

**KING SAUD UNIVERSITY**  
**COLLEGE OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**

**Semester 452 / MATH-244 (Linear Algebra) / Mid-term Exam 1**

**Max. Marks: 25**

**Max. Time: 1.5 hrs**

**Note:** Scientific calculators are not allowed.

**Question 1:** [Marks: 3 + 4 + 3]

(a). Let  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{bmatrix}$ . Compute  $A^2$  and then use  $A^2$  to find  $A^{-1}$ .

(b). Let  $A = \begin{bmatrix} 23 & 1 & -3 & -6 \\ 5 & 1 & 0 & -2 \\ -5 & 0 & 1 & 1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$ . Find  $A^{-1}$  and then use  $A^{-1}$  to find  $\text{adj}(A)$ .

(c). Let  $A$  be a  $3 \times 3$  matrix with  $\det(A) = 2$ . Evaluate  $\det(\text{adj}(A))$ .

**Question 2:** [Marks: 4 + 4]

(a). Let  $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$ . Find the matrix  $M = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$  such that  $AM = MA$ .

(b). Find the values of  $a$ ,  $b$  and  $c$  so that  $(1, -2, 3)$  is the solution of following system of linear equations:

$$2ax + 4by + 2cz = 6$$

$$ax + 6by + cz = -2$$

$$3ax + 4by + cz = 4.$$

**Question 3:** [Marks: 3 + 4]

(a). Use the Gauss-Jordan elimination method to solve the linear system  $AX = B$ , where:

$$A = \begin{bmatrix} 2 & -1 & -4 & 3 \\ 3 & -2 & -5 & 4 \\ 3 & -3 & -2 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(b). Find all the non-trivial solutions of the following homogeneous system:

$$2x + 2y + 4z = 0$$

$$w - y - 3z = 0$$

$$2w + 3x + y + z = 0$$

$$-2w + x + 3y - 2z = 0.$$

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