Half life and radioactive decay
Transmutation

• A process by which the nucleus of a radioactive atom undergoes decay into an atom with a different number of protons, until such time as a stable nucleus is produced.

• 3 types:
  a) Alpha decay
  b) Beta decay
  c) Gamma decay
Alpha decay

• An alpha particle (i.e., a helium nucleus) is released during alpha decay of a radioactive substance.
• An element with a lower mass is formed. Mass is not conserved.
• i.e., a helium nucleus
Beta decay

• (beta negative decay) occurs when a beta (negative) particle is released from the nucleus (i.e., electron).
• Mass is also not conserved in beta decay.
• In beta decay, the beta particle released originated in the nucleus of the atom, not in the electron orbital. A neutron is lost, and in its place a proton and an electron are formed.
Gamma decay

• Is the release of excess stored energy from the nucleus.
• No transmutation occurs.
• However, gamma decay often accompanies alpha and beta negative decay in a decay series.
• Gamma decay occurs when an excited nucleus (excited by photon or particle bombardment, or it may be a decay product in an excited state) returns to the ground state.
• An excited nucleus is heavier than the ground state, by a mass equal to the mass/energy equivalent of the energy of the emitted gamma ray.
• Alpha particles are stopped by a sheet of paper and cannot pass through unbroken skin
• Beta particles are stopped by an aluminium sheet
• Gamma rays are stopped by thick lead
Half-life $t_{1/2}$ and decay constant $\lambda$

- Half life is the time required for half of the original number of atoms to decay.

- $\lambda$ is the decay constant
Equations of radioactive decay

\[ \lambda = \frac{0.693}{t_{1/2}} \quad \text{or} \quad t_{1/2} = \frac{0.693}{\lambda} \]

- \( t_{1/2} = \) half life
- \( \lambda = \) decay constant
- \( N = \) the total number of radioactive atoms present at any given time
- \( N_0 = \) original number of radioactive atoms

\[ 2.3 \log \frac{N_0}{N} = \lambda t \]
Problem 1

1. Ca$^{45}$ has a half life of 163 days. Calculate
   A) the decay constant in terms of day$^{-1}$ and sec$^{-1}$
   B) The percent of the initial radioactivity remaining in a sample after 90 days
Solution 1

a) \( \lambda = \frac{0.693}{0.693} = 4.26 \times 10^{-3} \text{ day}^{-1} \)

\( t_{1/2} = 163 \)

\( \frac{0.693}{0.693} = 4.92 \times 10^{-8} \text{ sec}^{-1} \)

\( 163 \times 24 \times 60 \times 60 \)
b) \[ 2.3 \log N_0 = \lambda t \]

Let \( N_0 = 100\% \)

\[ 2.3 \log 100 = (4.26 \times 10^{-3}) (90) = 0.3834 \]

\[ \log 100 = 0.3834 = 0.167 \]

\[ \log 100 - \log N = 0.167 \]

\[ \log N = 2.000 - 0.167 = 1.83 \]

\[ N = 68.1\% \]
Problem 2

• C$^{14}$ has a half-life of 5700 years. Calculate the fraction of the C$^{14}$ atoms that decays.

(a) per year
(b) per minute
a) Calculate $\lambda$:

\[
\lambda = \frac{0.693}{5700} = 1.216 \times 10^{-4} \text{ yr}^{-1}
\]

To calculate the fraction of the C$^{14}$ atoms that decays per year:

\[
\frac{1}{1.216 \times 10^{-4}} = 8225
\]

This means that 1 out of every 8225 radioactive atom decay per year.
b) Home work !!
The curie

• The curie or Ci is the standard unit of radioactive decay
• It describes the rate at which a certain radioactive substance decay DPM (decay per min)
• \[ \text{DPM} = - \frac{\text{dN}}{\text{dt}} = \lambda N \]
• The negative sign indicates that the number of radioactive atoms decreases with time
• \[ 1 \, \mu \text{Ci} = 2.22 \times 10^6 \, \text{DPM} \]
Problem 3

• $^{40}$K has a half life of $1.3 \times 10^9$ yr and it constitutes 0.012% of the potassium in nature. The human body contains about 0.35% potassium by weight.

Calculate the total radioactivity resulting from $^{40}$K decay in a 75 kg human.
Solution 3

• Total $K^{40} = 0.012\% \times 0.35\% \times 75 \times 1000$
  $= 3.15 \times 10^{-2}$ g

• Number of atoms $= \frac{3.15 \times 10^{-2} \times 6.023 \times 10^{23}}{40}$
  $= 4.74 \times 10^{20}$ atom

• $\lambda = \frac{0.693}{1.3 \times 10^9 \times 365 \times 24 \times 60}$
  $= 1.014 \times 10^{-15}$ /min
• DPM = - dN/dt = \lambda N
  = 1.014 \times 10^{-15} \times 4.74 \times 10^{20}
  = 4.81 \times 10^5

1 \mu Ci = 2.22 \times 10^6 \text{ DPM}

So the total radioactivity is:
4.81 \times 10^5 = 0.217 \mu Ci
2.22 \times 10^6