Some Parameters & Their Statistics:

Parameter	Measure	**Statistic** or **the point estimator of a parameter**	Measure
μ	Mean of a single population	$\bar{X}$	Mean of a single sample
σ^2	Variance of a single population	s^2	Variance of a single sample
σ	Standard deviation of a single population	s	Standard deviation of a single sample
$p = \frac{D}{N}$ where D: the number of related units in population, N: population size.	Proportion of a single population	$\hat{p} = \frac{d}{n}$ where d: the number of related units in sample, n: sample size.	Proportion of a single sample
$\mu_1 - \mu_2$	Difference in means of two populations	$\bar{X}_1 - \bar{X}_2$	Difference in means of two samples
$p_1 - p_2$	Difference in proportions of two populations	$\hat{p}_1 - \hat{p}_2$	Difference in proportions of two samples
$\frac{\sigma_1^2}{\sigma_2^2}$	ratio of two variances of two populations	$\frac{s_1^2}{s_2^2}$	ratio of two variances of two samples
$\frac{\sigma_1}{\sigma_2}$	ratio of two Standard deviations of two populations	$\frac{s_1}{s_2}$	ratio of two Standard deviations of two samples

	$X_1, X_2, \ldots, X_n$ are independent random variables having **normal distributions** with means μ and **known variances $\boldsymbol{\sigma^2}$**	$X_1, X_2, \ldots, X_n$ are independent random variables having **normal distributions** with means μ and **unknown variances $\boldsymbol{\sigma^2}$**		$X_1, X_2, \ldots, X_n$ are independent random variables having **non-normal distributions** with means μ and **variances $\boldsymbol{\sigma^2}$**	
				unknown variances $\boldsymbol{\sigma^2}$	**known variances $\boldsymbol{\sigma^2}$**
		$n < 30$	$n \geq 30$ $n \to \infty$	$n \geq 30$ $n \to \infty$	
Sampling Distribution* of statistic $\bar{X}$	$\bar{X} \sim N\left(\mu_{\bar{X}} = \mu, \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\right)$ $\Rightarrow \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ where $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ is standard deviation of $\bar{X}$ or standard error of $\bar{X}$.	$\frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \sim t_{(n-1)}$ where $\mu_{\bar{X}} = \mu,$ $\hat{\sigma}_{\bar{X}}^2 = \frac{s^2}{n}$ estimated variance of $\bar{X}$, $\hat{\sigma}_{\bar{X}} = \frac{s}{\sqrt{n}}$ is estimated standard deviation of $\bar{X}$ or estimated standard error of $\bar{X}$, $s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{(n-1)}$ is the variances of random sample.	$\frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \sim N(0,1)$ where $\mu_{\bar{X}} = \mu,$ $\hat{\sigma}_{\bar{X}}^2 = \frac{s^2}{n}$ estimated variance of $\bar{X}$, $\hat{\sigma}_{\bar{X}} = \frac{s}{\sqrt{n}}$ is estimated standard deviation of $\bar{X}$ or estimated standard error of $\bar{X}$, $s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{(n-1)}$, is the variances of random sample.		$\bar{X} \approx N\left(\mu_{\bar{X}} = \mu, \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\right)$ $\Rightarrow \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \approx N(0,1)$ where $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ is standard deviation of $\bar{X}$ or standard error of $\bar{X}$.
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for μ	$\mu \in \bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ also, $e = Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $n = \left(Z_{1-\frac{\alpha}{2}} \frac{\sigma}{e}\right)^2$.	$\mu \in \bar{X} \pm t_{v,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ where $v = n - 1$.		$\mu \in \bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$	$\mu \in \bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

*The probability distribution of a statistic is called a sampling distribution.

	If two independent samples of size n_1 and n_2 are drawn at random from two **normal populations** with means μ_1 and μ_2 and **known variances σ_1^2 and σ_2^2** ,respectively	If two independent samples of size n_1 and n_2 are drawn at random from two **normal populations** with means μ_1 and μ_2 and **unknown variances σ_1^2 and σ_2^2 but equal**, respectively		If two independent samples of size n_1 and n_2 are drawn at random from two **non-normal populations** with means μ_1 and μ_2 and **variances σ_1^2 and σ_2^2** ,respectively	
				unknown variances σ_1^2 and σ_2^2	**known variances σ_1^2 and σ_2^2**
		n_1 and $n_2 < 30$	n_1 and $n_2 \geq 30$ n_1 and $n_2 \to \infty$	n_1 and $n_2 \geq 30$ n_1 and $n_2 \to \infty$	
Sampling Distribution* of statistic $\bar{X}_1 - \bar{X}_2$	$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2, \sigma^2_{\bar{X}_1-\bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$ $\Rightarrow \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ where $\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$	$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} \sim t_{(n_1+n_2-2)}$ where $\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2,$ $\hat{\sigma}^2_{\bar{X}_1-\bar{X}_2} = \frac{s_p^2}{n_1} + \frac{s_p^2}{n_2} = s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right),$ $\hat{\sigma}_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ $= \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ $= s_p\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)},$ $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$ is pooled estimate of the common variance , s_1^2 and s_2^2 are the variances of independent random samples.			$\bar{X}_1 - \bar{X}_2 \approx N\left(\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2, \sigma^2_{\bar{X}_1-\bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$ $\Rightarrow \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \approx N(0,1)$ where $\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for $\mu_1 - \mu_2$	$\mu_1 - \mu_2$ $\in (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$\mu_1 - \mu_2$ $\in (\bar{X}_1 - \bar{X}_2) \pm t_{v,\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ where $v = n_1 + n_2 - 2.$		$\mu_1 - \mu_2$ $\in (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$\mu_1 - \mu_2$ $\in (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

	Paired Observations: If two **related (non-independent)** samples of size n_1 and n_2 (where $n_1 = n_2 = n$)are drawn at random from two **normal populations** with means μ_1 and μ_2 and **unknown variances σ_1^2 and σ_2^2** , respectively
	n_1 and $n_2 < 30$
Sampling Distribution* of statistic	
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for $\mu_D = \mu_1 - \mu_2$	$\mu_D \in \overline{D} \pm t_{v,\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}$ where $v = n - 1.$

*The probability distribution of a statistic is called a sampling distribution.

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- 1−st population: $X_1, X_2, X_3, \ldots, X_n$ and with mean μ_1.
- 2−st population: $Y_1, Y_2, Y_3, \ldots, Y_n$ and with mean μ_2.

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We define the followings quantities:

- The differences (D-observations)

$$D_i = X_i - Y_i, i = 1, 2 \ldots, n$$

- Sample mean of the D-observations (differences)

$$\overline{D} = \frac{\sum_{i=1}^{n} D_i}{n} = \frac{D_1 + D_2 + \cdots + D_n}{n}$$

- Sample variance of the D-observations (differences)

$$S_D^2 = \frac{\sum_{i=1}^{n} (D_i - \overline{D})^2}{n-1}$$

- Sample standard deviation of the D-observations

$$S_D = \sqrt{S_D^2}$$

	$X_1, X_2, \ldots, X_n$ are independent random variables having **normal distributions** with and **known variances $\boldsymbol{\sigma^2}$**
Sampling Distribution* of statistic s^2	$$\frac{(n-1)s^2}{\sigma^2} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi^2_{(n-1)}$$ where $s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{(n-1)}$
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for σ^2	$$\frac{(n-1)s^2}{\chi^2_{v,\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{v,1-\frac{\alpha}{2}}}$$ where $v = n-1.$
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for σ	$$\sqrt{\frac{(n-1)s^2}{\chi^2_{v,\frac{\alpha}{2}}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{v,1-\frac{\alpha}{2}}}}$$ where $v = n-1.$

*The probability distribution of a statistic is called a sampling distribution.

	If two independent samples of size n_1 and n_2 are drawn at random from two **normal populations** with **known variances σ_1^2 and σ_2^2**
Sampling Distribution* of statistic $\frac{s_1^2}{s_2^2}$	$$\frac{\frac{s_1^2}{\sigma_1^2}}{\frac{s_2^2}{\sigma_2^2}} = \frac{s_1^2}{s_2^2}\frac{\sigma_2^2}{\sigma_1^2} \sim F_{(n_1-1),(n_2-1)}$$ where s_1^2 and s_2^2 are the variances of independent random samples.
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for $\frac{\sigma_1^2}{\sigma_2^2}$	$$\frac{s_1^2}{s_2^2}\frac{1}{F_{v_1,v_2,\frac{\alpha}{2}}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}F_{v_2,v_1,\frac{\alpha}{2}}$$ where $v_1 = n_1 - 1$, $v_2 = n_2 - 1$.
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for $\frac{\sigma_1}{\sigma_2}$	$$\sqrt{\frac{s_1^2}{s_2^2}\frac{1}{F_{v_1,v_2,\frac{\alpha}{2}}}} < \frac{\sigma_1}{\sigma_2} < \sqrt{\frac{s_1^2}{s_2^2}F_{v_2,v_1,\frac{\alpha}{2}}}$$ where $v_1 = n_1 - 1$, $v_2 = n_2 - 1$.

*The probability distribution of a statistic is called a sampling distribution.

	$X_1, X_2, \ldots, X_n$ are independent random variables having distributions.
	$n \geq 30$ $n \to \infty$ or $np \geq 5$ and $n(1-p) \geq 5$
Sampling Distribution* of statistic $\hat{p}$	$\hat{p} \approx N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}}^2 = \frac{p(1-p)}{n}\right)$ $\Rightarrow \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \approx N(0,1)$ where $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ is standard deviation of $\hat{p}$ or standard error of $\hat{p}$.
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for p	$p \in \hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ also, $e = Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, $n = \left(Z_{1-\frac{\alpha}{2}} \frac{1}{e}\right)^2 \hat{p}(1-\hat{p})$, $n = \left(Z_{1-\frac{\alpha}{2}} \frac{1}{e}\right)^2 \frac{1}{4}$.

*The probability distribution of a statistic is called a sampling distribution.

	If two independent samples of size n_1 and n_2 are drawn at random from two populations
	$n_1 \ and \ n_2 \geq 30$ $n_1 \ and \ n_2 \to \infty$ or $n_1 p_1 \geq 5$, $n_1(1-p_1) \geq 5$,$n_2 p_2 \geq 5$, and $n_2(1-p_2) \geq 5$
Sampling Distribution* of statistic $\hat{p}_1 - \hat{p}_2$	$\hat{p}_1 - \hat{p}_2 \approx N\left(\mu_{\hat{p}_1-\hat{p}_2} = p, \sigma^2_{\hat{p}_1-\hat{p}_2} = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$ $\Rightarrow \frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \approx N(0,1)$ where $\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ is standard deviation of $\hat{p}_1 - \hat{p}_2$ or standard error of $\hat{p}_1 - \hat{p}_2$.
$100(1-\alpha)\%$ **confidence interval** (or **Interval Estimation**) for $p_1 - p_2$	$p_1 - p_2 \in (\hat{p}_1 - \hat{p}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

*The probability distribution of a statistic is called a sampling distribution.