"إعادة صياغة حلول الباب السابع"

إختبار الفرضيات حول متوسط مجتمع μ

مثال في صفحة 130

Population: $\sigma = 8.9$

Sample: $n = 100$, $\bar{x} = 71.8$

① hypotheses: $H_0: \mu \leq 70$ Vs $H_1: \mu > 70$ ← هذا ما يسأل عنه

$\mu_0 = 70$, $\alpha = .05$

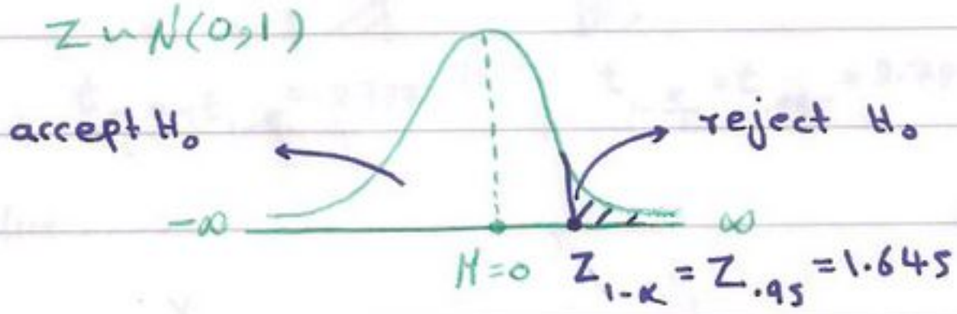
② test statistic (T.S.):

as we have a non-normal d's. + σ is know + $n \geq 30$, then

$$Z_{T.S.} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{71.8 - 70}{\frac{8.9}{\sqrt{100}}} = 2.02247 \sim N(0,1)$$

③ reject H_0 or accept H_0 by H_1 using:

(a) rejection region and acceptance region.....



(b) p-Value ....

$$\text{p-Value} = p(Z > Z_{T.S.}) = p(Z > 2.02)$$
$$= 1 - p(Z < 2.02) = 1 - .97831 = .02169$$

④ decision:

by ③$_a$: as $Z_{T.S.} = 2.02247 > Z_{.95} = 1.645$

So, we reject H_0 and accept H_1 at $\alpha = .05$.

by ③$_b$: as p-Value $= .02169 < \alpha = .05$

So, we have the same decision.

Population: $\mu_0 = 8$, normal dis.

Sample: $n = 25$, $\bar{x} = 7.8$, $S = .5$

① hypotheses: $H_0: \mu = 8$ vs $H_1: \mu \neq 8$

$\mu_0 = 8$, $\alpha = .01$

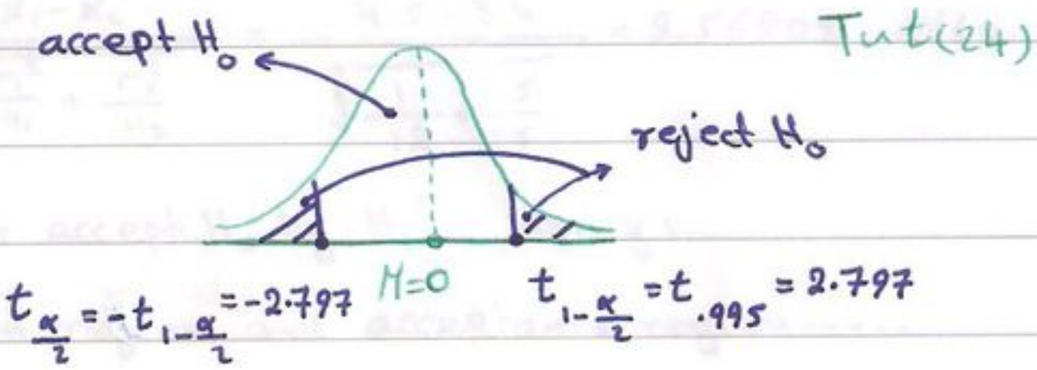
② test statistic (T.S.):

as we have a normal dis. + σ is unkown + $n < 30$, then

$$T_{T.S.} = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{7.8 - 8}{\frac{.5}{\sqrt{25}}} = -2 \sim t(n-1 = 24)$$

③ reject H_0 or accept H_0 by H_1 using:

ⓐ rejection region and acceptance region....



ⓑ p-value ....

X

④ decision:

by ③$_a$: as

$$t_{\frac{\alpha}{2}} = -2.797 < T_{T.S.} = -2 < t_{1-\frac{\alpha}{2}} = 2.797$$

So, we accept H_0 and reject H_1 at $\alpha = .01$

by ③$_b$: X

إختبار الفرضيات حول الفرق بين متوسطي مجتمعين $\mu_1 - \mu_2$ (عينات مستقلة)

مثال في صفحة 136

	Down's syndrome (1)	normal (2)
Population :	$\sigma_1^2 = 1$	$\sigma_2^2 = 1.5$
	normal dis.	normal dis.
sample :	$n_1 = 12$	$n_2 = 15$
	$\bar{x}_1 = 4.5$	$\bar{x}_2 = 3.4$

① hypotheses $H_0: \mu_1 = \mu_2$ Vs $H_1: \mu_1 - \mu_2 \neq 0$

or $H_0: \mu_1 - \mu_2 = 0$ Vs $H_1: \mu_1 - \mu_2 \neq 0$

$\alpha = .05$

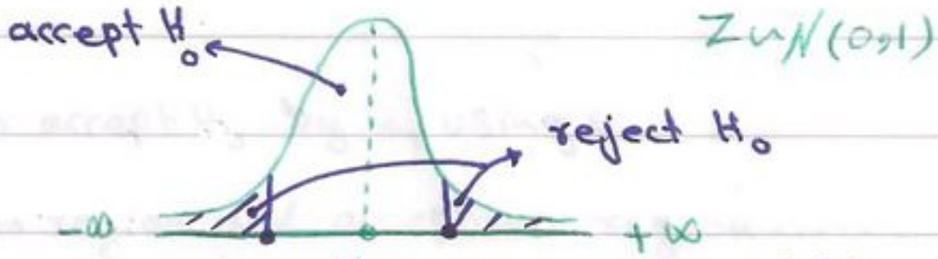
② test statistic (T.S.):

as we have a normal distributions + σ_1 and σ_2 are known, then

$$Z_{T.S.} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{4.5 - 3.4}{\sqrt{\frac{1}{12} + \frac{1.5}{15}}} = 2.56905 \sim N(0,1)$$

③ reject H_0 or accept H_0 by H_1 using:

ⓐ rejection region and acceptance region.....



$Z_{\frac{\alpha}{2}} = -Z_{1-\frac{\alpha}{2}} = -1.96$ $\quad Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$

ⓑ P-Value ....

$$\text{P-Value} = 2P(Z > |Z_{T.S.}|) = 2P(Z > 2.57) = 2[1 - P(Z < 2.57)]$$

$$= 2[1 - .99492] = .01016$$

④ decision: by ③$_a$: as $Z_{T.S.} = 2.56905 > Z_{.975} = 1.96$, so we reject H_0 and accept H_1 at $\alpha = .05$

by ③$_b$: as P-Value $= .01016 < \alpha = .05$, so we have the same decision.

③

مثال في صفحة 138

	material 1 (1)	material 2 (2)
population:	normal dis.	normal dis.
	$\sigma_1 = \sigma = ?$	$\sigma_2 = \sigma = ?$
sample:	$n_1 = 12$	$n_2 = 10$
	$\bar{x}_1 = 85$	$\bar{x}_2 = 81$
	$s_1 = 4$	$s_2 = 5$

① hypotheses: $H_0: \mu_1 \leq \mu_2$ Vs $H_1: \mu_1 > \mu_2$

or $H_0: \mu_1 - \mu_2 \leq 0$ Vs $H_1: \mu_1 - \mu_2 > 0$

$\alpha = .05$

② test statistic (T.S.):

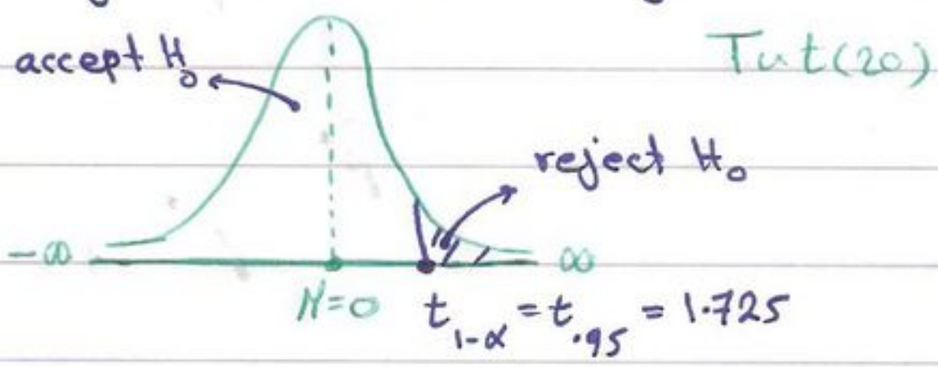
as we have normal distributions + $\sigma_1 = \sigma_2 = \sigma = ?$ + $n_1 < 30$, $n_2 < 30$, then

$$T_{T.S.} = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{85 - 81}{\sqrt{\frac{11(4)^2 + 9(5)^2}{12+10-2}} \sqrt{\frac{1}{12} + \frac{1}{10}}} = 2.08633 \sim t(n_1 + n_2 - 2 = 20)$$

where $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

③ reject H_0 or accept H_0 by H_1 using:

ⓐ rejection region and acceptance region.....



ⓑ p-value ----

X

④

④ decision:

by ③$_a$: as $T_{T.S.} = 2.08633 > t_{1-\alpha} = 1.725$, so we

reject H_0 and accept H_1 at $\alpha = .05$.

by ③$_b$: X

⑤

إختبار الفرضيات حول الفرق بين متوسطي مجتمعين $\mu_D = \mu_1 - \mu_2$ (عينات مترابطة أو غير مستقلة)

142 and 145 مثال في صفحة

weight before (1) — weight after (2)

i	Xi	Yi	$D_i = X_i - Y_i$
1	86.6	79.7	6.9
2	80.2	85.9	-5.7
3	91.5	81.7	9.8
4	80.6	82.5	-1.9
5	82.3	77.9	4.4
6	81.9	85.8	-3.9
7	88.4	81.3	7.1
8	85.3	74.7	10.6
9	83.1	68.3	14.8
10	82.1	69.7	12.4

من الآلة الحاسبة نجيب

$\bar{D} = 5.45$ (sample mean)

$S_D = 7.09$ (sample standard deviation)

$S_D^2 = 50.33$ (sample variance)

Population: normal dis.

Sample: $n=10$, $\bar{D} = 5.45$, $S_D = 7.09$

① $\mu_D = \mu_1 - \mu_2 \in \bar{D} \pm t_{1-\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}$ at $v = n-1$ and $\alpha = .05$

$\Rightarrow \mu_D \in 5.45 \pm t_{.975} \frac{7.09}{\sqrt{10}}$ ($t_{.975} = 2.262$)

$\Rightarrow \mu_D \in 5.45 \pm 2.262 \frac{7.09}{\sqrt{10}}$

$\Rightarrow \mu_D \in (.37847, 10.52153)$

(2)

(1) hypotheses: $H_0: \mu_1 \leq \mu_2$ Vs $H_1: \mu_1 > \mu_2$

or $H_0: \mu_1 - \mu_2 \leq 0$ Vs $H_1: \mu_1 - \mu_2 > 0$

or $H_0: \mu_D \leq 0$ Vs $H_1: \mu_D > 0$

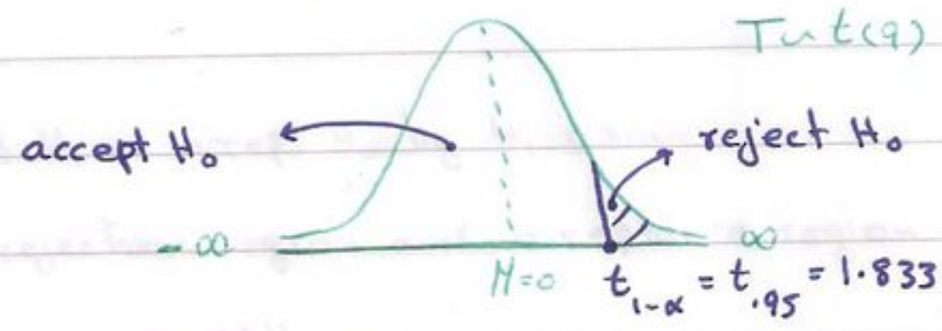
at $\alpha = .05$

(2) test statistic (T.S.):

$$T_{T.S.} = \frac{\bar{D}}{\frac{S_D}{\sqrt{n}}} = \frac{5.45}{\frac{7.09}{\sqrt{10}}} = 2.43081 \sim t(n-1=9)$$

(3) reject H_0 or accept H_0 by H_1 using:

(a) rejection region and acceptance region ....



(b) p-value ....

X

(4) decision:

by (3)a: as $T_{T.S.} = 2.43081 > t_{.95} = 1.833$

so, we reject H_0 and accept H_1 at $\alpha = .05$.

by (3)b: X

إختبار الفرضيات حول نسبة المجتمع p

147 مثال في صفحة

Population: —

Sample: $n = 45$ patients ⟶ 24 are females $\Rightarrow \hat{p} = \frac{24}{45}$ نسبة الإناث في المرضى

① hypotheses: $H_0: P = .7$ Vs $H_1: P \neq \frac{70}{100} = .7$, $P_0 = .7$

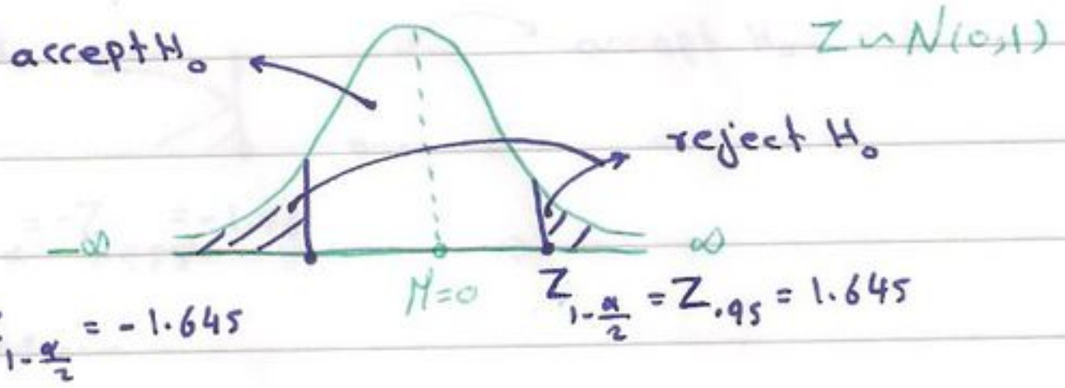
at $\alpha = .1$

② Test statistic (T.S.): as $n \geq 30$ then

$$Z_{T.S.} = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}} = \frac{\frac{24}{45} - .7}{\sqrt{\frac{.7(.3)}{45}}} = -2.43975 \sim N(0,1)$$

③ reject H_0 or accept H_0 by H_1 using:

ⓐ rejection region and acceptance region ...



$Z_{\frac{\alpha}{2}} = -Z_{1-\frac{\alpha}{2}} = -1.645$

ⓑ p-Value ....

$$\text{p-Value} = 2P(Z > |Z_{T.S.}|) = 2P(Z > 2.44)$$
$$= 2\left[1 - P(Z < 2.44)\right] = 2\left[1 - .99266\right] = .01468$$

④ decision:

by ③$_a$: as $Z_{T.S.} = -2.43975 < Z_{\frac{\alpha}{2}} = -1.645$

so, we reject H_0 and accept H_1 at $\alpha = .1$

by ③$_b$: as p-Value $= .01468 < \alpha = .1$

so, we have the same decision.

population: -

sample: $n = 200$ adults $\longrightarrow$ 60 who hesitate to take dental appointment

$$\Rightarrow \hat{p} = \frac{60}{200} = .3$$

① hypotheses: $H_0: P \geq .25$ Vs $H_1: P < .25$, $P_0 = .25$

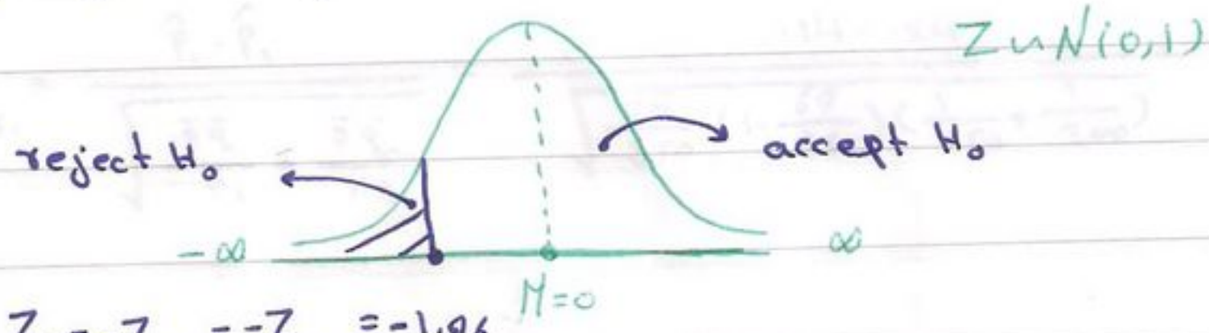
at $\alpha = .025$

② Test statistic (T.S.): as $n \geq 30$, then

$$Z_{T.S.} = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0 q_0}{n}}} = \frac{.3 - .25}{\sqrt{\frac{.25(.75)}{200}}} = 1.63399 \sim N(0,1)$$

③ reject H_0 or accept H_0 by H_1 using:

ⓐ rejection region and acceptance region ---



$$Z_\alpha = -Z_{1-\alpha} = -Z_{.975} = -1.96$$

ⓑ P-Value ---

$$\text{P-Value} = P(Z < Z_{T.S.}) = P(Z < 1.63) = .94845$$

④ decision:

by $③_a$: as $Z_{T.S.} = 1.63399 > Z_\alpha = -1.96$

so, we accept H_0 and reject H_1 at $\alpha = .025$

by $③_b$: as P-Value $= .94845 > \alpha = .025$

so, we have the same decision.

إختبار الفرضيات حول الفرق بين نسبتي مجتمعين $P_1 - P_2$

مثال في صفحة 152

	males (1)	females (2)
Population:	–	–
sample:	$n_1 = 150$	$n_2 = 200$
	↓	↓
	21 with obese	48 with obese
	↓	↓
	$\hat{P}_1 = \frac{21}{150} = .14$	$\hat{P}_2 = \frac{48}{200} = .24$

$$\therefore \overline{P} = \frac{21+48}{150+200} = \frac{69}{350}$$

① hypotheses: $H_0: P_1 = P_2$ Vs $H_1: P_1 \neq P_2$

or $H_0: P_1 - P_2 = 0$ Vs $H_1: P_1 - P_2 \neq 0$

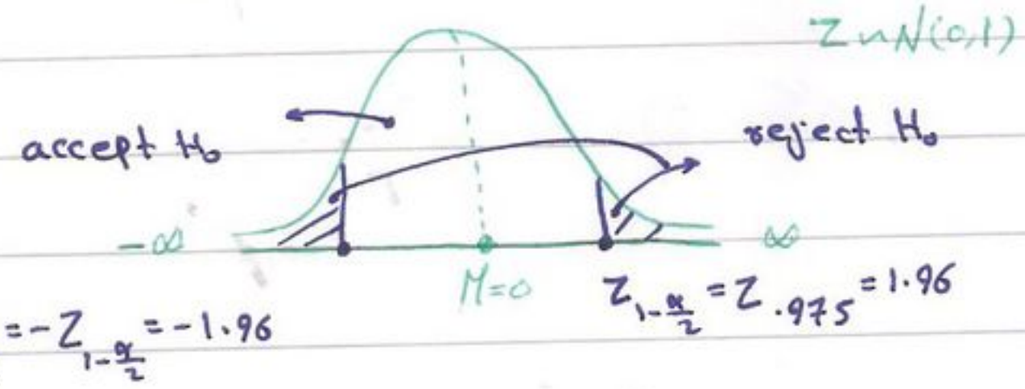
at $\alpha = .05$

② Test statistic (T.S.): as $n_1 \geq 30$ and $n_2 \geq 30$, then

$$Z_{T.S.} = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{\overline{P}\overline{q}}{n_1} + \frac{\overline{P}\overline{q}}{n_2}}} = \frac{.14 - .24}{\sqrt{\frac{69}{350}\left(1 - \frac{69}{350}\right)\left(\frac{1}{150} + \frac{1}{200}\right)}} = -2.32 \sim N(0,1)$$

③ reject H_0 or accept H_0 by H_1 using:

ⓐ rejection region and acceptance region ...



$Z_{\frac{\alpha}{2}} = -Z_{1-\frac{\alpha}{2}} = -1.96$

$Z_{1-\frac{\alpha}{2}} = Z_{.975} = 1.96$

ⓑ P-value ...

$$\text{P-value} = 2P(Z > |Z_{T.S.}|) = 2P(Z > 2.33)$$

$$= 2\left[1 - P(Z < 2.33)\right] = 2\left[1 - .99010\right]$$

$$= .0198$$

④ decision:

by ③$_a$: as $Z_{T.S.} = -2.32711 < Z_{\frac{\alpha}{2}} = -1.96$

so, we reject H_0 and accept H_1 at $\alpha = .05$

by ③$_b$: as p-Value $= .0198 < \alpha = .05$

so, we have the same decision.