5. Divide and conquer

Computer Science Department,
College of Computer and Information Sciences,
King Saud University.

CSC 311: Design and Analysis of Algorithms
Dr. Waleed Alsalih

5.4 Divide-and-conquer sorting algorithms

Merge sort

Merge sort is a divide-and-conquer sorting algorithm which is based on the idea of splitting the elements to be sorted into two parts of almost the same size, sorting each part, and then merging the two parts into one sorted list.

Algorithm 1: Merge Sort.

\begin{algorithm}
\begin{algorithmic}
    \enspace \textbf{Algorithm MergeSort}($A$, \textit{first}, \textit{last})
    \begin{algorithmic}
        \If{$\textit{first} == \textit{last}$}
        \State \textbf{return} $A[\textit{first}..\textit{last}]$;
        \EndIf
        \Else
        \State \textbf{return} \text{Merge(MergeSort($A$, $\textit{first}, \left\lceil \frac{\textit{first} + \textit{last}}{2} \right\rceil$) - 1,}
        \State \text{MergeSort($A$, $\left\lfloor \frac{\textit{first} + \textit{last}}{2} \right\rfloor$, \textit{last})});
        \EndIf
    \end{algorithmic}
\end{algorithm}
\end{algorithm}

Basic operation count:

$M(n) = \begin{cases} 
0 & \text{for } n = 1. \\
2M\left(\frac{n}{2}\right) + n & \text{for } n > 1. 
\end{cases}$

\footnote{This is a summary of the material we cover from the textbook: \textit{Introduction to the Design & Analysis of Algorithms}, A. Levitin, Second Edition, Pearson Addison-Wesley, 2006.}
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**Algorithm 2: Merge.**

Algorithm Merge\((B[0..p−1];C[0..q−1])\)

Array \(A[0..p+q−1]\);

\(i:=0;\)

\(j:=0;\)

\(k:=0;\)

while \(i < p \) and \(j < q\) do

if \(B[i] \leq C[j]\) then

\(A[k]:=B[i];\)

\(i:=i + 1;\)

else

\(A[k]:=C[j];\)

\(j:=j + 1;\)

end

\(k:=k + 1;\)

end

if \(i=p\) then

copy \(C[j..q−1]\) to \(A[k..p+q−1]\);

else

copy \(B[i..p−1]\) to \(A[k..p+q−1]\)

end

return \(A[0..p+q−1]\);

Exercise:

Solve this recurrence relation and show that \(M(n) = n \log n\).

You can do this by backward substitutions or by mathematical induction, and you can assume that \(n = 2^k\), for some positive integer \(k\).
Quick sort

Quick sort is another divide-and-conquer algorithm for sorting a list of elements. Merge sort has the following steps:

1. Pick any element and call it the pivot.
2. Rearrange the list so that all elements with values less than that of the pivot come before the pivot and all those with values larger than the pivot come after the pivot.
3. Recursively sort the sub-list before the pivot and the sub-list after the pivot.

Algorithm 3: Quick Sort.

Algorithm QuickSort(A[], l, r)
if l < r then
    s:=Partition(A[], l, r);
    QuickSort(A[], l, s - 1);
    QuickSort(A[], s + 1, r);
end
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Algorithm 4: Partition.

Algorithm Partition($A[]$, $l$, $r$) 
\[ p := A[l]; \]
\[ i := l + 1; \]
\[ j := r; \]
while $i < j$ do
  while $A[i] \leq p$ and $i < j$ do
    $i := i + 1;$
  end
  while $A[j] > p$ and $i < j$ do
    $j := j - 1;$
  end
  swap ($A, i, j);$
end
if $A[i] \leq p$ then
  swap ($A, i, l);$
  return $i;$
else
  swap ($A, i - 1, l);$
  return $i - 1;$
end