Propose

Importance of research:

The differential equation

\[
\frac{d^3y}{dx^3} + \beta_0 y'' y = 0, \; x \in [0, \infty)
\]  \hspace{1cm} (1)

With the boundary conditions

\[
y(0) = y'(0) = 0, \; y'(\infty) = 1,
\]  \hspace{1cm} (2)

models the two dimensional viscous flow over a semi-infinite flat plate. Equation (1) was derived by Blasius by using a similarity transformation on the well-known Navier-Stokes equation. The equation has numerous technical applications. Although various authors have found numerical solutions the problem yet finding analytical solutions to the problem remains a topic of current research.

Description of research problem:

The main hurdle in the solution of the problem, is the absence of the second derivative \(y''(0)\). Once this derivative has been correctly evaluated an analytical solution of the boundary value problem may be readily found. Blasius found a power-series solution containing the unknown parameter \(\sigma = y''(0)\). However the radius of convergence of the series is approximately 5.6 and it is impossible to correctly evaluate \(\sigma\) by using this series. Several authors have devised numerical algorithms to find good approximations to this parameter. Liao has used the homotopy analysis method to find \(\sigma\) analytically. Wang has used an ingenious idea to find \(\sigma\) analytically. He used the transformation

\[
t = y'(x), \; z = y''(x)
\]  to transform (1) to
\[
\frac{d^2 z}{dt^2} + \beta_0 \frac{t}{z} = 0 , 
\]  
(3)

Wang used the Adomian decomposition method (ADM) to solve equation (3). Hashim improved upon Wang's method by applying Pade' approximation to the truncated Taylor series found by ADM. Ahmad has developed a method which generates arbitrarily large number of terms in a series' solution of the above equation.

**Thesis Summary:**

Navier-Stokes equations for the fluid flow in a boundary layer are transformed, by a similarity transformation, into an ordinary differential equation, called the Blasius equation. The boundary value problem

\[
\frac{d^3 f}{d\eta^3} + \frac{1}{2} f'(\eta) f''(\eta) = 0 , 
0 \leq \eta < \infty 
\]

\[
f(0) = 0 , f'(0) = 0 , f'(') = 1 
\]

is called the Blasius problem. We discuss numerical solution of the problem and various attempts to solve it analytically. Wang's transformation

\[
x = f'(\eta) \\
y = f''(\eta) 
\]

transforms the problem into

\[
y \frac{d^2 y}{dx^2} + x = 0 , 
0 \leq x < 1 
\]

\[
y(0) = \alpha , y'(0) = 0 , y(1) = 0 . 
\]

Wang and Hashim solve the above problem by Adomian decomposition method and find approximate values of \( f''(0) = \alpha \) by solving the equation \( y(1)=0 \). We solve Wang's equation by a direct method which can
produce a series solution to an arbitrary number of terms. We use this solution to generate two sequences one of them increasing, the other decreasing, both converging to $\alpha$.

This determines $\alpha$ correct to six decimal positions. We show that

$$\int_{0}^{\infty} \left( \frac{d^2 f}{d \eta^2} \right)^2 d \eta = 0.37118$$

and use the above expression to find an asymptotic expression for $f(\eta)$.

We comment on the limitations of the Adomian and homotopy analysis methods. These methods produce series solutions which usually converge within a finite interval of convergence. To get useful information beyond this interval one must use asymptotic analysis.