Question # 1

The water mass flow rate \( (\rho = 998 \text{ kg/m}^3 \pm 0.2 \%) \) flowing in a pipe (diameter = 5cm \pm 2\%) is measured by an orifice with flow coefficient \( C = 0.6 \pm 0.02 \) (95\% confidence level) and is given by:

\[
m = C A \sqrt{\frac{2 \rho \Delta p}{\Delta V}} \text{ kg/sec}
\]

Where A is pipe area, and \( \Delta p \) is the pressure drop across the orifice in Pascal. The experimental results recorded for the pressure drop (\( \Delta p \)) are:

- 10.5, 10, 10.6, 10.2, 10.1, 9.8, 9.9, 10, 9.4, 9.5 kPa

- Calculate \( \Delta p \) and its uncertainty using 95\% confidence level.
- Calculate the percentage uncertainty in mass flow rate.
- Which parameter contributes more to the uncertainty in mass flow rate

Question # 2

If the flow velocity (U) is an input signal to a measuring system (Hot wire) and the output is E volt which can be represented by the relation:

\[
E^2 = 1.5 U^{0.5}
\]

- Find the static sensitivity of this measuring system at U = 25 m/s
- Transfer the above relation to a straight line relation.

Question # 3

Given the voltage signal \( E = 2 \sin (1000\pi t) \) which is sampled with \( \Delta t = 1.5 \) milisecond.

- Find the measured apparent frequency.
- Draw the frequency spectrum before and after sampling.
- Suggest how to avoid this aliasing.
**Question # (4)**

A thermocouple having a time constant of 2 seconds is used to minter the temperature of a fluid flowing in a conduit. The temperature of the fluid varies as under,

\[ T(t) = 50 + 5 \sin\left(\frac{\pi t}{10}\right) \]

Find

(a) Amplification ratio for the thermocouple
(b) Phase difference
(c) Time delay
(d) Output function of the thermocouple.

**Question # (5)**

(a) What are the basic functional elements of a measuring system and what is their main function?
(b) What is the principle of a piezo-electric transducer?
(c) What is the principle of a strain gage transducer? How does the temperature change affect the output of the strain gage?

**Question # (6)**

A load cell having a sensitivity of 1.5Ω/N is used as the sensing resistance of a four arm Wheatstone bridge. At no load all the resistance arms of the bridge have a resistance of 600Ωs. The excitation voltage of the bridge is 12 Volts. Find the output of the bridge when a load of 1 kN is applied to the load cell.
Mech. Measurement

\[ \begin{align*}
  \sigma &= 998 \text{ kPa} \\
  \varepsilon &= 0.5 \pm 0.2 \% \\
  \dot{\varepsilon} &= 0.2 \% = 0.002 \\
  \varepsilon_{\text{e}} &= 2 \% = 0.02 \\
  \varepsilon_{\text{e}} &= 0.033 \\
  \Delta P &= 10.5, 10.6, 10.7, 10.8, 10.9, 11.0, 11.1, 11.2, 11.3, 11.4, 11.5 \\
  n &= 10 \\
  \Delta P &= 10 \text{ kPa} \\
  \varepsilon_{\text{d}} &= 1.12 \\
  S_{\text{mp}} &= \sqrt{\frac{S_{\text{d}}}{n-1}} = 0.353 \\
  n &= 10 \\
  \Delta P &= 10 \pm 0.252 \text{ kPa} \\
  \Delta P &= 0.025 \\
  \Delta P &= 0.033 \\
  m &= c A \sqrt{2.9 \Delta P} \\
  m &= c \pi d^2 \frac{1}{2} \Delta P \\
  \frac{U_m}{m} &= \frac{\left[ \left( \frac{U_m}{m} \right)^2 + 12 \frac{U_m}{m} \frac{\Delta P}{4} + 4 \left( \frac{1}{2} \frac{U_m}{m} \frac{\Delta P}{4} \right)^2 \right]}{2} \\
  \frac{U_m}{m} &= \frac{0.033}{2} + \frac{(0.025)^2}{2} + \frac{(0.005)^2}{2} = 0.0533 \\
  \frac{U_m}{m} &= 0.0533 \\
  \frac{U_m}{m} &= 0.0533 \\
  \frac{U_m}{m} &= 5.33\% \\
\end{align*} \]
\[ E^2 = 1.5 U^{0.5} \]

\[ k = \frac{dE}{du} = \frac{2E dE}{u} = 0.5x1.5 U^{-0.5} du \]

\[ \frac{dE}{du} = \frac{0.5 \times 1.5}{2E} U^{-0.5} \]

\[ \frac{dE}{du} = \frac{0.5 \times 1.5}{U^{0.5}} = \frac{0.5}{0.25} = 2 \]

\[ E^2 = 1.5 U^{0.5} \]

\[ 2 \log E = \log 1.5 + 0.5 \log U \]

\[ \log E = \frac{1}{2} \log 1.5 + 0.5 \log U \]

\[ \log E = \log 1.5^{0.5} + 0.25 \log U \]

\[ \log E = \log 1.225 + 0.25 \log U \]

\[ \log E = 0.088 + 0.25 \log U \]
\[ E = 2 \sin(1000\pi t) = 2.8 \sin(3140 \pi t) \]

\[ f = 500 \text{ Hz} \quad \Rightarrow T = 1.5 \times 10^3 \text{ s} \]

\[ f_s = f + f \]
\[ = 666.67 - 500 = 166.67 \text{ Hz} \]

\[ E_a = 2.8 \sin(1046.7 \pi t) \]

The \( f_a \) is different than \( f \): Alasing occurs.

To avoid aliasing, \( f_s > 2f = 2 \times 500 \approx 1000 \text{ Hz} \).
\[ f_s = 1053 \text{ Hz} \]
\[ T(t) = 50 + 5 \sin \left( \frac{2 \pi t}{10} \right) \]

\[ \omega = \frac{2 \pi}{10} = 0.628 \text{ rad/s} \]

\[ \phi = 2 \text{ sec} \]

\[ T_s = 5 \frac{2}{\cos \phi} = 5 \frac{1}{\cos \phi} = 5 \frac{1}{0.9} = 5.56 \text{ sec} \]

\[ T_d = 5 \times 0.628 = 3.14 \text{ sec} \]

\[ \phi = \tan^{-1}(0.2) = 0.90 \]

\[ T_0 = 50 \pm 3.14 \sin \left[ \frac{2 \pi t - 0.9}{10} \right] \]

\[ t_d = \phi = 0.9 \times 0.628 = 1.43 \text{ sec} \]

5. Basic Functional Elements of a Measuring System are:

i. Primary Sensing Element - To sense the input
ii. Signal Conditioning Element - To modify \( V_0 \) for % compatibility
iii. Data Presentation Element - To present %

6. When a proper electrical crystal is compressed, it generates voltage.

- When the voltage is applied to its face, it expands.
Strain gauge is made of a resistance, when it is fixed to a loaded member, the load transfers to it. It elongates on tension in the same amount as the structural member on which it is fixed and compresses on compression. This change of length causes change in the resistance of strain gauge. This change unbalances the 4 arm bridge to which it is connected to. Therefore a change in output of the bridge is caused which is proportional to the load applied to the structural member.

\[ \frac{E_0}{E_1} = \left[ \frac{R_1}{R_1 + R_2} \right] \left[ \frac{R_4}{R_3 + R_4} \right] \]

\[ \text{Load} = 1 \text{KN} \quad K = 1.5 \frac{\Omega}{N} \]

\[ aR = 1.5 \times 1000 = 1500 \Omega \]

\[ \frac{E_0}{E_1} = \left[ \frac{R_2}{R_1 + R_2 + R_3 + R_4} \right] \left[ \frac{R_4}{R_1 + R_2 + R_3 + R_4} \right] = \frac{1500 - 600}{1500 + 1500 + 600 + 600} = 0.278 \]

\[ E_0 = E_1 = -3.33 \text{ volt} \]