

KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 462 / MATH-244 (Linear Algebra) / Mid-term Exam 1
Max. Marks: 25 **Max. Time: $1\frac{1}{2}$ hrs.**

Note: Scientific calculators are not allowed.

Question 1: [Marks: 5]

Determine whether the following statements are true or false and justify your answer:

- (i) If A and B are symmetric matrices compatible for the product AB , then AB is also symmetric.
- (ii) If the matrix A^2 is invertible, then A itself is invertible.
- (iii) If the matrix $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & x \end{bmatrix}$ is its own adjoint, then $x = 3$.
- (iv) If matrices A and B are compatible for the products AB and BA , then $|AB| = |BA|$.
- (v) If $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$, then $(-\frac{1}{11}, \frac{2}{11})$ is a solution of the equation $A^{-1} = xA + yI_2$.

Question 2: [Marks: 4 + 3 + 3]

- (a) Find the matrix A if $A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & -2 \end{bmatrix}$.
- (b) Show that $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ -(b+c-a) & -(c+a-b) & -(a+b-c) \end{vmatrix} = 0$.
- (c) Let A be a square matrix of size n with $|A| = 3$ and $|adj(A)| = 27$. Find n .

Question 3: [Marks: 3 + 3 + 4]

- (a) Solve the matrix equation: $XA = B$, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ -4 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 & 1 \\ 8 & 1 & -5 \\ 4 & 3 & -3 \end{bmatrix}$.
- (b) Solve the following system of linear equations by using Cramer's Rule:

$$\begin{aligned} x + 3y + 3z &= 1 \\ x + 3y + 5z &= -1 \\ x + 2y - z &= 2. \end{aligned}$$
- (c) Find the value of m for which the following system of linear equations admits a unique solution and then find this uniquely existing solution.

$$\begin{aligned} x + y + z &= 1 \\ x + y + 2z &= 0 \\ 2x - y - z &= -1 \\ x - 2y + z &= m. \end{aligned}$$

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