

إعادة صياغة حلول الباب السادس

إيجاد قيمة α أو $(1-\alpha)100\%$ Confidence level

مثال في صفحة 105 (A)

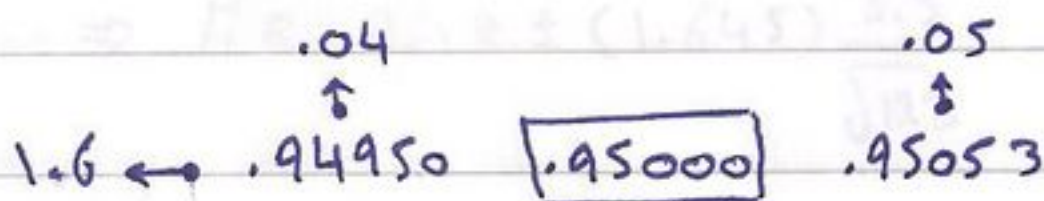
يعتمد على ما ذكر فيه من طريقة للحل في ملزمة

الجواب Z و t بدلالة α

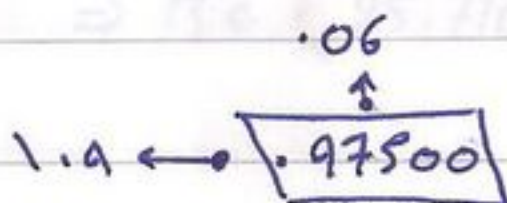
$Z \sim N(\mu=0, \sigma^2=1)$

مثال في صفحة 109

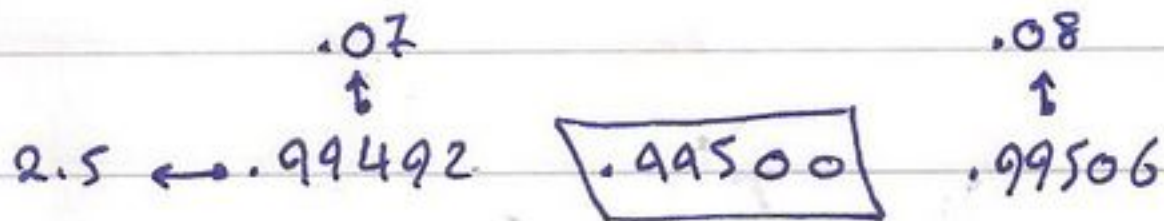
① $\alpha = .1 \Rightarrow 1 - \frac{\alpha}{2} = .95 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.95} = \frac{1.64 + 1.65}{2} = 1.645$



② $\alpha = .05 \Rightarrow 1 - \frac{\alpha}{2} = .975 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$



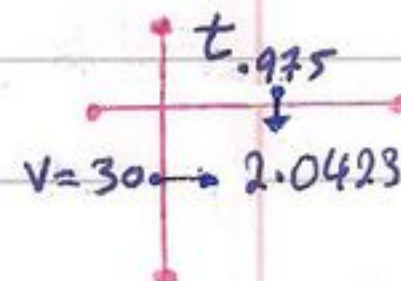
③ $\alpha = .01 \Rightarrow 1 - \frac{\alpha}{2} = .995 \Rightarrow Z_{1-\frac{\alpha}{2}} = Z_{.995} = \frac{2.57 + 2.58}{2} = 2.575$



$T \sim t(v=df=30)$

مثال في صفحة 109 (B)

$\alpha = .05 \Rightarrow 1 - \frac{\alpha}{2} = .975 \Rightarrow t_{1-\frac{\alpha}{2}} = t_{.975} = 2.0423$



①

الإحصاء، التقدير بنقطة، التقدير بفترة موثوق بها، التوزيع الطبيعي

① مثال في صيغة 110

Population: normal distribution, $\sigma = 3.3$

sample: $n = 123$, $\bar{x} = 26.2$

① point estimate for the population mean = $\hat{\mu} = \bar{x} = 26.2$

② 90% confidence interval for μ ?! $\rightarrow \alpha = \frac{10}{100} = .1$

as we have a normal distribution + σ 's known

so,

$$\mu \in \bar{x} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \Rightarrow \mu \in 26.2 \pm Z_{.95} \frac{3.3}{\sqrt{123}} \quad (\text{where } Z_{1-\frac{.1}{2}} = Z_{.95} = 1.645)$$

$$\Rightarrow \mu \in 26.2 \pm (1.645) \frac{3.3}{\sqrt{123}}$$

$$\Rightarrow \mu \in \left(L = 26.2 - (1.645) \frac{3.3}{\sqrt{123}}, U = 26.2 + (1.645) \frac{3.3}{\sqrt{123}} \right)$$

$$\Rightarrow \mu \in (25.71053, 26.68947)$$

$$\Rightarrow 25.71053 < \mu < 26.68947$$

②

Population: normal distribution

Sample: $n=21$, $\bar{x}=37$, $s=10$

① Point estimate of the mean $\mu = \hat{\mu} = \bar{x} = 37$

② 99% confidence interval for μ ? $\rightarrow \alpha = \frac{1}{100} = 0.01$

as we have a normal distribution & σ is unknown & $n < 30$, then

$$\mu \in \bar{x} \pm t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \text{ df} = v = n-1$$

$$\Rightarrow \mu \in 37 \pm t_{.995} \frac{10}{\sqrt{21}} \quad (\text{where } t_{1-\frac{0.01}{2}} = t_{.995} = 2.845 \text{ at } v=21-1=20)$$

$$\Rightarrow \mu \in 37 \pm (2.845) \frac{10}{\sqrt{21}}$$

$$\Rightarrow \mu \in \left(L = 37 - (2.845) \frac{10}{\sqrt{21}}, U = 37 + (2.845) \frac{10}{\sqrt{21}} \right)$$

$$\Rightarrow \mu \in (30.79170, 43.20830)$$

$$\Rightarrow 30.79170 < \mu < 43.20830$$

إيجاد التقدير بنقطة والتقدير بنقطة لحاصل فرق متوسطات $\mu_1 - \mu_2$
 مثال في الصفحة 116 ①

	A (1)	B (2)
Population:	$\sigma_1 = 8$	$\sigma_2 = 6$
sample:	$n_1 = 75$	$n_2 = 50$
	$\bar{x}_1 = 42$	$\bar{x}_2 = 36$

① point estimate for $\mu_1 - \mu_2 = (\hat{\mu}_1 - \hat{\mu}_2) = \bar{x}_1 - \bar{x}_2 = 42 - 36 = 6$

② 96% Confidence interval for $\mu_1 - \mu_2$?! $\rightarrow \alpha = \frac{4}{100} = 0.04$
 as σ_1 and σ_2 are known, then

$$\mu_1 - \mu_2 \in (\bar{x}_1 - \bar{x}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\Rightarrow \mu_1 - \mu_2 \in 6 \pm Z_{.98} \sqrt{\frac{8^2}{75} + \frac{6^2}{50}} \quad (\text{where } Z_{1-\frac{.04}{2}} = Z_{.98} = 2.055)$$

$$\Rightarrow \mu_1 - \mu_2 \in 6 \pm (2.055) \sqrt{\frac{8^2}{75} + \frac{6^2}{50}}$$

$$\Rightarrow \mu_1 - \mu_2 \in (L = 6 - (2.055) \sqrt{\frac{8^2}{75} + \frac{6^2}{50}}, U = 6 + (2.055) \sqrt{\frac{8^2}{75} + \frac{6^2}{50}})$$

$$\Rightarrow \mu_1 - \mu_2 \in (3.42236, 8.57764)$$

$$\Rightarrow 3.42236 < \mu_1 - \mu_2 < 8.57764$$

Since \leftarrow does not include 0, so $\mu_1 - \mu_2 \neq 0 \Leftrightarrow \mu_1 \neq \mu_2$
 i.e. the population means are not the same.

④

	A (1)	B (2)
Population:	normal distribution, $\sigma_1 = \sigma = ?$	normal distribution, $\sigma_2 = \sigma = ?$
Sample:	140, 138, 143, 142, 144, 137 $n_1 = 6$ $\bar{x}_1 = 140.67, S_1^2 = 7.87$	135, 140, 136, 142, 138, 140 $n_2 = 6$ $\bar{x}_2 = 138.5, S_2^2 = 7.1$

① point estimate for $\mu_1 - \mu_2 = (\hat{\mu}_1 - \hat{\mu}_2) = \bar{x}_1 - \bar{x}_2 = 2.17$

② 95% Confidence interval for $\mu_1 - \mu_2$?! $\rightarrow \alpha = \frac{5}{100} = 0.05$

as we have normal distributions + $\sigma_1 = \sigma_2 = \sigma$ but unknown
+ $n_1 < 30$ and $n_2 < 30$, then

$$\mu_1 - \mu_2 \in (\bar{x}_1 - \bar{x}_2) \pm t_{1-\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \quad v = n_1 + n_2 - 2$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$\Rightarrow \mu_1 - \mu_2 \in 2.17 \pm t_{.975} \sqrt{\frac{5(7.87) + 5(7.1)}{10} \left(\frac{1}{6} + \frac{1}{6} \right)}$$

(where $t_{1-\frac{0.05}{2}} = t_{.975} = 2.228$ at $v = 6+6-2=10$)

$$\Rightarrow \mu_1 - \mu_2 \in 2.17 \pm 2.228 \sqrt{\frac{5(7.87) + 5(7.1)}{10} \left(\frac{1}{6} + \frac{1}{6} \right)}$$

$$\Rightarrow \mu_1 - \mu_2 \in (L = -1.34925, U = 5.68925)$$

$$\Rightarrow -1.34925 < \mu_1 - \mu_2 < 5.68925$$

since include 0, so $\mu_1 - \mu_2 = 0 \Leftrightarrow \mu_1 = \mu_2$

i.e. the population means are the same.

5

إيجاد التقدير بنقطة والتقدير بفترة لمتباينة التباين P

① مثال في 120 أسئلة

Population: —

Sample: $n = 950$ women where we have 611 women who are obese

$\therefore \hat{p} = \frac{611}{950}$ is the proportion of women who are obese

① point estimate for $p = \hat{p} = \frac{611}{950}$ \rightarrow p is the proportion of women are obese in population

② 95% confidence interval for p ?! $\rightarrow \alpha = \frac{5}{100} = .05$

as we have that $n \geq 30$, then

$$p \in \hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Rightarrow p \in \frac{611}{950} \pm Z_{.975} \sqrt{\frac{\frac{611}{950} (1 - \frac{611}{950})}{950}} \quad (\text{where } Z_{1-\frac{.05}{2}} = Z_{.975} = 1.96)$$

$$\Rightarrow p \in \frac{611}{950} \pm (1.96) \sqrt{\frac{\frac{611}{950} (1 - \frac{611}{950})}{950}}$$

$$\Rightarrow p \in (L = .61269, U = .67362)$$

$$\Rightarrow .61269 < p < .67362$$

⑥

①

إيجاد التقدير بنقطة والتقدير بنقطة لخاصة الفرق بين نسبتين المجتمع $P_1 - P_2$

① مثال في صفحة 123

	A (1)	B (2)
Population:	—	—
Sample:	$n_1 = 1500$ people where 75 people who are have a cancer disease $\therefore \hat{P}_1 = \frac{75}{1500} = 0.05$ is the proportion of people who have a cancer disease	$n_2 = 2000$ people where 80 people who are have a cancer disease $\therefore \hat{P}_2 = \frac{80}{2000} = 0.04$ is the proportion of people who have a cancer disease

① point estimate for $P_1 - P_2 = (\hat{P}_1 - \hat{P}_2) = \frac{75}{1500} - \frac{80}{2000} = \frac{1}{100} = 0.01$
 is the proportion of people having cancer disease
 in two population

② 90% confidence interval for $P_1 - P_2$?! $\alpha = \frac{10}{100} = 0.1$
 as we have $n_1 \geq 30$ and $n_2 \geq 30$, then

$$P_1 - P_2 \in (\hat{P}_1 - \hat{P}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}$$

$$\Rightarrow P_1 - P_2 \in 0.01 \pm Z_{0.95} \sqrt{\frac{0.05(1-0.05)}{1500} + \frac{0.04(1-0.04)}{2000}} \quad (\text{where } Z_{1-\frac{\alpha}{2}} = Z_{0.95} = 1.645)$$

$$\Rightarrow P_1 - P_2 \in (L = -0.00173, U = 0.02173)$$

$$\Rightarrow -0.00173 < P_1 - P_2 < 0.02173$$

Since include 0, so $P_1 - P_2 = 0 \cong P_1 = P_2$

i.e the population proportions are the same.

⑦