KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Mid-term Exam II / MATH-244 (Linear Algebra) / Semester 452

Max. Marks: 25 **Max. Time: 1.5 hrs**

Note: Scientific calculators are not allowed.

Question 1: [Marks: (1+2+2) + (1+2)]

a) **Determine** whether the following statements are true. **Justify** your answers.

i) $\{(a, b, c) \mid a, b, c$ are non-negative real numbers$\}$ is a subspace of $\mathbb{R}^3$.

ii) For any fixed matrix $Y \in M_n(\mathbb{R})$, $\{A \in M_n(\mathbb{R}) \mid AY = YA\}$ is a subspace of the vector space $M_n(\mathbb{R})$ of all real matrices of type $n \times n$.

iii) Any **set of five** 2×2 matrices must be linearly dependent.

b) **Consider** the vector subspace $W = \{p(x) \in P_2 \mid p(1) = p(2)\}$ of P_2, where P_2 denotes the vector space of all real polynomials in x with degree at most 2 under the usual addition and scalar multiplication. Then:

i) **Show** that $P_2 - W \neq \emptyset$.

ii) **Show** that $\{1, (x-1)(x-2)\}$ is a linearly independent subset and **explain why** it must be a basis for W.

Question 2: [Marks: 3 + 2 + 2 + 2]

Let $B := \{u_1 = (2,1),\ u_2 = (5,2)\}$ and $C := \{v_1 = (1,-2), v_2 = (-3,7)\}$. Then:

i) **Show** that both B and C are bases for the vector space $\mathbb{R}^2$.

ii) **Construct** the transition matrix ${}_CP_B$ from the basis B to the basis C.

iii) **Use** the matrix ${}_CP_B$ to **find** the transition matrix ${}_BP_C$.

iv) **Find** the coordinate vectors $[u_1]_C$ and $[v_2]_B$ by **using** the matrices ${}_CP_B$ and ${}_BP_C$, respectively.

Question 3: [Marks: 2 + 3 + 3]

Let $RREF(A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $RREF(A^T) = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ be the reduced row-echelon forms of a matrix A and its transpose A^T, respectively. Then, **find**:

i) $rank(A)$ and $nullity(A)$.

ii) A basis for each of the vector spaces: $col(A)$, $row(A)$, $N(A)$.

iii) Three vectors u, v and w such that $u \in \mathbb{R}^3 \setminus row(A)$, $v \in \mathbb{R}^4 \setminus col(A)$ and $w \in \mathbb{R}^3 \setminus N(A)$.

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