A note on “Simultaneously scheduling n jobs and the preventive maintenance on the two-machine flow shop to minimize the makespan”

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In Allaoui H., Lamouri S., Artiba A., Aghezzaf E., Simultaneously scheduling n jobs and the preventive maintenance on the two-machine flow shop to minimize the makespan. International Journal of Production Economics 2008, 112, 161–167, several results are proposed. We show in this note that some of them are false. We also extend the analysis for some other results.

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1. Introduction

Allaoui et al. (2008) consider the problem of simultaneously scheduling a set \( J = \{j_1, j_2, \ldots, j_n\} \) of n jobs and a single maintenance period on a two-machine flow shop. Each job consists of a sequence of two operations to be processed on two machines \( M_1 \) and \( M_2 \). We denote by \( a_i \) and \( b_i \) the processing times of \( j_i \) on \( M_1 \) and \( M_2 \) respectively. Furthermore, a single maintenance period (hole for short) is to be scheduled on one of the two machines. The hole has a constant length \( g \) and must start at \( S_k \) on machine \( M_k \), \( k = 1, 2 \) such that \( 0 \leq S_k \leq T \) where \( T \) is a fixed date. The objective is to minimize the makespan \( C_{\text{max}} \).

Allaoui et al. (2008) consider only the nonresumable (nr) model in which if a job is interrupted by the hole, it has to fully restart when the machine becomes available again. We recall that authors also recognize the semiresumable (sr) model in which the job has to partially restart its processing, and the resumable (r) model where the job can proceed without any penalty. We denote the problem by \( F_2|\text{nr} - a(M_1), T|C_{\text{max}} \) where \( x \in \{r, sr, nr\} \) denotes the considered model and \( k \in \{1, 2\} \) refers to the machine concerned with the hole. We add the variable \( T \) to differentiate the considered problem from the \( F_2|x - a(M_k)|C_{\text{max}} \) problem in which the starting time of the hole is fixed (see for example Hadda et al., 2010 and Rapine, 2013). For a given schedule \( \pi = (\pi(1), \ldots, \pi(n)) \), we designate by \( \pi(i) \) the ith job of the sequence. We also denote by \( C^*_\text{max} \) the optimal makespan.

2. The maintenance period on the first machine

Allaoui et al. (2008) state the two following propositions for the \( F_2|\text{nr} - a(M_1), T|C_{\text{max}} \) problem.

Proposition 1 (Allaoui et al., 2008). If we denote any given sequence of jobs by \( \sigma \), we have \( \forall \sigma, C_{\text{max}}'(s_1 = 0) > C_{\text{max}}''(s_1 > 0) \).

Proposition 2 (Allaoui et al., 2008). There exists an optimal solution such that the maintenance period starts at the end time of a given job on the first machine.

Proposition 1 is not totally accurate. Indeed let us consider an instance with two jobs where \( a_1 = a_2 = b_1 = b_2 = g = 10 \) and \( T = 10 \). The sequence \( \sigma = (j_1, j_2) \) with \( s_1 = 0 \) gives \( C_{\text{max}}'(s_1 = 0) = 40 \). On the other hand it should be clear that \( C_{\text{max}}''(0 < s_1 < 10) = s_1 + 40 \), which contradicts with Proposition 1. In fact the proposition would be true if the hole is scheduled directly at the end of an operation as explained by the authors themselves in the proof. We now present a stronger version for Proposition 2.

Proposition 3. There exists an optimal solution such that the hole starts at the end time of the last job that finishes before \( T \) on \( M_1 \).

Proof. Let \( \pi \) be an optimal solution and let \( \pi(p) \) be the last job that finishes before the hole. Let \( \pi(q) \) be the last job that finishes before \( T \) on \( M_1 \). If \( p = q \) then we are done, otherwise we construct schedule
\( \pi' \) obtained from \( \pi \) by shifting backward all the jobs \( \pi(i), p + 1 \leq i \leq q \) such that they directly start after the completion of \( \pi(p) \). We then schedule the hole after \( \pi(q) \). Schedule \( \pi' \) is clearly feasible and optimal which concludes the proof. □

On other terms, given a permutation \( \pi \), Proposition 3 suggests to schedule the hole directly after the last job that finishes before \( T \). Furthermore, since that it is always possible to transform any schedule such that no job is interrupted by the hole, then the considered model becomes nonrelevant, and we can derive the same results for both resumable and semiresumable scenarios.

Allaoui et al. (2008) also propose a heuristic (H1) based on Johnson rule (JR).

**Heuristic H1.** (i) Use JR to schedule the jobs.
(ii) For equal processing times on the first machine, order the corresponding jobs on the decreasing order of processing times on the second machine.
(iii) Evaluate the different situations where the maintenance period is inserted after each job in the sequence. Then get the best position.

Given Proposition 3, we can see that step (iii) of H1 becomes unnecessary. We establish now the worst case behavior of H1.

**Theorem 1.** For the \( F_2 | n r - a(M_2) - T | C_{\text{max}} \) problem, the relative worst-case error bound of H1 is given by \( C_{\text{max}}(\pi_{H1})/C_{\text{max}} \leq 2 \), and the bound is tight.

**Proof.** Let \( \pi_{H1} \) be the schedule generated by H1. We have \( C_{\text{max}}(\pi_{H1}) \leq (\sum_{i=1}^{n} a_{i} + g) + (\sum_{j=1}^{m} b_{j}) \leq 2C_{\text{max}}^\star \). To demonstrate that the bound is tight, we consider the following problem instance with \( n = 2 \) and \( T = 4 \). Let \( a_{1} = 2 \), \( b_{1} = 3 \), \( a_{2} = 3 \) and \( b_{2} = g = w \) where \( w > 3 \).

It is clear that H1 will generate schedule \( \pi_{H1} = (j_1, j_2) \) with \( s_1 = 2 \) which gives \( C_{\text{max}}(\pi_{H1}) = 2w + 5 \). However the optimal solution is \( \pi^\star = (j_2, j_1) \) with \( s_1 = 3 \) which gives \( C_{\text{max}}^\star = w + 8 \). We see that \( C_{\text{max}}(\pi_{H1})/C_{\text{max}}^\star \) goes to 2 as \( w \) tends to infinity. □

We note here that for any arbitrary permutation \( \pi \) we have \( C_{\text{max}}(\pi)/C_{\text{max}}^\star \leq 2 \). This means that more sophisticated heuristics are required to achieve a better error bound. We recall here that Hadda et al. (2010) proposed a (\( \frac{3}{2} \))-approximation algorithm for the \( F_2 | nr - a(M_2) - T | C_{\text{max}} \) problem, and it would be interesting to see if this result could be extended for the problem under consideration.

3. The maintenance period on the second machine

Allaoui et al. (2008) state that \( F_2 | nr - a(M_2) - T | C_{\text{max}} \) is NP-hard, and yet omit the main part of the proof. We now show that this statement is false and that the problem is actually polynomial.

**Theorem 2.** The schedule given by JR with \( s_2 = 0 \) is optimal for \( F_2 | nr - a(M_2) - T | C_{\text{max}} \).

**Proof.** Let us consider an optimal schedule \( \pi \) such that \( s_2 \neq 0 \), and let \( \pi(p) \) be the first job to be processed after the hole. We construct schedule \( \pi' \) obtained from \( \pi \) as follows:

- Maintain all operation on \( M_1 \) in their positions.
- On \( M_2 \), maintain also all the operations of the jobs \( \pi(i), p \leq i \leq n \) in their positions (which makes the makespan unchanged).
- Schedule the maintenance period at the beginning (i.e. \( s_2 = 0 \)).
- On \( M_2 \), schedule the operations of the jobs \( \pi(i), 1 \leq i \leq p - 1 \) as late as possible (i.e. between the hole and \( \pi(p) \)).

Such schedule \( \pi' \) is clearly feasible and optimal. We then conclude that there exists an optimal schedule where the hole is scheduled at the beginning (i.e. \( s_2 = 0 \)). It is known that JR is optimal for \( F_2 | C_{\text{max}} \) even if one of the two machines is not available at the beginning of the horizon which concludes the proof.

An other way to look at the problem is to consider the problem of scheduling (\( n + 1 \)) jobs where \( j_{n+1} \) corresponds to the maintenance period with \( a_{n+1} = 0 \) and \( b_{n+1} = g \). JR will in this case generate an optimal schedule where the hole is scheduled first on \( M_2 \). □

Here too, we can see that Theorem 2 remains true under the resumable and semiresumable models. We finally add that studying this problem under the assumption that the hole has to be started within an interval \([T_1, T_2]\) where \( T_1 > 0 \) would be more realistic and interesting as suggested by Gara-Ali et al. (2014).

**References**


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