

Short Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Linear Algebra	الجبر الخطي	اسم المقرر
Course Code	MATH 244	244 رياض	رمز المقرر
Exam Date	2023-05-23	1444-11-03	تاريخ الامتحان
Exam Time	08: 20 AM		وقت الامتحان
Exam Duration	45 min	٤٥ دقيقة	مدة الامتحان
Classroom No.	F114		رقم قاعة الاختبار
Instructor Name	د. هدى الرشيدى		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.	55620	رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculator is permitted

- عدد صفحات الامتحان صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- يمكن استخدام الآلة الحاسبة

هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.2 Distinguish vector spaces and subspaces and their basis elements and inner product spaces, linear transformations and their kernels and images and know their basic properties for them.	QII(1)	2.5	10
2	22.2 Solve a system of linear equations by using different methods..	QI(1)	1	
3	2.3 Decide whether a set of vectors is linearly dependent or independent or basis of a vector space; whether a set of vectors is orthogonal or orthonormal; whether a transformation between vector spaces is linear or not.	QI(2-3-4-5)_QII(2)	6.5	
4				
5				
6				
7				
8				

(8:00—09:05)

King Saud University

College of Sciences

Department of Mathematics

Semester 443 / MATH-244 / Second Quiz (23-5-2023)

Max. Marks: 10

Max. Time: 45 Min.

Student's Name	Student's ID	Section No

Question No.	I	II	Total
Mark			

Question No	(1)	(2)	(3)	(4)	(5)
Answer					

Question I [5 Marks]: Choose the correct answer and write it in the top table.

(1) For $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$, the solution space of $Ax = 0$ in \mathbb{R}^3 is

Handwritten row reduction for matrix A:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix} \xrightarrow{-2R_2+R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R_1+R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) A line through the origin

(b) A plane through the origin

(c) The origin

(2) The following vectors span \mathbb{R}^3

Handwritten determinant calculation:

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ -1 & -2 & 0 \end{vmatrix} = -1(-2+2) = 0 \text{ (Not)}$$

Handwritten determinant calculation:

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 4 & -2 \end{vmatrix} = 2(-6-4) + (-1-3) \neq 0 \text{ (Yes)}$$

(a) $\{(1,0, -1), (2,0, -2), (1,1,0)\}$

(b) $\{(2,0,1), (-1,3,4), (1,1,-2)\}$

(c) $\{(2,0,-2), (2,1,-1), \}$

(3) Which of the following is a linear combination of the vectors $v_1 = (1, -2, 0)$, $v_2 = (0, 1, 2)$, $v_3 = (5, -6, 8)$?

Handwritten row reduction for linear combination:

$$\begin{bmatrix} 1 & 0 & 5 & | & a \\ -2 & 1 & -6 & | & b \\ 0 & 2 & 8 & | & c \end{bmatrix} \xrightarrow{2R_1+R_2} \begin{bmatrix} 1 & 0 & 5 & | & a \\ 0 & 1 & -4 & | & 2a+b \\ 0 & 2 & 8 & | & c \end{bmatrix} \xrightarrow{-2R_2+R_3} \begin{bmatrix} 1 & 0 & 5 & | & a \\ 0 & 1 & -4 & | & 2a+b \\ 0 & 0 & 0 & | & -4a-2b+c \end{bmatrix} \Rightarrow C = 4a + 2b$$

(a) $(2, 1, 1)$

(b) $(2, -1, 7)$

(c) $(2, -1, 6)$

(4) If $v = (m, 2, 0)$, $u = (0, 1, 2)$ and $w = (2, 3, m)$. Then the set $\{u, v, w\}$ linearly dependent if

Handwritten determinant calculation:

$$\begin{vmatrix} m & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 2 & m \end{vmatrix} = m(m-6) + 2(4) = m^2 - 6m + 8 = 0$$

(a) $m = -2, m = 4$

(b) $m = 2, m = 4$

(c) $m = -2, m = -4$

(5) If The coordinate vector $(w)_S = (2, -1, 1)$ where $S = \{(1, 0, 1), (2, 3, 0), (-1, 3, 2)\}$. Then $w =$

Handwritten vector calculation:

$$2(1, 0, 1) - (2, 3, 0) + (-1, 3, 2)$$

(a) $(-1, 0, 4)$

(b) $(3, 6, 4)$

(c) $(5, 0, 0)$

Question II [5 Marks]:

(1) Let $W = \{(a, b, c) \in \mathbb{R}^3 : a = b + c\}$. Show that W is a subspace of \mathbb{R}^3 .

① $W \neq \emptyset$ since $(0, 0, 0) \in W$ ($0 = 0 + 0$)

② let $u = (a, b, c) \in W \Rightarrow a = b + c \rightarrow \textcircled{1}$
 $v = (x, y, z) \in W \Rightarrow x = y + z \rightarrow \textcircled{2}$

$u + v = (a + x, b + y, c + z)$, $a + x \stackrel{??}{=} b + y + c + z$
L.H.S = $a + x = b + c + y + z$ from $\textcircled{1}$ & $\textcircled{2}$
 $= (b + y) + (c + z) = \text{R.H.S}$

$\therefore u + v \in W$

③ let $u = (a, b, c) \in W \Rightarrow a = b + c \rightarrow \textcircled{1}$

$\Rightarrow ku = (ka, kb, kc)$, $ka \stackrel{??}{=} kb + kc$

R.H.S = $ka = k(b + c) = kb + kc = \text{R.H.S}$

$\therefore ku \in W$

$\therefore W$ subspace of \mathbb{R}^3 .

(2) Determine whether the following is True or False. Justify your answer.

i. The functions $f_1(x) = \cos x$, $f_2(x) = \sin x$ are linearly dependent.

(F)

$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1 \neq 0$

$\Rightarrow \{f_1, f_2\}$ linearly independent

ii. $p_1 = 1 - x^2$, $p_2 = -3 + x + 3x^2$, $p_3 = 9 - 6x - 8x^2$ form a basis for P_2 .

(T)

$A = \begin{bmatrix} 1 & -3 & 9 \\ 0 & 1 & -6 \\ -1 & 3 & -8 \end{bmatrix} \Rightarrow \det(A) = 1(-8 + 18) - (+18 - 9)$
 $= 10 - 9 = 1 \neq 0$

$\Rightarrow Ax = 0$ has a trivial solution \Rightarrow linearly independent

$\Rightarrow Ax = b$ consistent \Rightarrow span

$\Rightarrow \{p_1, p_2, p_3\}$ is a basis

Good Luck ☺