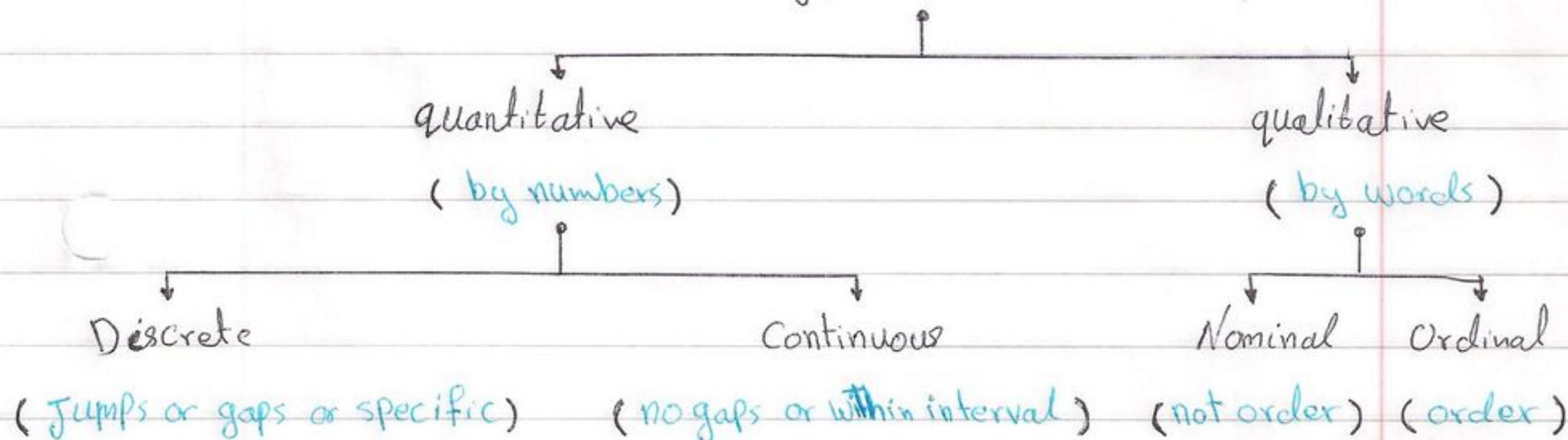


"Basic Concepts"

- * population: is the largest collection of entities (elements or individuals) in which we are interested at particular time and about which we want to draw some conclusions ($N = \text{population size}$).
- * Sample: is a part of population ($n = \text{sample size}$).
- * Variable: characteristic to be measured (number or words) on the elements (in population or in sample).




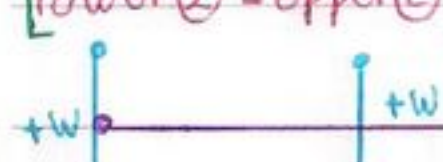
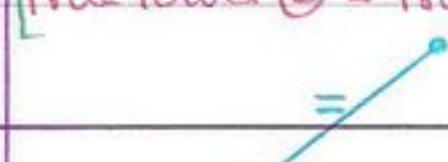

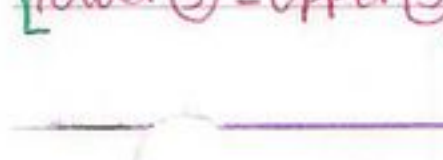
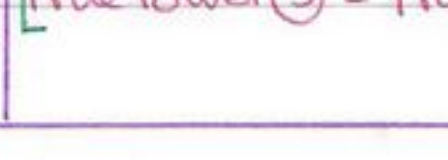
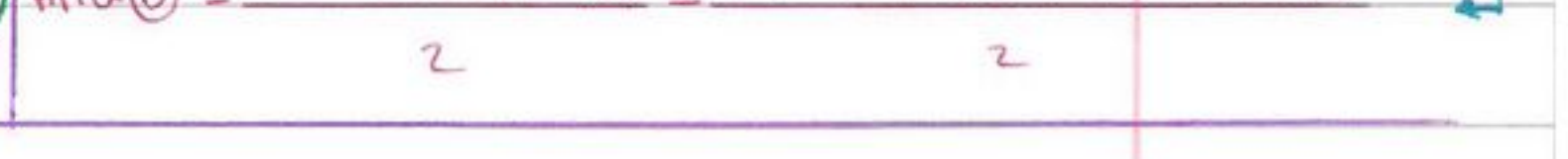
* types of Variables



- * data: raw material of statistics.
- * statistics: field of study concerned with.
- * descriptive statistics: collection, organization, summarization, describe, and analysis of data.
- * inferential statistics: reach decisions, inference, and conclusions about a large body of data (population) by only part of the data (sample) is observed.

"Frequency Distribution"

* Ordered array or set: listing of values in order from the smallest to the largest values with frequency.

class interval	true class interval	mid point or mid interval
$[lower(1) - upper(1)]$ 	$[true\ lower(1) - true\ upper(1)]$ 	$mid(1) = \frac{lower(1) + upper(1)}{2} = \frac{true\ lower(1) + true\ upper(1)}{2}$ 
$[lower(2) - upper(2)]$ 	$[true\ lower(2) - true\ upper(2)]$ 	$mid(2) = \frac{lower(2) + upper(2)}{2} = \frac{true\ lower(2) + true\ upper(2)}{2}$ 
$[lower(3) - upper(3)]$ 	$[true\ lower(3) - true\ upper(3)]$ 	$mid(3) = \frac{lower(3) + upper(3)}{2} = \frac{true\ lower(3) + true\ upper(3)}{2}$ 

The method to find the following:

* class interval \Rightarrow true class interval:

$$d = lower(2) - upper(1) = lower(3) - upper(2) \quad (\text{gaps})$$

$$\begin{cases} true\ lower(1) = lower(1) - \frac{d}{2} \\ true\ upper(1) = upper(1) + \frac{d}{2} \\ true\ lower(2) = lower(2) - \frac{d}{2} \\ true\ upper(2) = upper(2) + \frac{d}{2} \\ true\ lower(3) = lower(3) - \frac{d}{2} \\ true\ upper(3) = upper(3) + \frac{d}{2} \end{cases}$$

* true class interval \Rightarrow class interval (where d or $\frac{d}{2}$ is known):

$$\left[\begin{array}{l} \text{lower } ① = \text{true lower } ① + \frac{d}{2} \\ \text{upper } ① = \text{true upper } ① - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ② = \text{true lower } ② + \frac{d}{2} \\ \text{upper } ② = \text{true upper } ② - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{lower } ③ = \text{true lower } ③ + \frac{d}{2} \\ \text{upper } ③ = \text{true upper } ③ - \frac{d}{2} \end{array} \right.$$

* midpoint \Rightarrow true class interval:

$$\begin{aligned} w = \text{width} &= \text{lower } ② - \text{lower } ① = \text{lower } ③ - \text{lower } ② && \left. \begin{array}{l} \text{by using the class} \\ \text{interval} \end{array} \right\} \\ &= \text{upper } ② - \text{upper } ① = \text{upper } ③ - \text{upper } ② \end{aligned}$$

$$= \text{true upper } ① - \text{true lower } ①$$

$$= \text{true upper } ② - \text{true lower } ②$$

$$= \text{true upper } ③ - \text{true lower } ③$$

$$= \text{mid } ② - \text{mid } ① = \text{mid } ③ - \text{mid } ② \left. \begin{array}{l} \text{by using the mid point} \end{array} \right\}$$

$$\left[\begin{array}{l} \text{true lower } ① = \text{mid } ① - \frac{w}{2} \\ \text{true upper } ① = \text{mid } ① + \frac{w}{2} \end{array} \right. =$$

$$\left[\begin{array}{l} \text{true lower } ② = \text{mid } ② - \frac{w}{2} \\ \text{true upper } ② = \text{mid } ② + \frac{w}{2} \end{array} \right. =$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

$$\left[\begin{array}{l} \text{true lower } ③ = \text{mid } ③ - \frac{w}{2} \\ \text{true upper } ③ = \text{mid } ③ + \frac{w}{2} \end{array} \right.$$

جزء الثاني

* Frequency \cong Freq.

* Relative \cong r. (relative \cong proportion)

* Percentage \cong per. (percentage \cong percent)

* Cumulative \cong Cum.

Freq.	Cum. Freq.	r. Freq.	Cum. r. Freq.	Per. Freq.	Cum. Per. Freq.
Total = n	n	Total = 1	1	Total = 100%	100%

Diagram illustrating the relationships between different frequency measures. The table shows columns for Frequency, Cumulative Frequency, Relative Frequency, Cumulative Relative Frequency, Percentage Frequency, and Cumulative Percentage Frequency. The bottom row shows the total values for each column: Total = n, n, Total = 1, 1, Total = 100%, and 100%. Arrows indicate the operations used to derive these values: $\div n$ (from Freq. to Cum. Freq. and from r. Freq. to Cum. r. Freq.) and $\times 100\%$ (from Cum. Freq. to Per. Freq. and from Cum. r. Freq. to Cum. Per. Freq.).

We have another Method to Find the Following :

* Cumulative Frequency :

Cum. Freq. of 1st class interval = Freq.

Cum. Freq. of any class interval = Freq. + Cum. Freq. of preceding class interval

* Cumulative relative Frequency :

Cum. r. Freq. of 1st class interval = r. Freq.

Cum. r. Freq. of any class interval = r. Freq. + Cum. r. Freq. of preceding class interval

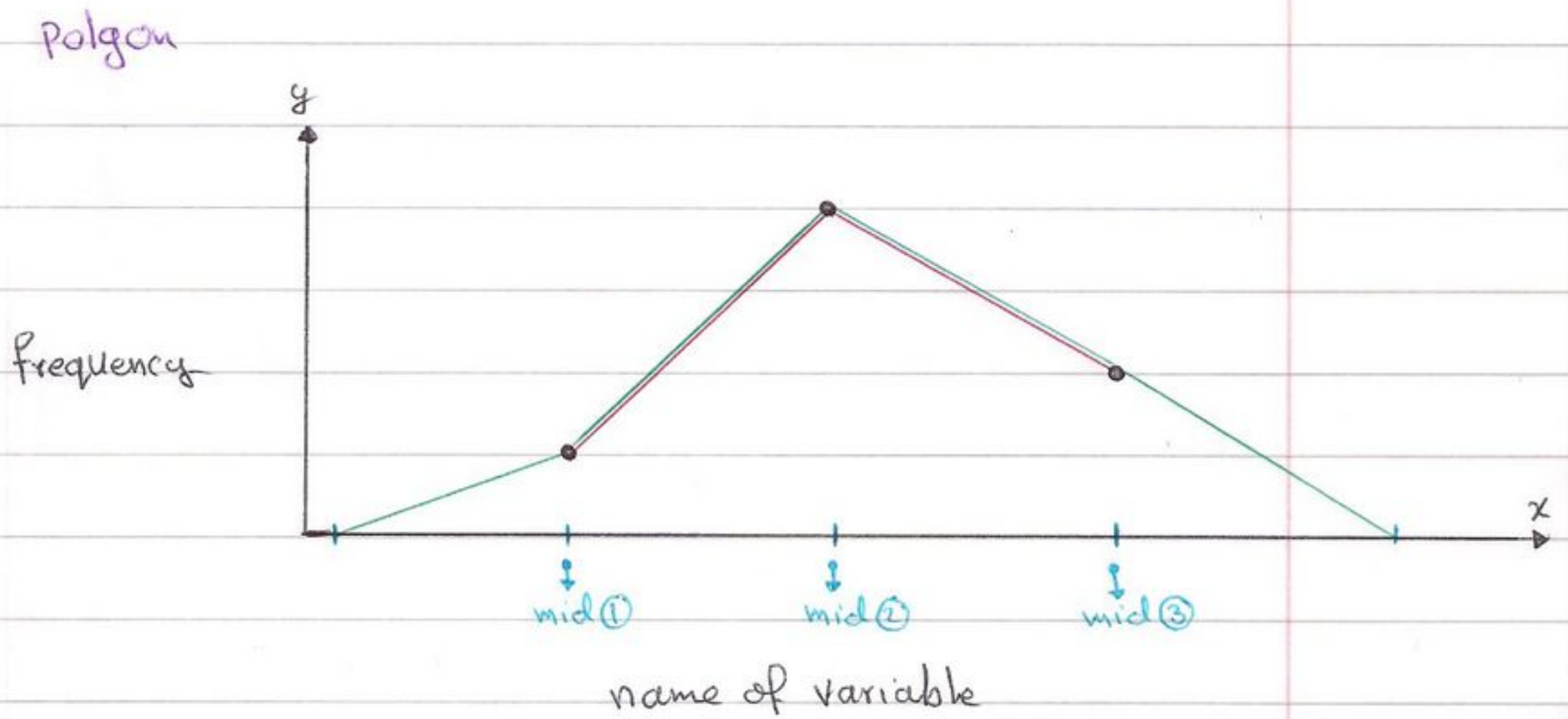
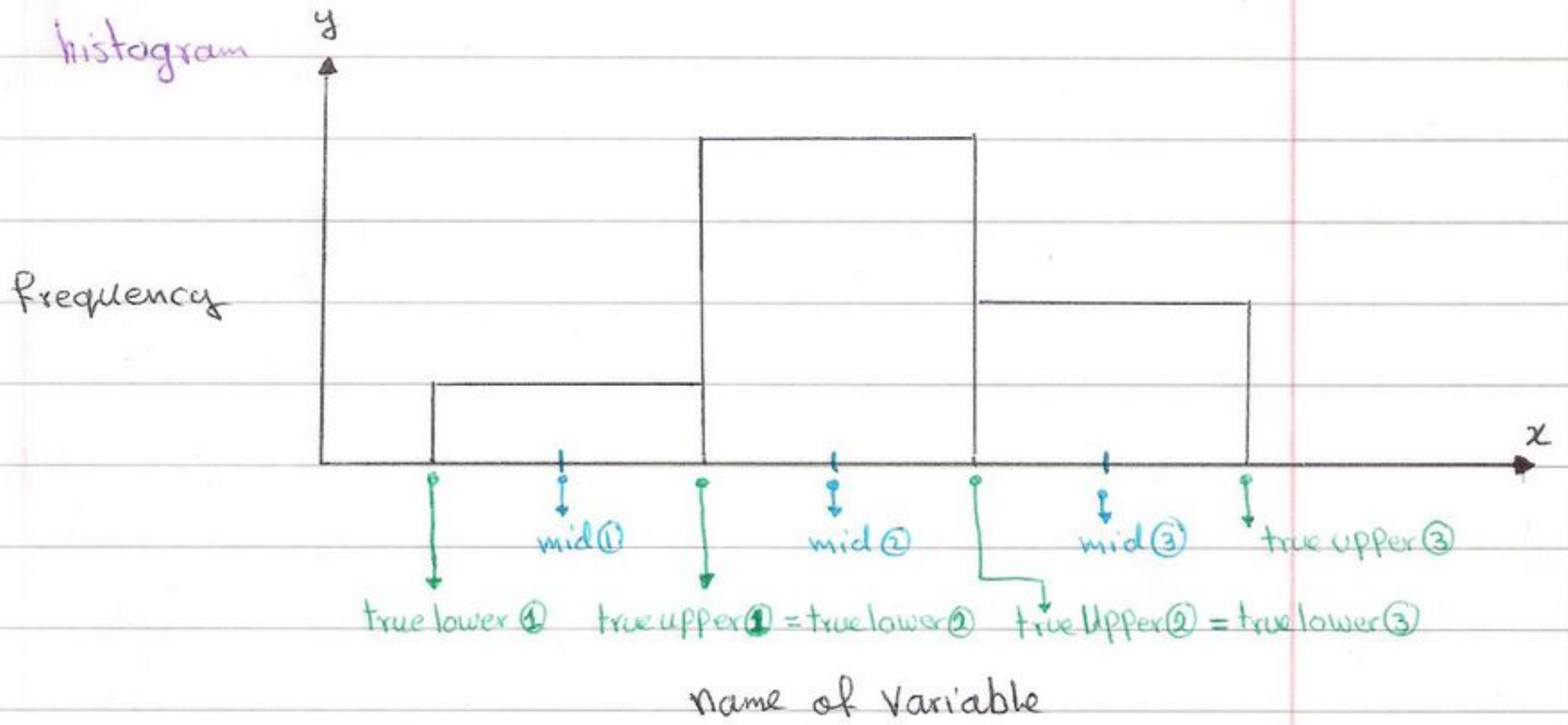
* Cumulative Percentage Frequency :

Cum. per. Freq. of 1st class interval = per. Freq.

Cum. per. Freq. of any class interval = per. Freq. + Cum. per. Freq. of preceding class interval

جزء الثاني

* displaying the frequency distribution:



≡ polygon closed

≡ polygon open

"Descriptive Statistics"

measures of

Central Tendency (Location)

- * mean (unit)
- * median (unit)
- * mode (unit)

Dispersion (Variation)

- * range (unit)
- * Variance = (standard deviation)² (unit)²
- * standard deviation = $\sqrt{\text{variance}}$ (unit)
- * Coefficient of variation (unit-less)
C.V.

population

* X_1, X_2, \dots, X_N
* any measure here
it called "parameter"

sample

* X_1, X_2, \dots, X_n
* any measure here
it called "statistic"

	Population	sample
mean	$M = \frac{\sum_{i=1}^N X_i}{N}$	$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
Variance	$\sigma^2 = \frac{\sum_{i=1}^N (X_i - M)^2}{N}; \sigma^2 \geq 0$	$S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}; S^2 \geq 0$
Standard deviation	$\sigma = \sqrt{\sigma^2}; \sigma \geq 0$	$S = \sqrt{S^2}; S \geq 0$
size	N	n

"Probability"

- * **Experiment**: some procedure or process that we do.
- * **Sample space (Ω)**: set of all possible outcomes of experiment; where $n(\Omega)$ is the number of outcomes (elements) in Ω .
- * **Event (E)**: any subset of Ω ; where $n(E)$ is the number of outcomes in E .
 - $E \subseteq \Omega$
 - $\emptyset \subseteq \Omega$ (impossible event)
 - $\Omega \subseteq \Omega$ (sure event)
- * **Equally likely outcomes of experiment**: if the outcomes have the same chance of occurrence.
- * **Probability**: measure used to measure the chance of occurrence of event; which is between 0 and 1.
- * **Probability of event E** : $0 \leq P(E) = \frac{n(E)}{n(\Omega)} \leq 1$
 - $P(\emptyset) = 0$
 - $P(\Omega) = 1$

* Operations on events:

① Union:

- $A \cup B \cong A \text{ or } B \cong A + B$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B) \Leftrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $A \cup B = \Omega$ or $P(A \cup B) = 1 \Leftrightarrow A$ and B are exhaustive
- $A \cup A^c = \Omega$ (exhaustive)
- $P(A \cup B) = \frac{n(A \cup B)}{n(\Omega)}$

② intersection:

- $A \cap B \cong A \text{ and } B$
- $P(A \cap B) = \frac{n(A \cap B)}{n(\Omega)}$
- $A \cap B = \emptyset$ or $P(A \cap B) = 0 \Leftrightarrow A \text{ and } B \text{ are disjoint (mutually exclusive)}$
- $A \cap A' = \emptyset$ (disjoint or mutually exclusive)

③ Complement:

- $\bar{A} \cong A^c \cong A' \cong \text{not } A$
- $n(A) + n(A') = n(\Omega) \Leftrightarrow P(A) + P(A') = 1$

* Marginal probability:

① table 2x2:

	B	B ^c	
A	$n(A \cap B)$	$n(A \cap B^c)$	$= n(A)$
	+	+	+
A ^c	$n(A^c \cap B)$	$n(A^c \cap B^c)$	$= n(A^c)$
	$n(B)$	$n(B^c)$	$= n(\Omega)$

⇔

	B	B ^c	
A	$P(A \cap B)$	$P(A \cap B^c)$	$= P(A)$
	+	+	+
A ^c	$P(A^c \cap B)$	$P(A^c \cap B^c)$	$= P(A^c)$
	$P(B)$	$P(B^c)$	$= 1$

الدمج عامودياً أو عرضياً للتقاطع يساوي الأجزاء عامودياً أو عرضياً على التوالي...

② tables of 2×3 ; 3×3 ; 2×4 ; 3×4 , 4×4 , ... : for example if we take

	B_1	B_2	B_3	
A_1	$n(A_1 \cap B_1)$	$+ n(A_1 \cap B_2)$	$+ n(A_1 \cap B_3)$	$= n(A_1)$
	+	+	+	+
A_2	$n(A_2 \cap B_1)$	$+ n(A_2 \cap B_2)$	$+ n(A_2 \cap B_3)$	$= n(A_2)$
	+	+	+	+
A_3	$n(A_3 \cap B_1)$	$+ n(A_3 \cap B_2)$	$+ n(A_3 \cap B_3)$	$= n(A_3)$
	$n(B_1)$	$+ n(B_2)$	$+ n(B_3)$	$= n(\mathcal{U})$

	B_1	B_2	B_3	
A_1	$p(A_1 \cap B_1)$	$+ p(A_1 \cap B_2)$	$+ p(A_1 \cap B_3)$	$= p(A_1)$
	+	+	+	+
A_2	$p(A_2 \cap B_1)$	$+ p(A_2 \cap B_2)$	$+ p(A_2 \cap B_3)$	$= p(A_2)$
	+	+	+	+
A_3	$p(A_3 \cap B_1)$	$+ p(A_3 \cap B_2)$	$+ p(A_3 \cap B_3)$	$= p(A_3)$
	$p(B_1)$	$+ p(B_2)$	$+ p(B_3)$	$= 1$

الجمع عامودياً أو عرضياً للتقاطع يساوي الأجزاء عامودياً أو عرضياً على التوالي

• $A_1 \cap A_2 = A_1 \cap A_3 = A_2 \cap A_3 = A_1 \cap A_2 \cap A_3 = \emptyset$;

$B_1 \cap B_2 = B_1 \cap B_3 = B_2 \cap B_3 = B_1 \cap B_2 \cap B_3 = \emptyset$.

• $A_1' = A_2 \cup A_3$; $A_2' = A_1 \cup A_3$; $A_3' = A_1 \cup A_2$; $B_1' = B_2 \cup B_3$; $B_2' = B_1 \cup B_3$; $B_3' = B_1 \cup B_2$.

• $A_1' \cap B_1 = (A_2 \cup A_3) \cap B_1 = (A_2 \cap B_1) \cup (A_3 \cap B_1) \Leftrightarrow p(A_1' \cap B_1) = p(A_2 \cap B_1) + p(A_3 \cap B_1)$

$A_1 \cap B_1' = A_1 \cap (B_2 \cup B_3) = (A_1 \cap B_2) \cup (A_1 \cap B_3) \Leftrightarrow p(A_1 \cap B_1') = p(A_1 \cap B_2) + p(A_1 \cap B_3)$

وعلى هذا يقاس الأمر لأي حادثتين

* conditional probability:

• $A|B \cong A$ given $B \cong A$ knowing B

• $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)}$$

• A and B independent $\Leftrightarrow P(A \cap B) = P(A)P(B)$

$$\Leftrightarrow P(A|B) = P(A)$$

$$\Leftrightarrow P(B|A) = P(B)$$