Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $y^2 - 2y + 4x = 3$, then sketch it. [4]
2. Find the standard equation of the ellipse with vertices at $(-4, 2)$, $(6, 2)$ and one of its two focci at $(5, 2)$, then sketch it. [4]
3. Calculate, whenever it is possible, the products $2AB$ and BA of matrices [4]

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} x - 2y + z &= 4 \\ -x + 2y + z &= -2 \\ 2x - 3y - z &= 3 \end{cases}$$

(a) Solve this system by using Cramer' rule. [4]

(b) Solve this system by using Gauss elimination method. [4]

5. Evaluate the integrals

(a) $\int (3x-1)\sqrt{3x^2 - 2x + 1}\, dx.$ [2]

(b) $\int (5x+4)^5\, dx.$ [2]

(c) $\int x^3 \ln x\, dx.$ [3]

(d) $\int \frac{3\cos(3x) + 2\sin(2x)}{\sin(3x) - \cos(2x)}\, dx.$ [3]

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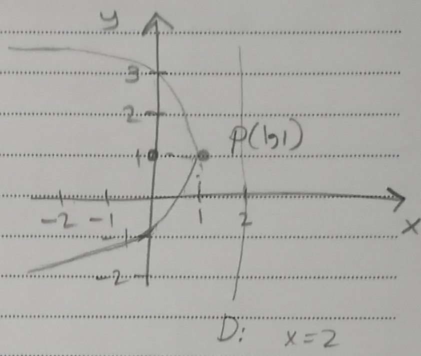
1

$$y^2 - 2y = -4x + 3$$

$$y^2 - 2y + 1 = -4x + 3 + 1$$

$$(y-1)^2 = -4(x-1)$$



Parabola

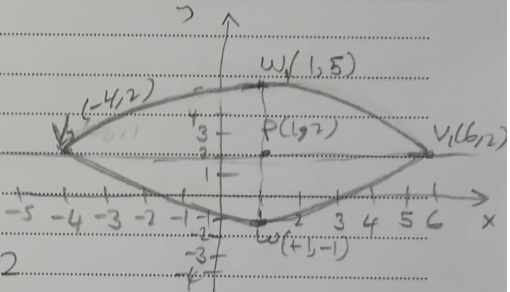
$h = 1,\ k = 1,\ a = 1$

$F(h-a, k) = F(0, 1)$

$D:\ x = h + a \Rightarrow x = 2$

2

$V(h \pm a, k)$



$h + a = 6$

$h - a = -4$

$2h = 2 \Rightarrow h = 1,\ k = 2$

$\Rightarrow a = 5$

$F_1(h+c, k) = F_1(5, 2)$

$h + c = 5 \Rightarrow 1 + c = 5$

$c = 4$

$c^2 = a^2 - b^2 \Rightarrow 16 = 25 - b^2$

$\Rightarrow b^2 = 9 \Rightarrow b = 3$

$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$$

3

$$2AB = \begin{bmatrix} 6 & -2 \\ 2 & 0 \end{bmatrix} \quad BA = \begin{bmatrix} 0 & -2 & 1 \\ -1 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix}$$

$\boxed{4}$ (a) $A=\begin{bmatrix} 1 & -2 & 1 \\ -1 & 2 & 1 \\ 2 & -3 & -1 \end{bmatrix} \Rightarrow \det(A) = -2$

$A_1=\begin{bmatrix} 4 & -2 & 1 \\ -2 & 2 & 1 \\ 3 & -3 & -1 \end{bmatrix} \Rightarrow \det(A_1) = 2$

$A_2=\begin{bmatrix} 1 & 4 & 1 \\ -1 & -2 & 1 \\ 2 & 3 & -1 \end{bmatrix} \Rightarrow \det(A_2) = 4$

$A_3=\begin{bmatrix} 1 & -2 & 4 \\ -1 & 2 & -2 \\ 2 & -3 & 3 \end{bmatrix} \Rightarrow \det(A_3) = -2$

$x = \frac{2}{-2} = -1 \quad , \quad y = \frac{4}{-2} = -2 \; , \; z = \frac{-2}{-2} = 1$

$X = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

(b) $[A|B] = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ -1 & 2 & 1 & -2 \\ 2 & -3 & -1 & 3 \end{array}\right] \xrightarrow[-2R_1+R_3]{1R_1+R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 1 & -3 & -5 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 2 & 2 \end{array}\right]$

$\xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$z = 1$

$y - 3(1) = -5$

$\Rightarrow y = -2$

$x - 2(-2) + 1(1) = 4$

$\Rightarrow x = -1$

$X = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

$\boxed{5}$ (a) $\frac{1}{2}\int 2(3x-1)\,(3x^2-2x+1)^{\frac{1}{2}}\,dx = \frac{1}{\cancel{2}}\,\frac{(3x^2-2x+1)^{\frac{3}{2}}}{\frac{3}{\cancel{2}}}+c$

(b) $\frac{1}{5}\int 5(5x+4)^5\,dx$

$= \frac{1}{5}\,\frac{(5x+4)^6}{6}+c$

$= \frac{(5x+4)^6}{30}+c$

OR

$u=5x+4 \Rightarrow du=5dx$

$\Rightarrow \frac{du}{5}=dx$

$\int u^5\,\frac{du}{5} = \frac{1}{5}\,\frac{u^6}{6}+c$

$= \frac{(5x+4)^6}{30}+c$

(c) $I=\int x^3 \ln x\,dx$

$u=\ln x$ $\Rightarrow$ $du=\frac{1}{x}dx$

$dv=x^3$ $\quad v=\int x^3\,dx = \frac{x^4}{4}$

$I = \frac{x^4}{4}\ln x - \int \frac{x^{\cancel{4}\,3}}{4}\,\frac{1}{\cancel{x}}\,dx$

$= \frac{x^4}{4}\ln x - \frac{1}{4}\,\frac{x^4}{4}+c$

$= \frac{x^4}{4}\ln x - \frac{x^4}{16}+c$

(d) $\int \frac{3\cos(3x)+2\sin(2x)}{\sin(3x)-\cos(2x)}\,dx = \ln\left|\sin(3x)-\cos(2x)\right|+c$