

**KING SAUD UNIVERSITY**  
**COLLEGE OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**

Mid-term Exam I / MATH-244 (Linear Algebra) / Semester 451

**Max. Marks: 25**

**Max. Time: 1.5 hrs**

**Note:** Scientific calculators are not allowed.

**Question 1:** [Marks: 4+2+3]:

- a) Find the reduced row echelon form of the matrix  $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ -1 & 2 & -3 & -1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$  and use it to

find non-trivial solutions of the linear system  $AX = O$ , where  $O = [0 \ 0 \ 0 \ 0]^T$ .

- b) Let  $B$  be a  $3 \times 3$  matrix with  $\det(B) = 2$ . Compute  $\det(B^{-1} + \text{adj}(B))$ .

- c) Let  $P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ . Compute  $\text{adj}(P)$  and use it to find  $P^{-1}$ .

**Question 2:** [Marks: 2+3+3]:

- a) Give example of an invertible matrix  $A$  with  $\text{tr}(A) = 0$ .

- b) Find the values of  $\lambda$  for which the matrix  $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda \\ 1 & -1 & 3 - 2\lambda \end{bmatrix}$  is not invertible.

- c) Solve the matrix equation  $AZ = X + Y$  for  $Z$ , where  $A$  is an invertible matrix of size 3,

$$X = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, Y = \begin{bmatrix} 5 \\ 0 \\ -4 \end{bmatrix}, AX = \frac{1}{3}X \text{ and } AY = \frac{1}{2}Y.$$

**Question 3:** [Marks: 4+4]

- a) Find the values of  $\delta$  for which the following linear system of equations

$$x + y + z + t = 4$$

$$x + \delta y + z + t = 4$$

$$x + y + \delta z + (3 - \delta)t = 6$$

$$2x + 2y + 2z + (\delta - 5)t = 6$$

has: (i) no solution (ii) infinitely many solutions.

- b) Use Cramer's rule to solve the following linear system of equations:

$$x + y = 1$$

$$x + 2y + z = -1$$

$$x + 3y - z = 2$$