Flux-limiting solution techniques for simulation of reaction–diffusion–convection system

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Abstract

The aim of this work is to analyze the use of a number of flux-limiters to simulate numerically the behavior of a homogeneous tubular reactor which exhibits steep moving fronts. The strength and limitations of five different flux-limiters are examined for different test cases. All flux-limiters are found successful in capturing the steep concentration profiles. The simulations show that the minmod and MUSCL flux-limiter are more appropriate for the convection–diffusion case. On the other hand, the superbee limiter shows the best performance in capturing discontinuities in the convection–diffusion–reaction case.

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1. Introduction

Systems in which chemical reactions are coupled to diffusion and convection transports arise in a wide range of chemical engineering applications and require quite sophisticated mathematical tools for their rigorous study. Conservation laws describing such systems support solutions that have discontinuities, shocks and moving fronts. Unless special treatment is taken, the numerical solutions will lead to excessive dissipation, incorrect phase speeds, spurious oscillations, see [1] for an extensive comparison of many numerical methods applied to solve simple linear and nonlinear advection and Euler equations.

The preservation of monotonicity properties—like positivity—is often essential for numerical schemes to approximate nonsmooth solutions in a qualitatively correct manner. It is, however, very difficult to design a smoothing numerical scheme that does not degrade the overall accuracy of the solution. Typically, the choice is between having shocks that are smeared and having oscillations near the shocks [2]. Applying first-order

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finite difference methods results in monotonic and stable solutions but the solution become smeared out and inaccurate. On the other hand high-order accurate difference methods are less dissipative which are prone to numerical oscillations. This has led to the development of different high-resolution technique. These include the Godunov methods, flux limiting, and slope limiting methods.

One common scheme to reduce nonphysical oscillations resulted in convective-dominant problems is to use “artificial diffusion” to smooth out the discontinuities. The numerical diffusion should be tuned so that it is large enough near discontinuities and small enough elsewhere to maintain high-order accuracy. Based on this approach different methods are developed. Among these methods is the flux-corrected transport method of Boris and Book [3] is one of the earliest flux-limiter methods. The general idea behind designing a suitable flux-limiting method is to cast it in a flux form, and make it a combination of a low-order method and a high-order antidiffusive correction. One of the most important properties for the designed flux-limiter is to be a total-variation diminishing. This means that as the time progress the numerical scheme generate solutions with nonincreasing total variation which prevent nonoscillatory behavior near steep profiles.

The aim of the present work is to simulate dynamic behaviors of a reaction–diffusion–convection system using different high-resolution flux-limiter schemes. In our previous work we have studied different traditional and advanced finite difference schemes for the convection-reaction case [4]. We have shown that the high-resolution techniques such as the flux-corrected transport must be used in order to obtain nonoscillatory and accurate solutions. In this work we study different flux-limiters that can provide stable and accurate solutions for the convection–diffusion–reaction system, assuming that the convective mass transfer is predominant compared to diffusive mass transfer. Two different cases will be examined, in the first case, we neglect the source term which make our problem is equivalent to a single linear PDE while in the second case we consider the reaction which results in a system of three nonlinear PDEs.
The relevant partial differential equation is given as follows:

\[
\frac{\partial U}{\partial t} + V \frac{\partial U}{\partial x} - D_{xx} \frac{\partial^2 U}{\partial x^2} = f(U)
\]  

(1)

\(U(x,t)\) represents the concentration of the species in both space and time. \(V_x\) is the velocity which advects the species concentration and \(D_{xx}\) is the diffusivity. It is assumed that the velocity and diffusivity are positives and constants. The centered finite volume discretization method is applied to Eq. (1). Node \(i\) is typically used to represent a center point of the control volume. The control volume boundaries are located at positions \(x_{i+1/2}\) and \(x_{i-1/2}\). The discretized form is obtained by integrating Eq. (1) in space.

\[
\int_{x_{i-1/2}}^{x_{i+1/2}} \left[ \frac{\partial U}{\partial t} + V \frac{\partial U}{\partial x} - D_{xx} \frac{\partial^2 U}{\partial x^2} \right] dx = \int_{x_{i-1/2}}^{x_{i+1/2}} f(U) \, dx
\]  

(2)

The mass transfer flux is given by the form

\[
J = VU - D \frac{\partial U}{\partial x}
\]  

(3)

Now we can use the flux definition and the Mean-Value Theorem to obtain the following discretized form:

\[
\Delta x_i \frac{\partial U_i}{\partial t} + [J_{i+1/2} - J_{i-1/2}] = \Delta x_i F_i
\]  

(4)

where \(F_i\) is the integral of the source term over the control volume. In this work the time marching is performed using the fully explicit method which allows to rewrite Eq. (4) as

\[
\Delta x_i [U_{i+1}^{n+1} - U_i^n] + \Delta t [J_{i+1/2}^{n+1} - J_{i-1/2}^{n+1}] = \Delta t \Delta x_i F_i
\]  

(5)

To estimate the fluxes at the control volume faces given by Eq. (3) we make use of the following approximations of the derivatives used for the diffusion term;

\[
\frac{\partial U}{\partial x}_{i+1/2} = \frac{U_{i+1} - U_i}{\Delta x_i}, \quad \frac{\partial U}{\partial x}_{i-1/2} = \frac{U_i - U_{i-1}}{\Delta x_i}
\]  

(6)

The convection term is approximated using the flux-limiter method

\[
U_{i+1/2} = U_i + \frac{\sigma}{2} (U_{i+1} - U_i)
\]

\[
U_{i-1/2} = U_i - \frac{\sigma}{2} (U_i - U_{i-1})
\]  

(7)

Thus, the flux takes the following definition

\[
J_{i+1/2} = V \left[ U_i + \frac{\sigma}{2} (U_{i+1} - U_i) \right] - D \frac{U_{i+1} - U_i}{\Delta x}
\]

\[
J_{i-1/2} = V \left[ U_{i-1} + \frac{\sigma}{2} (U_i - U_{i-1}) \right] - D \frac{U_i - U_{i-1}}{\Delta x}
\]  

(8)

where \(\sigma\) is a flux-limiter. The flux-limiter is calculated using a sensor which is based on the consecutive gradient parameter. The consecutive gradient parameter is the gradient of the concentration of the upstream control volume face divided by the gradient of concentration of the downstream face:

\[
\theta = \frac{\frac{dU}{dx} |_{i-1/2}}{\frac{dU}{dx} |_{i+1/2}}
\]  

(9)

For the linear case with uniform grid spacing, the sensor can be rewritten as

\[
\theta = \frac{U_i - U_{i-1}}{U_{i+1} - U_i}
\]  

(10)
This sensor can be thought as a smoothness indicator near the cell interface. If the data is smooth we expect the sensor to be approximately 1 while near discontinuities we expect the ratio to be far away from 1.

The flux-limiter is introduced to prevent the unwanted oscillations and to ensure that the results are physically realistic. It is mainly used to limit the amplitude of the original higher-order difference scheme around steep gradients by multiplying the flux terms with the flux-limiters. If the flux-limiter is close to zero, the first-order upwind method is applied in the spatial domain while if the flux-limiter approaches one, a second-order averaging scheme is used. The limits on $\sigma(\theta)$ are indicated by the shaded region in Fig. 1. This region is said to be TVD (total variance diminishing). A further constraint on the limiter function is the preferred order of accuracy of the resulting scheme. If the scheme can be shown to be an interpolation of the Lax–Wendrof method ($\sigma(\theta) = 1$) and Warming and Beam method (which corresponds to $\sigma(\theta) = \theta$) then the scheme will be of second-order accuracy. $\sigma(\theta)$ must therefore fall in the region indicated in Fig. 2 to be second-order accu-

![Fig. 1. The bounds on the limiter function for the scheme to be TVD.](image1)

![Fig. 2. Special limiter functions: Lax–Wendroff flux $\sigma(\theta) = 1$ and the Warming-Beam flux $\sigma(\theta) = \theta$.](image2)
rate. Finally, if \( \sigma(\theta) \) is chosen to fall in the intersection of these two regions, second order and TVD, then the resulting scheme will be both TVD and second-order accurate.

There are many possible limiters but the most widely used are:

1. The van Leer flux-limiter is given by the formula [5]:
   \[
   \sigma(\theta) = \frac{\theta + |\theta|}{1 + \theta}
   \]

2. The second limiter is the minmod limiter which is bounded between 0 and 1 and takes the form [6]:
   \[
   \sigma(\theta) = \max[0, \min(\theta, 1)]
   \]

3. Another form for the flux-limiter is the superbee limiter due to Roe [7]:
   \[
   \sigma(\theta) = \max\left[0, \min\left(\frac{2\theta}{3} + \frac{1}{3}, 2\right)\right]
   \]

4. Koren limiter [8] and is defined as
   \[
   \sigma(\theta) = \max\left[0, \min\left(\frac{2\theta}{3} + \frac{1}{3}, 2\right)\right]
   \]

5. MUSCL (monotone upstream scheme for conservations laws) limiter defined [9]:
   \[
   \sigma(\theta) = \max\left[0, \min\left(\frac{2\theta}{3} + \frac{1}{3}, 2\right)\right]
   \]

### 3. Test problem

The test problem is a chemical tubular reactor chemical species flow from the inlet to the outlet in one pass. The chemical reaction model consists of a cubic autocatalytic reaction where the autocatalyst is assumed to undergo a mutation process giving another form that can also undergo an autocatalytic reaction, thus competing with the original autocatalyst [10]. The autocatalytic reaction consists of three reagents (substrate \( A \), autocatalyst \( B \), and mutant \( C \)) reacting according to the following reaction scheme:

- Replication of \( B \) : \( A + 2B \xrightarrow{k_1} 3B \)
- Death of \( B \) : \( B \xrightarrow{k_2} P_1 \)
- Mutation of \( B \) into \( C \) : \( A + 2B \xrightarrow{\eta k_1} 2C + B \)
- Replication of \( C \) : \( A + 2C \xrightarrow{k_3} 3C \)
- Death of \( C \) : \( C \xrightarrow{k_{1/2}} P_2 \)

where \( k_1, k_2, \eta, \) and \( \lambda \) are a set of constants defining the kinetics of the reaction and mutation efficiency. If we assume dispersion to be unidirectional along the reactor axis, mass balance equations for species \( A, B, \) and \( C \) can be written in a dimensionless form as follows:

\[
\frac{\partial U_1}{\partial T} + V \frac{\partial U_1}{\partial X} = D_1 \frac{\partial^2 U_1}{\partial X^2} + (1 + \eta)(1 - U_1)U_2^2 + \lambda(1 - U_1)U_3^2
\]

\[
\frac{\partial U_2}{\partial T} + V \frac{\partial U_2}{\partial X} = D_2 \frac{\partial^2 U_2}{\partial X^2} + (1 - \eta)(1 - U_1)U_2^2 - \gamma U_2
\]

\[
\frac{\partial U_3}{\partial T} + V \frac{\partial U_3}{\partial X} = D_3 \frac{\partial^2 U_3}{\partial X^2} + \lambda(1 - U_1)U_2^2 + 2\eta(1 - U_1)U_2^2 - \frac{\gamma}{\lambda} U_3
\]
where

\[ U_1 = \frac{u_1 - u_f}{u_f}, \quad U_2 = \frac{u_2}{u_f}, \quad U_3 = \frac{u_3}{u_f}, \]

\[ X = \frac{x}{L}, \quad T = k_1u_f^2t, \]

\[ D_i = \frac{D_i}{k_1u_f^2L^2}, \quad D_2 = \frac{D_2}{k_1u_f^2L^2}, \quad D_3 = \frac{D_3}{k_1u_f^2L^2}, \quad V = \frac{v_1}{k_1u_f^2L}, \quad \gamma = \frac{k_2}{k_1u_f^2} \]

\( u_f \) is the feed substrate concentration, \( U_i (i = 1, 2, 3) \) is the dimensionless concentration of species \( i \), \( X \) is the dimensionless reactor length, \( T \) is the dimensionless time, \( D_i (i = 1, 2, 3) \) is the dimensionless diffusion parameter, \( V \) is the dimensionless convection parameter, and \( \gamma \) is a dimensionless kinetic parameter.

The system equations was solved using the following boundary conditions at the reactor inlet and outlet, respectively:

\[ U_i = U_{if}; \quad X = 0 \quad \text{(Dirichlet-boundary condition)} \]  
\[ \frac{\partial U_i}{\partial X} = 0; \quad X = 1 \quad \text{(Reactor assumed to be long enough)} \]

Eqs. (11)–(13) with boundary conditions (14) and (15) constitute a complete set of equations for the reaction–diffusion–convection system in the tubular reactor as a function of the diffusion coefficients \( (D_i) \), convection parameter \( (V) \), and kinetic parameters \( (\eta, \lambda, \gamma) \). The assumption of the Dirichlet condition is in accordance with the large values of the estimated Peclet number, defined as the ratio of convective flux to diffusion flux, viz., \( Pe_i = V/D_i \). That is, convective mass transfer is predominant compared to diffusive mass transfer.

4. Results and discussion

The simulation results correspond to the case of an initially clean system for which the concentrations of all species inside the reactor are initially set to zero which give the following initial conditions:

\[ U_1(X, 0) = 1, \quad U_2(X, 0) = 0, \quad U_3(X, 0) = 0 \]
Two cases are examined; in the first we neglect the reaction source term and in the second we consider the reaction term with the following kinetic parameters:

$$\eta = 6.5, \quad \lambda = 2.0, \quad \gamma = 0.025$$

For all runs, the inlet feed conditions are fixed as

$$U_{1f} = 0, \quad U_{2f} = 1.0, \quad U_{3f} = 0$$

In all simulation the reactant concentration at the reactor center ($X = 0.5$) will be plotted. The integration time consists of 5000 time steps of 0.002s, which produce simulation results for the time period $[0, 1]$.

Fig. 4. Dynamic simulation of the reactant ($U_1$) at the reactor center ($X = 0.5$) for the convection-diffusion case using $N = 50, (a) Pe = 100, (b) Pe = 1000, (c) Pe = 10000.$
4.1. Convection–diffusion case

The model becomes equivalent to a single convection–diffusion PDE, and the exact solution can be shown analytically to be

\[ U(x, t) = \frac{U_i}{2} \left( \text{Erfc} \left( \frac{x - H t}{2\sqrt{D t}} \right) + e^{\alpha^2} \text{Erfc} \left( \frac{x + H t}{2\sqrt{D t}} \right) \right) \]

Fig. 5. Dynamic simulation of the reactant \((U_1)\) at the reactor center \((X = 0.5)\) for the convection–diffusion case using \(N = 200\), (a) \(Pe = 100\), (b) \(Pe = 1000\), (c) \(Pe = 10000\).
For our problem a precise solution can be obtained using the QUICK method with large grid number. The Quick scheme is a third-order finite difference scheme derived by Leonard [11]. It avoids the instability problem associated with the central differencing of the convection term and it greatly reduces the numerical damping exhibited when using first-order upwind method. For the case $Pe = 10000$, the dynamic behavior of the reactant $A$ and autocatalyst $B$ at the reactor center ($X = 0.5$) is obtained using QUICK method with 100 grids. Fig. 3 compares this solution with the exact solution. For the reactant $A$ and the autocatalyst $B$, the QUICK scheme exhibits unacceptable numerical oscillations behind and after the front which means that this scheme is

<table>
<thead>
<tr>
<th>Method</th>
<th>$Pe = 100$</th>
<th>$Pe = 1000$</th>
<th>$Pe = 10000$</th>
<th>$Pe = 100$</th>
<th>$Pe = 1000$</th>
<th>$Pe = 10000$</th>
<th>$Pe = 100$</th>
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<th>$Pe = 10000$</th>
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<tbody>
<tr>
<td>Van Leer</td>
<td>4.66</td>
<td>3.89</td>
<td>3.77</td>
<td>0.10</td>
<td>0.275</td>
<td>0.525</td>
<td>0.37</td>
<td>0.51</td>
<td>0.56</td>
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<tr>
<td>Superbee</td>
<td>4.72</td>
<td>3.98</td>
<td>3.81</td>
<td>0.098</td>
<td>0.246</td>
<td>0.506</td>
<td>0.22</td>
<td>0.38</td>
<td>0.40</td>
</tr>
<tr>
<td>Minmod</td>
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<td>3.88</td>
<td>3.72</td>
<td>0.11</td>
<td>0.268</td>
<td>0.516</td>
<td>0.28</td>
<td>0.44</td>
<td>0.48</td>
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<td>3.87</td>
<td>3.71</td>
<td>0.11</td>
<td>0.268</td>
<td>0.517</td>
<td>0.26</td>
<td>0.37</td>
<td>0.40</td>
</tr>
<tr>
<td>Koren</td>
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<td>3.76</td>
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<td>0.534</td>
<td>0.30</td>
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Table 2
Comparison of different flux-limiters for $N = 200$

<table>
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<tr>
<th>Method</th>
<th>$Pe = 100$</th>
<th>$Pe = 1000$</th>
<th>$Pe = 10000$</th>
<th>$Pe = 100$</th>
<th>$Pe = 1000$</th>
<th>$Pe = 10000$</th>
<th>$Pe = 100$</th>
<th>$Pe = 1000$</th>
<th>$Pe = 10000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Leer</td>
<td>1.29</td>
<td>1.08</td>
<td>1.15</td>
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<td>0.074</td>
<td>0.243</td>
<td>3.98</td>
<td>4.56</td>
<td>4.94</td>
</tr>
<tr>
<td>Superbee</td>
<td>1.29</td>
<td>1.13</td>
<td>1.09</td>
<td>0.026</td>
<td>0.066</td>
<td>0.213</td>
<td>3.06</td>
<td>3.52</td>
<td>3.79</td>
</tr>
<tr>
<td>Minmod</td>
<td>1.28</td>
<td>1.05</td>
<td>0.95</td>
<td>0.027</td>
<td>0.074</td>
<td>0.224</td>
<td>4.94</td>
<td>5.71</td>
<td>6.07</td>
</tr>
<tr>
<td>MUSCL</td>
<td>1.28</td>
<td>1.05</td>
<td>0.93</td>
<td>0.027</td>
<td>0.074</td>
<td>0.224</td>
<td>3.45</td>
<td>3.96</td>
<td>4.32</td>
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<tr>
<td>Koren</td>
<td>1.29</td>
<td>1.11</td>
<td>1.22</td>
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<td>0.083</td>
<td>0.261</td>
<td>4.88</td>
<td>5.72</td>
<td>6.07</td>
</tr>
</tbody>
</table>

Fig. 6. Dynamic simulations of the reactant $U_1$ at the reactor center obtained using QUICK scheme for the convection–diffusion–reaction case.
not capable of ensuring the monotone solutions for species concentrations. We can improve the accuracy of the QUICK method with increasing the number of grids but it still gives negative results.

In the following we present the results of employing the flux-limiting techniques to simulate the problem. Fig. 4 shows the results of using five different flux-limiters for the cases $Pe = 100, 1000$ and $10000$ using 50 grids. The sharpness of the moving fronts increase with increasing the $Pe$ number and the numerical solutions become more complicated. For these cases all tested limiters schemes converge and capture the steep gradient without unphysical oscillations. All schemes show similar behavior and it is not clear which limiter scheme is the most accurate. It can be also seen that the accuracy of all limiters decrease with increasing the $Pe$ number and more nodes need to be added to increase the simulations accuracies. Fig. 5 exhibits the simulation results for the flux-limiter methods using 200 nodes. The accuracies of the flux-limiting methods are improved by increasing the grids number. Again, the numerical solutions for the low $Pe$ case are the most accurate. For

![Figure 4](image)

![Figure 5](image)

Fig. 7. Dynamic simulation of the reactant ($U_1$) at the reactor center ($X = 0.5$) for the convection–diffusion–reaction case using $N = 50$, (a) $Pe = 100$, (b) $Pe = 1000$, (c) $Pe = 10000$. 
the high Pe number case, the limiters schemes solutions are the least accurate which suggests using more nodes to enhance their accuracy.

The errors of the computed solutions are estimated by comparing them with the exact solution obtained by

\[
\text{Relative error\%} = \frac{\sum_j |U_j - U_{j,\text{exact}}|}{\sum_j |U_{j,\text{exact}}|} \times 100
\]

and

\[
\text{Maximum error\%} = \max_j |U_j - U_{j,\text{exact}}|
\]

Fig. 8. Dynamic simulation of the reactant (\(U_1\)) at the reactor center (\(X = 0.5\)) for the convection–diffusion–reaction case using \(N = 200\), (a) Pe = 100, (b) Pe = 1000, (c) Pe = 10000.
The errors of various flux-limiters methods are listed in Tables 1 and 2 for different Pe cases for $N = 50$ and 200 respectively. The computation times are also tabulated in the last three columns. The predictions of all schemes for the low grids number case are not very different giving around 4% relative error. Increasing the grids number to 200 nodes reduces the relative errors to around 1% but the CPU times increases significantly. Table 1 shows that for the low Pe case, where the solution is smoother, the sharp superbee limiter gives the maximum error while the minmod and MUSCL schemes show the best performances in terms of relative errors. When increasing the grids number (Table 2), one cannot notice big differences between all schemes for low Pe number case. When considering the maximum deviation from the exact solution (the maximum error) the Superbee limiter is the most accurate for all Pe values. All schemes finish the computations for 50 grids in less than 0.6 s while for $N = 200$, the computational time has increased to be more than 3 s. The superbee limiter is found to be the fastest scheme for simulating the sharp front of the convection-diffusion model.

4.2. Convection-diffusion-reaction case

There is no analytical solution for this case. The simulations are compared with the most accurate solutions obtained using QUICK scheme with large nodes number ($N > 1000$). The Quick scheme results are depicted in Fig. 6 for different Pe numbers. The moving fronts take different forms than the convection-diffusion case. The reactant conversion decreases due to the convective transport mechanism then it increases according the reaction mechanism. The sharpness of the fronts increases with increasing the Pe number. Fig. 7 shows the results of various flux-limiters using 50 nodes. The predictions of all methods are not so different but the solutions from Superbee are the most accurate. However, these plots show the need for more grid refinement in order to obtain better results. Fig. 8 shows the performances of the flux-limiters for smaller grid size ($N = 200$). The solutions accuracies are improved significantly.

Tables 3 and 4 report the errors for the numerical simulations of various flux-limiters using 50 and 200 nodes respectively. Both tables show that the closest agreement with the exact solution is obtained using the Superbee flux-limiter scheme especially for the steep profiles exhibited by the high Pe case. It can be seen that for the smoother case ($Pe = 100$) and $N = 200$, the accuracies obtained by all flux-limiters are similar. For $Pe = 10000$, the solutions from Koren and van Leer are the least accurate. The minmod and MUSCL predictions are very similar but the MUSCL consumes less computational time. For the low girds number case the

<table>
<thead>
<tr>
<th>Method</th>
<th>Relative error%</th>
<th>Maximum error</th>
<th>CPU time</th>
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<tbody>
<tr>
<td></td>
<td>Pe = 100</td>
<td>Pe = 1000</td>
<td>Pe = 10000</td>
</tr>
<tr>
<td>Van Leer</td>
<td>0.90</td>
<td>1.30</td>
<td>1.77</td>
</tr>
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<td>Superbee</td>
<td>0.73</td>
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<td>Minmod</td>
<td>0.88</td>
<td>1.22</td>
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</tr>
<tr>
<td>MUSCL</td>
<td>0.88</td>
<td>1.22</td>
<td>1.65</td>
</tr>
<tr>
<td>Koren</td>
<td>0.86</td>
<td>1.27</td>
<td>1.72</td>
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<tr>
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<tr>
<td></td>
<td>Pe = 100</td>
<td>Pe = 1000</td>
<td>Pe = 10000</td>
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<tr>
<td>Van Leer</td>
<td>0.21</td>
<td>0.24</td>
<td>0.36</td>
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<tr>
<td>Superbee</td>
<td>0.20</td>
<td>0.22</td>
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<tr>
<td>Koren</td>
<td>0.21</td>
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</tbody>
</table>
van Leer scheme and the MUSCL are the fastest while for $N = 200$, the superbee exhibits the fastest response. The Koren and minmod schemes consume more computational times for all runs.

We have found that including the reaction source term has resulted in improving the accuracies and the speed of the flux-limiter methods. For example, for $N = 200$, the superbee flux-limiter predicts 1–1.29% relative error for the convection–diffusion case (Table 2) while for the second case the superbee flux-limiter relative error is less than 0.29.

5. Conclusion

A comparative study, with focus on the accuracy, of five different flux-limiters: van Leer, superbee, minmod, MUSCL, Koren is made using a tubular reactor model operating with different $Pe$ number. The flux-limiters are tested and found successful in solving the cases with steep concentration profiles without giving negative concentrations. For the convection–diffusion case, the most accurate results are obtained using the minmod and the MUSCL flux-limiters especially for the smooth cases (low $Pe$ numbers). For the convection–diffusion–reaction case, the results are different and the closest agreement with the exact solution is obtained using the superbee flux-limiter scheme especially for the steep profiles exhibited by the high $Pe$ case. The superbee limiter is found to be the fastest scheme for simulating the sharp front of the model for all cases.

References