KING SAUD UNIVERSITY
College of Science/Department of Mathematics
(Math 151) Discrete Mathematics
Summer Semester 2009/2010

First Internal Exam
Date: 19/08/1431  Time: 07:00 P.M – 09:00 P.M  Time allowed: 2 Hours

STUDENT NAME (IN ENGLISH)  
Registration Number  
Lecture Time

- There are 8 multiple choice questions in part A and 4 questions in part B. The maximum score is 35 marks.
- Please do not forget to put your name and registration number on your paper.

Put your answers in the following table, please.

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<th>QUESTION</th>
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<tr>
<td>ANSWER</td>
<td>A</td>
<td>B</td>
<td>D</td>
<td>A</td>
<td>C</td>
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**PART - A**

Q1. The proposition \( [(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r) \) is

A. Tautology  
B. Contradiction  
C. Contingency  
D. None of above

Q2. The proposition \( \neg(p \rightarrow (q \rightarrow r)) \) is logically equivalent to

A. \( (p \rightarrow q) \lor r \)  
B. \( q \rightarrow (p \lor r) \)  
C. \( (r \rightarrow p) \rightarrow q \)  
D. \( p \leftrightarrow (q \land r) \)
Q3. The contrapositive of the proposition $p \to (q \lor r)$ is equivalent to

A. $\neg p \to \neg(q \lor r)$
B. $(q \lor r) \to p$
C. $(\neg q \lor \neg r) \to \neg p$
D. $(\neg q \land \neg r) \to \neg p$

Q4. The following argument $p \to q, q \to r; p \to r$ is

A. Valid
B. Invalid
C. Tautology
D. Contingency

Q5. If you want to give a proof by contradiction for the proposition $p \to q$ then you suppose first that

A. $p$ true and $q$ true.
B. $p$ false and $q$ true.
C. $p$ true and $q$ false.
D. $p$ false and $q$ false.

Q6. If $R$ is a relation from a set $A = \{1,2\}$ to a set $B = \{3,4,5\}$ and $S$ is a relation from $B$ to a set $C = \{6,7\}$ and the representing matrices are $M_R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$, $M_S = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ respectively then $S \times R$ is the relation:

A. $A \times C$
B. $\{(1,6),(2,7)\}$
C. $\{(6,1),(7,2)\}$
D. $\{(2,6),(1,7)\}$

Q7. If $R = \{(1,1),(1,2),(2,2),(2,1),(3,3),(3,4),(4,3)\}$ be a relation on $A = \{1,2,3,4\}$ then $R$ is

A. Reflexive and symmetric
B. Reflexive and not transitive
C. Symmetric and transitive
D. Symmetric and not transitive

Q8. If $S$ be a relation defined on the set of positive integers $\mathbb{N}$ as: $x\mathrel{S}y \iff xy > 10$ then $S$ is

A. Symmetric and transitive
B. Symmetric and not transitive
C. Not symmetric and transitive
D. Not symmetric and not transitive.
PART -B

Q1. Without using truth tables, determine whether the following arguments is valid or invalid

\[ \begin{align*}
- p \lor q \\
p \rightarrow (r \land s) \\
s \rightarrow q \\
\therefore q \lor r
\end{align*} \]

We suppose that \( q \) is false and \( r \) is also false. Then the conclusion \( q \lor r \) is false.

If we take \( p \) false and \( s \) false then all premises:

\[ \neg p \lor q \]; \( p \rightarrow (r \land s) \); \( s \rightarrow q \) are true.

Thus this argument form is invalid.

Q2. Using a proof by way of contraposition, show that for two real numbers \( x \) and \( y \), if \( 2x - 3y > 12 \) then \( x > 3 \) or \( y < -2 \).

We put \( p : 2x - 3y > 12 \)

\( q : x > 3 \) or \( y < -2 \)

By contrapositive proof, we suppose that \( \neg q \) is true. It means \( \neg q : x \leq 3 \) and \( y \geq -2 \) and we will show that \( \neg p \) \( (2x - 3y \leq 12) \) is true.

As \( x \leq 3 \), \( 2x \leq 6 \) (1)

and \( y \geq -2 \) then \(-3y \leq 6 \) (2)

(1) + (2) we get \( 2x - 3y \leq 12 \).
Q3. Use mathematical induction to prove that for all integers \( n \geq 1 \), \( 2^n - 1 \) is divisible by 7.

We put \( P(n) : \frac{7}{2^n - 1} \text{ for } n \geq 1 \).

- Base step: \( n = 1 \), \( 2^1 - 1 = 1 \).

\( P(1) \) is true.

- Inductive step: Let \( k \geq 1 \).

We suppose that \( P(k) \) is true, (that means \( \frac{7}{2^k - 1} \)).

We will show that \( P(k+1) \) remains true (\( \frac{7}{2^{k+1} - 1} \)).

\( \frac{2^{k+1} - 1}{2^k - 1} = \frac{2^k \cdot 2 - 1}{2^k - 1} = 2 \).

As \( \frac{7}{2^k - 1} \) is true, there exist \( m \in \mathbb{N} \) such that \( 2^k - 1 = 7m \).

So \( 2^k = 7m + 1 \). It follows that \( 2^{k+1} - 1 = (7m + 1) \cdot 2 - 1 \).

Therefore, \( \frac{7}{2^{k+1} - 1} \) is true.

\( \frac{7}{2^{k+1} - 1} = \frac{7}{7m + 2} \cdot \frac{1}{7} \).

Conclusion: \( \forall n \geq 1 \), \( \frac{7}{2^n - 1} \) is true.

Q4. Using a proof by contradiction and well-ordering property to show that \( 2^n > 3n + 18 \) for all \( n \geq 6, n \in \mathbb{N} \).

We suppose that there exists \( n_0 \geq 6 \) such that

\( 2^{n_0} \leq 3n_0 + 18 \).

Now, let \( S = \{ n \geq 6, n \in \mathbb{N} / 2^n \leq 3n + 18 \} \).

- \( S \) is nonempty (\( n_0 \in S \)).

By well-ordering property, there exists a least element in \( S \).

We denote it \( m \geq 6 \) that satisfies \( 2^m \leq 3m + 18 \). (1)

As \( (m-1) < S \), then \( 2^{m-1} > 3(m-1) + 18 \). (2)

Multiplying (2) by two, we get \( 2^m > 6(m-1) + 36 \). (3)

Comparing (1) and (2), we have \( 3m + 18 > 6m - 15 \)

It follows that \( 3m + 12 > 6m - 15 \)

It is contradiction (\( m \geq 6 \)).

Conclusion: \( \forall n \geq 6 \), \( 2^n > 3n + 18 \).