Question # 1(a) (6 Marks)
The cables AB and AC are attached to the top of the transmission tower as shown in the figure. If the tension in cable AB is 8 kN, determine

a) The required tension T in the cable AC such that the resultant (R) of the two cable tensions is along y-axis.

b) The magnitude of the resultant force R.

Solution
The angles α and β can be obtained from the above figure as
\[ \alpha = \tan^{-1} \left( \frac{40}{50} \right) = 38.7^\circ; \text{ and } \beta = \tan^{-1} \left( \frac{50}{30} \right) = 59^\circ \]
The angle γ = 180° − (α + β) = 180° − (38.7° + 59°) = 82.3°
Applying sine law on the second figure yields,
\[ \frac{T}{\sin \alpha} = \frac{R}{\sin \gamma} \Rightarrow \frac{T}{\sin 38.7^\circ} = \frac{R}{\sin 82.3^\circ} = \frac{8}{\sin 59^\circ} \]
\[ \Rightarrow \frac{T}{\sin 38.7^\circ} = \frac{8}{\sin 59^\circ} \Rightarrow T = 5.83 \text{ kN} \quad \text{Ans.} \]
Similarly, \[ \frac{R}{\sin 82.3^\circ} = \frac{8}{\sin 59^\circ} \Rightarrow R = 9.25 \text{ kN} \quad \text{Ans.} \]
Alternatively,
Note \( R_x = 0 \); and \( R_y = R \)
\[ R_x = \sum F_x = T \sin \beta - 8 \sin \alpha = T \sin 59^\circ - 8 \sin 38.7^\circ = 0 \]
\[ \Rightarrow T = 5.83 \text{ kN} \quad \text{Ans.} \]
\[ R_y = \sum F_y = T \cos \beta + 8 \cos \alpha = 5.83 \cos 59^\circ + 8 \cos 38.7^\circ \]
\[ \Rightarrow R_y = 9.25 \text{ kN} = R \quad \text{Ans.} \]
Question # 1(b) (2 Marks)

Replace the force-couple system at point $O$ by a single force. Specify the $y$-coordinate through which the line of action of this single force passes.

Solution

$$y_A = \frac{M}{R_x} = \frac{-1}{-4} = 0.25 \text{ m} \quad \text{Ans.}$$

Question # 1(c) (2 Marks)

Compute the moment of the two 10-kN forces about the
a) Point $O$; and
b) Point $A$.

Solution

The two 10-kN forces are forming a couple. As couple moment is independent of the moment centers, the moment produced by the two 10-kN forces about points $O$ and $A$ will be the same.

Therefore, $M_O = M_A = Fd = -10 \times 8 = -80 \text{ kN.m (CW)} \quad \text{Ans.}$
Question #2 (10 Marks)

For the force-system shown in the figure:

i. Replace the three forces and one couple by an equivalent force-couple system \((R, M)\) at point \(O\).

ii. Determine the direction of \(R\).

iii. Sketch the single resultant force \(R\) that represents the force-couple system and find its intersection with the \(x\)- and \(y\)-axes.

Solution

(i)

\[
\rightarrow R_x = \sum F_x = 2 \cos 70^\circ + 1.2 \cos \theta = 2 \cos 70^\circ + 1.2 \times (4/5) = 1.64 \text{ kN} \rightarrow
\]

\[
\uparrow R_y = \sum F_y = -5 + 2 \sin 70^\circ - 1.2 \sin \theta = -5 + 2 \sin 70^\circ - 1.2 \times (3/5) = -3.84 \text{ kN} \downarrow
\]

Therefore, \(R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1.64)^2 + (-3.84)^2} = 4.18 \text{ kN} \quad \text{Ans.}\)

\(CCW (+)M_O = -5 \times 0.25 - 2 \cos 70^\circ \times 0.3 + 2 \sin 70^\circ \times 1.0 - 1.2 \sin \theta \times 0.5 - \left(\frac{500}{1000}\right)\)

\[\Rightarrow M_O = -1.25 - 0.205 + 1.879 - 1.2 \times (3/5) \times 0.5 \times 0.5 = -0.44 \text{ kN.m (CW)} \quad \text{Ans.}\)

(ii)

\[
\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-3.84}{1.64}\right) = -66.9^\circ \quad \text{Ans.}\)

(iii)

\[|M_O| = Rd \Rightarrow d = \frac{|M_O|}{R} = \frac{0.44}{4.18} = 0.10 \text{ m} \]

\(x\)-intercept, \(x = \frac{M_O}{R_y} = \frac{-0.44}{-3.84} = 0.11 \text{ m} \quad \text{Ans.}\)

\(y\)-intercept, \(y = -\frac{M_O}{R_x} = -\frac{-0.44}{1.64} = 0.27 \text{ m} \quad \text{Ans.}\)

\[\text{Ans.}\]
Question # 3 (10 Marks)

A force of 5 kN is acting along the line BC as shown in the figure. Determine the following:

(i) The moment about point O ($M_O$).
(ii) The moment about line OA ($M_{OA}$).
(iii) The moment about line CD ($M_{CD}$).

![Diagram of the force and points O, A, B, and C]

Solution

The coordinates of points O, A, B and C are: O(0, 0, 0); A(1, 0, 2.5); B(1, 0, 0) and C(0, 3, 2.5)

(i) $\vec{M}_O = \vec{r}_{OB} \times \vec{F}$, where $\vec{r}_{OB} = \vec{i}$ and $\vec{F} = 5\vec{n}_{BC} = 5\left(\frac{-\vec{i} + 3\vec{j} + 2.5\vec{k}}{\sqrt{(-1)^2 + (3)^2 + (2.5)^2}}\right) = -1.24\vec{i} + 3.72\vec{j} + 3.1\vec{k}$ kN

Therefore, $\vec{M}_O = \vec{r}_{OB} \times \vec{F} = \vec{i} \times (-1.24\vec{i} + 3.72\vec{j} + 3.1\vec{k}) = -3.1\vec{j} + 3.72\vec{k}$ kN.m \quad \text{Ans.}

And the magnitude of $\vec{M}_O$ is, $M_O = |\vec{M}_O| = \sqrt{(-3.1)^2 + (3.72)^2} = 4.84$ kN.m \quad \text{Ans.}

(ii) $M_{OA} = \vec{M}_O \cdot \vec{n}_{OA}$, where $\vec{n}_{OA}$ is the unit vector along the line OA.

$\vec{n}_{OA} = \frac{(x_2-x_1)\vec{i} + (y_2-y_1)\vec{j} + (z_2-z_1)\vec{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}} = \frac{\vec{r}_{OA}}{|\vec{r}_{OA}|} = \frac{i + 2.5k}{\sqrt{1^2 + 2.5^2}} = 0.37\vec{i} + 0.928\vec{k}$

Therefore, $M_{OA} = \vec{M}_O \cdot \vec{n}_{OA} = (-3.1\vec{j} + 3.72\vec{k}) \cdot (0.37\vec{i} + 0.928\vec{k}) = 3.45$ kN.m \quad \text{Ans.}

The above moment can be expressed in a vector form as

$\vec{M}_{OA} = M_{OA}\vec{n}_{OA} = 3.45(0.37\vec{i} + 0.928\vec{k}) = 1.28\vec{i} + 3.20\vec{k}$ kN.m \quad \text{Ans.}

(iii) Since the line of action of the force is passing through the line CD, the moment of the force about line CD will be zero. That is, $M_{CD} = 0 \quad \text{Ans.}$