"إعادة صياغة حلول الباب السادس"

إيجاد قيمة α من "$(1-\alpha)100\%$ confidence level"

① مثال في صفحة 105(A)

يعتمد على ماذكر فيه من طريقة للحل في ملزمة

إيجاد $Z_{1-\frac{\alpha}{2}}$ و $t_{1-\frac{\alpha}{2}}$ بدلالة α

① مثال في صفحة 109

$Z \sim N(\mu=0, \sigma^2=1)$

① $\alpha = .1 \Rightarrow 1-\frac{\alpha}{2} = .95 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.95} = \frac{1.64+1.65}{2} = 1.645$

$$\begin{array}{cccc} & .04 & & .05 \\ & \uparrow & & \updownarrow \\ 1.6 \leftarrow & .94950 & \boxed{.95000} & .95053 \end{array}$$

② $\alpha = .05 \Rightarrow 1-\frac{\alpha}{2} = .975 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$

$$\begin{array}{cc} & .06 \\ & \updownarrow \\ 1.9 \leftarrow & \boxed{.97500} \end{array}$$

③ $\alpha = .01 \Rightarrow 1-\frac{\alpha}{2} = .995 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.995} = \frac{2.57+2.58}{2} = 2.575$

$$\begin{array}{cccc} & .07 & & .08 \\ & \updownarrow & & \uparrow \\ 2.5 \leftarrow & .99492 & \boxed{.99500} & .99506 \end{array}$$

$T \sim t(v = df = 30)$

② مثال في صفحة 109

$\alpha = .05 \Rightarrow 1-\frac{\alpha}{2} = .975 \Rightarrow t_{1-\frac{\alpha}{2}} = t_{.975} = 2.0423$

$$\begin{array}{cc} & t_{.975} \\ & \updownarrow \\ V=30 \rightarrow & 2.0423 \end{array}$$

إيجاد التقدير بنقطة والتقدير بفترة لمتوسط المجتمع μ

① مثال في صفحة ١١٥

Population: normal distribution, $\sigma = 3.3$

Sample: $n = 123$, $\bar{x} = 26.2$

① point estimate for the population mean $= \hat{\mu} = \bar{x} = 26.2$

② 90% confidence interval for μ ?! $\rightarrow \alpha = \frac{10}{100} = .1$

as we have a normal distribution + σ is known

so,

$$\mu \in \bar{x} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \Rightarrow \mu \in 26.2 \pm Z_{.95} \frac{3.3}{\sqrt{123}} \quad (\text{where } Z_{1-\frac{.1}{2}} = Z_{.95} = 1.645)$$

$$\Rightarrow \mu \in 26.2 \pm (1.645) \frac{3.3}{\sqrt{123}}$$

$$\Rightarrow \mu \in \left(L = 26.2 - (1.645)\frac{3.3}{\sqrt{123}} , U = 26.2 + (1.645)\frac{3.3}{\sqrt{123}} \right)$$

$$\Rightarrow \mu \in (25.71053, 26.68947)$$

$$\Rightarrow 25.71053 < \mu < 26.68947$$

② مثال في صفحة ١١٢

population : normal distribution

Sample : $n = 21$, $\bar{x} = 37$, $s = 10$

① Point estimate of the mean $\mu = \hat{\mu} = \bar{x} = 37$

② 99% confidence interval for μ ?! $\rightarrow \alpha = \frac{1}{100} = .01$

as we have a normal distribution + σ is unknown + $n < 30$, then

$$\mu \in \bar{x} \pm t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \quad , df = v = n-1$$

$$\Rightarrow \mu \in 37 \pm t_{.995} \frac{10}{\sqrt{21}} \quad \left(\text{where } t_{1-\frac{.01}{2}} = t_{.995} = 2.845 \text{ at } v = 21-1 = 20 \right)$$

$$\Rightarrow \mu \in 37 \pm (2.845) \frac{10}{\sqrt{21}}$$

$$\Rightarrow \mu \in \left(L = 37 - (2.845) \frac{10}{\sqrt{21}} , U = 37 + (2.845) \frac{10}{\sqrt{21}} \right)$$

$$\Rightarrow \mu \in (30.79170, 43.20830)$$

$$\Rightarrow 30.79170 < \mu < 43.20830$$

إيجاد التقدير بنقطة والتقدير بفترة لحاصل فرق متوسطي مجتمعين $\mu_1-\mu_2$

① مثال في صفحة 116

	A (1)	B (2)
Population:	$\sigma_1=8$	$\sigma_2=6$
sample:	$n_1=75$	$n_2=50$
	$\bar{x}_1=42$	$\bar{x}_2=36$

① point estimate for $\mu_1-\mu_2 = (\widehat{\mu_1-\mu_2}) = \bar{x}_1-\bar{x}_2 = 42-36=6$

② 96% Confidence interval for $\mu_1-\mu_2$?! $\longrightarrow \alpha = \frac{4}{100} = .04$

as σ_1 and σ_2 are known, then

$$\mu_1-\mu_2 \in (\bar{x}_1-\bar{x}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$\Rightarrow \mu_1-\mu_2 \in 6 \pm Z_{.98} \sqrt{\frac{8^2}{75}+\frac{6^2}{50}} \quad (\text{where } Z_{1-\frac{.04}{2}} = Z_{.98} = 2.055)$$

$$\Rightarrow \mu_1-\mu_2 \in 6 \pm (2.055) \sqrt{\frac{8^2}{75}+\frac{6^2}{50}}$$

$$\Rightarrow \mu_1-\mu_2 \in \left(L=6-(2.055)\sqrt{\frac{8^2}{75}+\frac{6^2}{50}},\ U=6+(2.055)\sqrt{\frac{8^2}{75}+\frac{6^2}{50}}\right)$$

$$\Rightarrow \mu_1-\mu_2 \in (3.42236, 8.57764)$$

$$\Rightarrow 3.42236 < \mu_1-\mu_2 < 8.57764$$

Since does not include 0, so $\mu_1-\mu_2 \neq 0 \cong \mu_1 \neq \mu_2$

i.e. the population means are not the same.

② مثال في صفحة ١١٧

	A (1)	B (2)
population :	normal distribution, $\sigma_1 = \sigma = ?$	normal distribution, $\sigma_2 = \sigma = ?$
sample :	140, 138, 143, 142, 144, 137	135, 140, 136, 142, 138, 140
	$n_1 = 6$ (من الآلة الحاسبة)	$n_2 = 6$ (من الآلة الحاسبة)
	$\bar{x}_1 = 140.67$, $S_1^2 = 7.87$	$\bar{x}_2 = 138.5$, $S_2^2 = 7.1$

① point estimate for $\mu_1 - \mu_2 = (\widehat{\mu_1 - \mu_2}) = \bar{x}_1 - \bar{x}_2 = 2.17$

② 95% Confidence interval for $\mu_1 - \mu_2$?! $\rightarrow \alpha = \frac{5}{100} = .05$

as we have normal distributions + $\sigma_1 = \sigma_2 = \sigma$ but unknown + $n_1 < 30$ and $n_2 < 30$, then

$$\mu_1 - \mu_2 \in (\bar{x}_1 - \bar{x}_2) \pm t_{1-\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad , V = n_1 + n_2 - 2$$

$$, S_p = \sqrt{\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}}$$

$$\Rightarrow \mu_1 - \mu_2 \in 2.17 \pm t_{.975} \sqrt{\frac{5(7.87) + 5(7.1)}{10}} \sqrt{\frac{1}{6} + \frac{1}{6}}$$

$$\left(\text{where } t_{1-\frac{.05}{2}} = t_{.975} = 2.228 \text{ at } V = 6 + 6 - 2 = 10 \right)$$

$$\Rightarrow \mu_1 - \mu_2 \in 2.17 \pm 2.228 \sqrt{\frac{5(7.87) + 5(7.1)}{10}} \sqrt{\frac{1}{6} + \frac{1}{6}}$$

$$\Rightarrow \mu_1 - \mu_2 \in (L = -1.34925, U = 5.68925)$$

$$\Rightarrow -1.34925 < \mu_1 - \mu_2 < 5.68925$$

Since include 0, so $\mu_1 - \mu_2 = 0 \cong \mu_1 = \mu_2$

i.e. the population means are the same.

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إيجاد التقدير بنقطة والتقدير بفترة لنسبة المجتمع P

(i) مثال في صفحة 120

Population: —

Sample: $n = 950$ women where we have 611 women who are obese

$\therefore \hat{p} = \frac{611}{950}$ is the proportion of women who are obese

(1) point estimate for $p = \hat{p} = \frac{611}{950}$, p is the proportion of women are obese in population

(2) 95% confidence interval for p ?! $\rightarrow \alpha = \frac{5}{100} = .05$

as we have that $n \geq 30$, then

$$p \in \hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Rightarrow p \in \frac{611}{950} \pm Z_{.975} \sqrt{\frac{\frac{611}{950}\left(1-\frac{611}{950}\right)}{950}} \quad \left(\text{where } Z_{1-\frac{.05}{2}} = Z_{.975} = 1.96\right)$$

$$\Rightarrow p \in \frac{611}{950} \pm (1.96) \sqrt{\frac{\frac{611}{950}\left(1-\frac{611}{950}\right)}{950}}$$

$$\Rightarrow p \in (L = .61269, U = .67362)$$

$$\Rightarrow .61269 < p < .67362$$

إيجاد التقدير بنقطة والتقدير بفترة لحاصل الفرق بين نسبتي المجتمع $P_1 - P_2$

① مثال في صفحة 123

	A (1)	B (2)
Population:	—	—
Sample:	$n_1 = 1500$ people where 75 people who are have a cancer disease $\therefore \hat{P}_1 = \frac{75}{1500} = .05$ is the proportion of people who have a cancer disease	$n_2 = 2000$ people where 80 people who are have a cancer disease $\therefore \hat{P}_2 = \frac{80}{2000} = .04$ is the proportion of people who have a cancer disease

① point estimate for $P_1 - P_2 = (\widehat{P_1 - P_2}) = \hat{P}_1 - \hat{P}_2 = \frac{75}{1500} - \frac{80}{2000} = \frac{1}{100} = .01$

is the proportion of people having cancer disease in two population

(b) 90% confidence interval for $P_1 - P_2$?! $\rightarrow \alpha = \frac{10}{100} = .1$

as we have $n_1 \geq 30$ and $n_2 \geq 30$, then

$$P_1 - P_2 \in (\hat{P}_1 - \hat{P}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}$$

$$\Rightarrow P_1 - P_2 \in .01 \pm Z_{.95} \sqrt{\frac{.05(1-.05)}{1500} + \frac{.04(1-.04)}{2000}} \quad (\text{where } Z_{1-\frac{.1}{2}} = Z_{.95} = 1.645)$$

$$\Rightarrow P_1 - P_2 \in (L = -.00173 , U = .02173)$$

$$\Rightarrow -.00173 < P_1 - P_2 < .02173$$

Since include 0, so $P_1 - P_2 = 0 \cong P_1 = P_2$

i.e the population proportions are the same.