1)$$
$$= k^2+2k+1 = (k+1)^2.$$

3

$\Rightarrow P(k+1)$ is true.

$\Rightarrow \forall n \geq 1\ P(n)$ is true.

3) $P(n): u_n = 3n+6;\ \forall n \geq 0.$

B.S: $P(0): u_0 = 3\times 0+6 = 6$ T

$P(1) = u_1 = 3+6 = 9$ T.

I.S: Let $k \geq 1$, we assume that $(P(0), P(1), .., P(k))$ are true and we prove that $P(k+1)$ is true.

$P(k+1): u_{k+1} = 3(k+1)+6 = 3k+9.$

3

$u_{k+1} = 2u_k - u_{k-1}$

$P(k)$ true $\Rightarrow u_k = 3k+6.$

$P(k+1)$ true $\Rightarrow u_{k-1} = 3(k-1)+6 = 3k+3.$

$\Rightarrow u_{k+1} = 6k+12-3k-3 = 3k+9$

$\Rightarrow P(k+1)$ true $\Rightarrow \forall n \geq 0\ P(n)$ true.

③ ⑥

Q_3)

1) a) S_1 : False , $\{\{1\},\{\phi\}\}$ not in A.

b) S_2 : True : $\{1\} \in A$; $\{\phi\} \in A$.

c) S_3 : False $\{2\} \notin A$. ②

d) S_4 : False $\{1,\{\phi\}\} \notin \{1,2,\{\{1\},\{\phi\}\}\}$.

2). $(B \cap C) = \{1, a, \{1\}\}$.

i) $(B \cap C) \times C = \{(1,1),(1,2),(1,a),(1,\{1\}),(a,1),(a,2),(a,a),(a,\{1\})$
$(\{1\},1),(\{1\},2)(\{1\},a);(\{1\},\{1\})\}$. ①

ii) $(B - C) = \phi$; $(C - B) = \{2\}$,
$(B - C) \times (C - B) = \phi$ ①

iii) $\phi \times C = \phi$. ①

iv) $B \times \{\phi\} = \{(a, \phi), (1, \phi), (\{1\}; \phi)\}$ ①

Q_4)

1) $R = \{(1,1),(1,2),(1,3),(1,4),(3,3),(3,4)\}$. ①

2) domain$(R) = \{1,3\}$.
image$(R) = \{1,2,3,4\}$. ①

3) $R^{-1} = \{(1,1),(2,1),(3,1),(4,1),(3,3),(4,3)\}$. ①

$S^{-1} = \{(1,0),(1,1),(1,2),(5,3)\}$. ①

4).
$R \cup S^{-1} = \{(1,1),(1,2),(1,3),(1,4),(3,3),(3,4),(1,0),(5,3)\}$. ①

$S \cap R^{-1} = \{(1,1),(2,1)\}$. ①

5).
$S \circ R = \{(1,1),(1,5),(3,5)\}$. (1r)

$R \circ S = \{(0,1),(0,2),(0,3),(0,4),(1,1),(1,2),(1,3),(1,4)$
$(2,1),(2,2),(2,3),(2,4)\}$. (1r)