Assignment #2

1. Show that for any constants \( a \in \mathbb{R}, \ k \in \mathbb{Z}^+ \) (positive integer), then
\[
(n + a)^k = \Theta(n^k).
\]

2. If \( f(n) = n^{\log_5 m} \) and \( g(n) = m^{\log_2 n} \), indicate which of these relations are true and prove your answers: \( f(n) = O(g(n)), f(n) = \Omega(g(n)), \) and \( f(n) = \Theta(g(n)) \).

3. Consider the following list of functions and arrange them in ascending (increasing) order of growth rate.
\[
\begin{align*}
  f_1(n) &= 10^n \\
  f_2(n) &= n^{\frac{1}{3}} \\
  f_3(n) &= n^n \\
  f_4(n) &= n \log_2 n \\
  f_5(n) &= 2^{\sqrt{\log n}}.
\end{align*}
\]

4. Show that
\[
\sum_{k=1}^{n} \frac{1}{k} = O(\log n).
\]

5. Suppose that an algorithm \( A \) runs in worst-case time \( f(n) \) and that algorithm \( B \) runs in worst-case time \( g(n) \). Is \( A \) faster than \( B \) for all \( n \) greater than some \( n_0 \) if \( g(n) = \Omega(f(n) \log n) \)? Answer either yes, no, or can’t tell and explain why.

Reading Assignment (Introduction to Algorithms, 2/e).
- The Rabin-Karp string matching algorithm (Section 32.2, pp. 911-5).
- Heapsort (Sections 6.1-6.4, pp. 127-37).