$X_i \sim f(x_i;\theta)$, $i = 1, 2, \dots, n$

$\underline{X} = (X_1, \dots, X_n)$

reject H_0	accept H_0

$\rightarrow S$ sample space

$\Omega_0 = \{\theta_0\}$	$\Omega_1 = \{\theta_1\}$

$\rightarrow \theta \in \Omega$ Parameter space

$$\underset{\text{test}}{\gamma} = \begin{cases} H_0: \theta \in \Omega_0 \quad \underline{\text{or}} \quad \theta = \theta_0 \\ \text{vs} \\ H_1: \theta \in \Omega_1 \quad \underline{\text{or}} \quad \theta = \theta_1 \end{cases}$$

1. $0 \le$ type of errors ≤ 1

   α: type 1 error

   (+)ve error

   size of the test

   level of significance

   reject H_0 | H_0 ✓

   β: type 2 error

   (−)ve error

   accept H_0 | H_0 ✗

2. $0 \le$ power function ≤ 1 $\longrightarrow$ $\pi_\gamma = \pi = 1 - \beta$

   reject H_0 | H_0 ✗

(3) most powerful test γ_{MP} with size α_{MP} if

$\forall$ other γ with size α and

- $\alpha \le \alpha_{MP}$
- $\Pi_{\gamma_{MP}} \ge \Pi_{\gamma} \iff \beta_{MP} \le \beta$

How to find γ_{MP} for given α_{MP} ?

or

by Neyman-Pearson lemma

(1) $$\lambda = \frac{L(\underline{x};\theta_0)}{L(\underline{x};\theta_1)} = \frac{\prod_{i=1}^{n} f(x_i;\theta_0)}{\prod_{i=1}^{n} f(x_i;\theta_1)}$$

Likelihood Ratio (LR)

(2) reject H_0 if $\lambda < k$, $k = ?$

(3) find k for given α_{MP}. i.e.,

$$\alpha_{MP} = P(\underbrace{\lambda < k}_{\text{reject } H_0} \mid \underbrace{\theta_0}_{H_0 \checkmark})$$

EF

$$f(x_i;\theta) = e^{a(\theta) + b(x_i) + c(\theta)d(x_i)}$$

(1)

	$\theta_0 < \theta_1$	$\theta_0 > \theta_1$
$c(\theta)\nearrow$	$\sum_{i=1}^{n} d(x_i) > k$	$\sum_{i=1}^{n} d(x_i) < k$
$c(\theta)\searrow$	$\sum_{i=1}^{n} d(x_i) < k$	$\sum_{i=1}^{n} d(x_i) > k$

reject H_0

(2) find k for given α_{MP}. i.e.,

$$\alpha_{MP} = P(\underbrace{\qquad}_{\text{reject } H_0} \mid \underbrace{\theta_0}_{H_0 \checkmark})$$