Entropy (Section 7-1)

Clausius Inequality
For any given system, the following inequality holds:

\[ \int \frac{\delta Q}{T} \leq 0 \rightarrow \begin{cases} \int \frac{\delta Q}{T} < 0, & \text{irreversible process} \\ \int \frac{\delta Q}{T} \_{\text{int rev}} = 0, & \text{internally reversible process} \end{cases} \]

Proof of this inequality is found in the book for the student’s reference

Definition of Entropy (S)

\[ dS = \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \quad \text{(kJ/K)} \quad \rightarrow \quad \Delta S = S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \]

- Entropy is a property
- The integral of \( \frac{\delta Q}{T} \) along an irreversible process path is not entropy

Entropy can also be given as: \( \Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}} \)

where \( S_{\text{gen}} \) is called entropy generation and it is always a positive quantity

What does \( S_{\text{gen}} \) mean?
It means that entropy is generated or created during an irreversible process, and this generation is due entirely to the presence of irreversibilities (friction, etc.)

\[ S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases} \]

Increase of Entropy Principle (Section 7-2)

Special Case: Isolated System (Adiabatic and Closed System)
\( \Delta S_{\text{isolated}} \geq 0 \)

Example of an Isolated System: A System and its Surroundings
\( \Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{surr}} = S_{\text{gen}} \geq 0 \)

Increase of Entropy Principle
The entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant
Notes on Entropy
1. A process can only proceed in the direction that makes $S_{\text{gen}} \geq 0$
2. Entropy is a nonconserved property. There is no such thing as the conservation of entropy principle.
3. Entropy generation is a measure of the magnitude of the irreversibilities present during a process. High irreversibilities mean higher entropy generation.

Entropy Change of Pure Substances (Section 7-3)
Specific entropy can be found from property tables in the same way specific volume ($v$), specific internal energy ($u$), and specific enthalpy ($h$) are found (See Chapter 2)

Isentropic Processes (Section 7-4)
- Entropy can be changed by: (1) heat transfer (2) irreversibilities
  $\Rightarrow$ entropy will not change during a process that is both adiabatic and internally reversible
- An isentropic process is a process during which the entropy remains constant
  $\Delta s = 0$ or $s_2 = s_1$
- Devices such as pumps, turbines, and nozzles are usually adiabatic
- They perform best when the irreversibilities are minimized
- Isentropic processes provide the best performance
  $\Rightarrow$ isentropic processes can be used to define efficiencies of actual devices

Property Diagrams Involving Entropy (Section 7-5)

$$dS = \left( \frac{\delta Q}{T} \right)_{\text{int\ rev}} \quad \Rightarrow \quad \delta Q_{\text{int\ rev}} = T dS \quad \Rightarrow \quad Q_{\text{int\ rev}} = \int_1^2 T dS$$
- This means that the area under the process curve on a $T$-$S$ diagram represents heat transfer during an internally reversible process.
- On a $T$-$S$ diagram, an isentropic process is represented by a vertical line
  $\Rightarrow$ The area under this curve is 0 $\Rightarrow$ Heat transfer during an isentropic process is 0

What is Entropy? (Section 7-6)
- Entropy can be viewed as a measure of molecular disorder or molecular randomness
- The more disorganized molecules are, the higher entropy they possess.
- Heat is a form of disorganized energy and some disorganization (entropy) will flow with heat
- Work is a highly organized way to transfer energy $\Rightarrow$ Work does not produce entropy

The Third Law of Thermodynamics
The entropy of a pure crystalline substance at 0 K is zero (because there is no motion)
The $T dS$ Relations (Section 7-7)

The First $T dS$ Relation (Gibbs Equation)
The differential form of the first law of thermodynamics for a closed system for an internally reversible process can be written as:

$$T dS = dU + P dV \quad \text{or} \quad T ds = du + P dv$$

$$\Rightarrow ds = \frac{du}{T} + \frac{P dv}{T}$$

The Second $T dS$ Relations

$$h = u + P v \rightarrow \left\{ \begin{array}{l} dh = du + P dv + v dP \\ T ds = du + P dv \end{array} \right\} \rightarrow T ds = dh - v dP \quad \Rightarrow ds = \frac{dh}{T} - \frac{vdP}{T}$$

Entropy Change of Liquids and Solids (Section 7-8)

- Liquids and solids are considered incompressible substances: $dv \approx 0$
- Therefore, the first $T dS$ relation becomes:

$$ds = \frac{du}{T} = \frac{C dT}{T} \quad \Rightarrow s_2 - s_1 = \int_{T_1}^{T_2} C(T) \frac{dT}{T} \approx C_{av} \ln \frac{T_2}{T_1}$$

Entropy Change of Ideal Gases (Section 7-9)

- For ideal gases: $du = C_v dT$ and $P = RT/v$.
- The first $T dS$ relation then becomes:

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v} \quad \Rightarrow s_2 - s_1 = \int_{T_1}^{T_2} C_v(T) \frac{dT}{T} + R \ln \frac{V_2}{V_1}$$

Approximate Analysis: Constant Specific Heat

$$s_2 - s_1 = C_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

and

$$s_2 - s_1 = C_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Exact Analysis: Variable Specific Heat

$$s_2 - s_1 = s^o_2 - s^o_1 = R \ln \frac{P_2}{P_1}$$

where $s^o$ is the “absolute” entropy relative the absolute zero given by:

$$s^o = \int_{T_1}^{T_2} C_p(T) \frac{dT}{T}$$

Values of $s^o$ can be found in Table A-17.
Isentropic Process of Ideal Gases (Approximate Analysis)
An isentropic process is a process where no entropy change occurs, i.e.: \( ds = 0 \) (or, \( s_2 = s_1 \)).
For an isentropic process, the equations in the approximate analysis above can be written as:

\[
0 = C_{v,av} \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \rightarrow \ln \frac{T_2}{T_1} = \ln \left( \frac{v_1}{v_2} \right)^{R/C_v} \rightarrow \left( \frac{T_2}{T_1} \right)_{s \text{-const}} = \left( \frac{v_1}{v_2} \right)^{k-1} \rightarrow \left( \frac{P_2}{P_1} \right)_{s \text{-const}} = \left( \frac{v_1}{v_2} \right)^k
\]

\[
0 = C_{p,av} \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \rightarrow \ln \frac{T_2}{T_1} = \ln \left( \frac{P_2}{P_1} \right)^{R/C_p} \rightarrow \left( \frac{T_2}{T_1} \right)_{s \text{-const}} = \left( \frac{P_2}{P_1} \right)^{k-1} \rightarrow \left( \frac{\rho_2}{\rho_1} \right)_{s \text{-const}} = \left( \frac{v_1}{v_2} \right)^k
\]

This also means that:
\( T_s^{k-1} = \text{constant} \)
\( TP^{(k-1)/k} = \text{constant} \)
\( P \nu^k = \text{constant} \)

Isentropic Process of Ideal Gases (Exact Analysis)

- We have already seen that: \( s_2 - s_1 = s_2^o - s_1^o - R \ln \frac{P_2}{P_1} \)

- When a process is isentropic, \( s_2 - s_1 = 0 \), and the above equation becomes:

\[
0 = s_2^o - s_1^o - R \ln \frac{P_2}{P_1} \rightarrow \frac{P_2}{P_1} = \exp \left( \frac{s_2^o - s_1^o}{R} \right) \rightarrow \frac{P_2}{P_1} = \exp \left( \frac{s_2^o}{R} \right) \exp \left( \frac{s_1^o}{R} \right)
\]

- The quantity \( (s^o/R) \) is defined as the relative pressure \( P_r \). Therefore, we can write:

\[
\left( \frac{P_2}{P_1} \right)_{s \text{-const}} = \frac{P_{r_2}}{P_{r_1}}
\]

- Let us now write the ideal gas equation of state for the isentropic process 1-2:

\[
\frac{P_1 \nu_1}{T_1} = \frac{P_2 \nu_2}{T_2} \rightarrow \frac{\nu_2}{\nu_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r_1}}{P_{r_2}} = \frac{T_2}{P_{r_2}} \frac{P_{r_1}}{P_{r_1}}
\]

- The quantity \( (T/P_r) \) is a function of temperature only and is defined as relative specific volume \( \nu_r \). Therefore, we can write:

\[
\left( \frac{\nu_2}{\nu_1} \right)_{s \text{-const}} = \frac{\nu_{r_2}}{\nu_{r_1}}
\]

- The quantities, \( P_r \) and \( \nu_r \), are found in Table A-17

Entropy Change in the Surroundings

- So far, we have discussed entropy change within the system.
- Likewise, the surroundings can experience entropy change, and to calculate this change, we need to return to the definition of entropy:

\[
\Delta S = \int_{1}^{2} \left( \frac{\partial Q}{T} \right)_{\text{int rev}}
\]

- Since the temperature of the surroundings is constant, the equation can be simplified to: \( \Delta S = S_2 - S_1 = \frac{Q}{T_{surr}} \)
Reversible Steady-Flow Work (Section 7-10)

- We had seen that the moving boundary work for a closed system is given by:
  \[ W_b = \int P \, dV \]
- For a steady-flow device, this expression is not valid.
- To express the work associated with a reversible steady-flow device, we need to write the first law of thermodynamics as follows:
  \[ \delta q_{rev} - \delta w_{rev} = dh + dke + dpe \]
  \[ \Rightarrow \quad \delta q_{rev} = T \, ds \rightarrow \delta q_{rev} = dh - v \, dP \]
  \[ \Rightarrow - \delta w_{rev} = v \, dP + dke + dpe \quad \rightarrow \quad w_{rev} = -\int v \, dP - \Delta ke - \Delta pe \]
- If the kinetic energy and potential energy changes are negligible, reversible work becomes:
  \[ w_{rev} = -\int v \, dP \]
- For incompressible fluids (i.e. \( v \approx \text{constant} \))
  \[ w_{rev} = -v \left( P_2 - P_1 \right) - \Delta ke - \Delta pe \]
- This means that the larger the specific volume, the larger the reversible work produced or consumed by the steady-flow device.

**STEADY-FLOW DEVICES DELIVER THE MOST AND CONSUME THE LEAST WORK WHEN THE PROCESS IS REVERSIBLE**

Minimizing the Compressor Work (Section 7-11)

- Compressor work is work input. Therefore, it is desirable to minimize it.
- The closer we get to a reversible compression process, the less work the compressor consumes.

Ideas for minimizing compressor work
1. Reduce friction, etc to make the process closer to being internally reversible
2. Keep the specific volume of the gas as small as possible during the compression process
   \[ \Rightarrow \quad \text{To keep (v) small, temperature has to be kept low} \]
   \[ \Rightarrow \quad \text{Cooling is needed} \]

Isentropic Efficiencies of Steady-Flow Devices (Section 7-12)

- Many steady-flow devices are intended to operate under adiabatic conditions
- The model process should be adiabatic AND should involve no irreversibilities
  \[ \Rightarrow \quad \text{The model process should be an isentropic process} \]

Isentropic (Adiabatic) Efficiency
A measure of the deviation of actual processes from the corresponding idealized ones

Isentropic Efficiency of Turbines
The ratio of the actual work output of the turbine to the work output that would be achieved if the process between the inlet state and the exit pressure were isentropic:
\[ \eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \approx \frac{h_1 - h_{2s}}{h_l - h_{2s}} \]

**Isentropic Efficiency of Compressors**

The ratio of the work input required to raise the pressure of a gas to a specified value in an isentropic manner to the actual work input:

\[ \eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \approx \frac{h_{2s} - h_l}{h_{2a} - h_l} \]

**Isentropic Efficiency of Pumps**

The ratio of the work input required to raise the pressure of a liquid to a specified value in an isentropic manner to the actual work input:

\[ \eta_P = \frac{\text{Isentropic pump work}}{\text{Actual pump work}} = \frac{w_s}{w_a} \approx \frac{h_{2s} - h_l}{h_{2a} - h_l} \]

For pumps, changes in potential and kinetic energies are negligible. Therefore, the isentropic efficiency of pumps can be written as:

\[ \eta_P = \frac{w_s}{w_a} = \frac{\sqrt{P_s - P_i}}{h_{2a} - h_l} \]

**Isentropic Efficiency of Nozzles**

The ratio of the actual kinetic energy of the fluid at the nozzle exit to the kinetic energy value at the exit of an isentropic nozzle for the same inlet state and exit pressure:

\[ \eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{\frac{\nu^2}{2}}{\frac{\nu_{2s}^2}{2}} \]

**Entropy Balance (Section 7-13)**

\[ \left( \text{Total entropy entering} \right) - \left( \text{Total entropy leaving} \right) + \left( \text{Total entropy generated} \right) = \left( \text{Change in the total entropy of the system} \right) \]

or: \[ S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}} \]

or: \[ \dot{S}_{\text{in}} - \dot{S}_{\text{out}} + \dot{S}_{\text{gen}} = \Delta \dot{S}_{\text{system}} \]

Rate of net entropy transfer by heat and mass \[ \text{Rate of entropy generation} \]

Rate of change in entropy
Entropy Balance for Closed Systems

Closed system: \[ \sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \]

Adiabatic closed system: \[ S_{\text{gen}} = \Delta S_{\text{adiabatic system}} \]

System + Surroundings: \[ S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \]

Entropy Balance for Open Systems (Control Volumes)

Control Volume: \[
\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = \left(S_2 - S_1 \right)_{\text{CV}}
\]
\[
\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = \Delta \dot{S}_{\text{CV}}
\]

Steady Flow: \[
\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = 0
\]
\[
\rightarrow \dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}
\]

Steady Flow, Single-Stream: \[
\dot{S}_{\text{gen}} = \dot{m}_e (s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}
\]

Steady-Flow, Single-Stream, Adiabatic: \[
\dot{S}_{\text{gen}} = \dot{m}_e (s_e - s_i) \]