College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Short Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Linear Algerbra**	الجبر الخطي	اسم المقرر
Course Code	**MATH 244**	244 ريض	رمز المقرر
Exam Date	**2023-05-23**	**1444-11-03**	تاريخ الامتحان
Exam Time	**10: 50 AM**		وقت الامتحان
Exam Duration	45 min	٤٥ دقيقة	مدة الامتحان
Classroom No.	**G051**		رقم قاعة الاختبار
Instructor Name	د.هدى الرشيدي		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.	74962	رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 2 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculator is permitted

تعليمات عامة:

- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
- يمكن استخدام الآلة الحاسبة

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.2 Distinguish vector spaces and subspaces and their basis elements and inner product spaces, linear transformations and their kernels and images and know their basic properties for them.	QII(1)	2.5	
2	22.2 Solve a system of linear equations by using different methods..	QI(1)	1	
3	2.3 Decide whether a set of vectors is linearly dependent or independent or basis of a vector space; whether a set of vectors is orthogonal or orthonormal; whether a transformation between vector spaces is linear or not.	QI(2-3-4-5)_QII(2)	6.5	
4				
5				10
6				
7				
8				

EXAM COVER PAGE

(*10:30—11:35*)

King Saud University
College of Sciences
Department of Mathematics
Semester 443 / MATH-244 / Second Quiz (23-5-2023)

Max. Marks: 10 **Max. Time: 45 Min.**

Student's Name	Student's ID	Section No

Question No.	I	II	Total
Mark			

Question No	(1)	(2)	(3)	(4)	(5)
Answer					

Question I [5 Marks]**: Choose** the correct answer and **write** it in the top table**.**

(1) For $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$, the **solution space** of $A\boldsymbol{x} = \boldsymbol{0}$ in $\mathbb{R}^3$ is

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \xrightarrow{-2R1+R3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R2+R1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) A line through the origin (b) A plane through the origin n (c) The origin

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(2) The following vectors **span** $\mathbb{R}^3$

$$\begin{vmatrix} 2 & 1 & 0 \\ 0 & 3 & 6 \\ 0 & 1 & 2 \end{vmatrix} = 2(6-6) = 0$$ not span

$$\begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 0 \\ -1 & 2 & 4 \end{vmatrix} = 1(4+1) = 5$$ span

(a) $\{(0,1,-2),(3,1,-1)\}$ (b) $\{(2,0,0),(1,3,1),(0,6,2)\}$ (c) $\{(1,0,-1),(5,1,2),(1,0,4)\}$

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(3) Which of the following is a **linear combination** of the vectors $\boldsymbol{v_1} = (1,0,-1)$, $\boldsymbol{v_2} = (0,1,2)$, $\boldsymbol{v_3} = (-1,1,3)$?

$$a+c-2b=0 \Leftarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & 0 & a+c-2b \end{array}\right] \xleftarrow{-2R2+R3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ 0 & 2 & 2 & a+c \end{array}\right] \xleftarrow{R1+R3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ -1 & 2 & 3 & c \end{array}\right]$$

(a) $(1,1,1)$ (b) $(2,4,5)$ (c) $(0,1,5)$

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(4) If $v = (m,0,1), u = (2,1,-2)$ *and* $w = (2,1,m)$. Then the set $\{u,v,w\}$ linearly dependent if

$$\begin{vmatrix} m & 2 & 2 \\ 0 & 1 & 1 \\ 1 & -2 & m \end{vmatrix} = m(m+2) + (2-2)$$

$m(m+2) = 0$

$m = 0$

$m = -2$

(a) $m = 1, m = 1$ (b) $m = 0, m = -2$ (c) $m = -1, m = 2$

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(5) If The coordinate vector $(\boldsymbol{w})_S = (2,-1,1)$ where $S = \{(1,2.1), (2,1.0), (3,3.4)\}$. Then $\boldsymbol{w} =$

$2(1,2,1) - (2,1,0) + (3,3,4)$

(a) $(3,6,6)$ (b) $(2,-5,1)$ (c) $(7,6,6)$

Question II [5 Marks]**:**

(1) Let $W = \{(a, b, c) \in \mathbb{R}^3 : c = a - b\}$. Show that W is a subspace of $\mathbb{R}^3$.

① $W \neq \phi$ since $(0,0,0) \in W$ $(0 = 0 - 0)$

② let $u = (u_1, u_2, u_3) \in W \Rightarrow \boxed{u_3 = u_1 - u_2} \to$ ①

$v = (v_1, v_2, v_3) \in W \Rightarrow \boxed{v_3 = v_1 - v_2} \to$ ②

Now

$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$, $u_3 + v_3 \overset{??}{=} (u_1 + v_1) - (u_2 + v_2)$

L.H.S $= u_3 + v_3 = u_1 - u_2 + v_1 - v_2$ from ① & ②

$= (u_1 + v_1) - (u_2 + v_2) =$ R.H.S.

$\therefore u + v \in W$

③ let $u = (u_1, u_2, u_3) \in W$, $u_3 = u_1 - u_2 \to$ ①

Now $ku = (ku_1, ku_2, ku_3)$, $ku_3 \overset{??}{=} ku_1 - ku_2$

L.H.S $= ku_3 = k(u_1 - u_2)$ from ①

$= ku_1 - ku_2$

$=$ R.H.S

$\Rightarrow ku \in W$

(2) Determine whether the following is True or False. Justify your answer.

i. The functions $f_1(x) = e^x$, $f_1(x) = xe^x$ are linearly independent. (T)

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix} = e^x(xe^x + e^x) - xe^{2x}$$

$$= \cancel{xe^{2x}} + e^{2x} - \cancel{xe^{2x}} = e^{2x} \neq 0$$ since it is exponential function

$\Rightarrow \{f_1, f_2\}$: linearly independent.

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ii. $p_1 = 2 - x + 3x^2$, $p_2 = 1 + 2x - x^2$, $p_3 = -x + 2x^2$ form a basis for P_2. (T)

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & -1 \\ 3 & -1 & 2 \end{bmatrix} \Rightarrow \det A = 2(4-1) - (-2+3)$$

$= 2(3) - (1)$

$= 6 - 1$

$= 5$

$\neq 0$

$\Rightarrow Ax = 0$ has a trivial solution $\Rightarrow$ independent

$\Rightarrow Ax = b$ consistent $\Rightarrow$ span

$\therefore \{P_1, P_2, P_3\}$ is a basis

Good Luck ☺