Chapter 8: Fundamental Sampling Distributions and Data Descriptions:

8.1 Random Sampling:
Definition 8.1:
A population consists of the totality of the observations with which we are concerned. (Population=Probability Distribution)

Definition 8.2:
A sample is a subset of a population.

Note:
- Each observation in a population is a value of a random variable X having some probability distribution f(x).
- To eliminate bias in the sampling procedure, we select a random sample in the sense that the observations are made independently and at random.
- The random sample of size $n$ is:
  \[ X_1, X_2, \ldots, X_n \]
  It consists of $n$ observations selected independently and randomly from the population.

8.2 Some Important Statistics:
Definition 8.4:
Any function of the random sample $X_1, X_2, \ldots, X_n$ is called a statistic.

Central Tendency in the Sample:
Definition 8.5:
If $X_1, X_2, \ldots, X_n$ represents a random sample of size $n$, then the sample mean is defined to be the statistic:

\[
\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n} \quad \text{(unit)}
\]

Note:
- $\bar{X}$ is a statistic because it is a function of the random sample $X_1, X_2, \ldots, X_n$.
- $\bar{X}$ has same unit of $X_1, X_2, \ldots, X_n$.
- $\bar{X}$ measures the central tendency in the sample (location).
Variability in the Sample:

**Definition 8.9:**
If $X_1, X_2, \ldots, X_n$ represents a random sample of size $n$, then the sample variance is defined to be the statistic:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 = \frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \cdots + (X_n - \bar{X})^2}{n-1} \text{ (unit)}^2$$

**Theorem 8.1:** (Computational Formulas for $S^2$)

$$S^2 = \frac{n}{n-1} \frac{\sum_{i=1}^{n} X_i^2 - n \bar{X}^2}{n} = \frac{n}{n-1} \left( \frac{S^2}{n} - \frac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)} \right)$$

Note:
- $S^2$ is a statistic because it is a function of the random sample $X_1, X_2, \ldots, X_n$.
- $S^2$ measures the variability in the sample.

**Definition 8.10:**
The sample standard deviation is defined to be the statistic:

$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}} \text{ (unit)}$$

Example 8.1: Reading Assignment
Example 8.8: Reading Assignment
Example 8.9: Reading Assignment

8.4 Sampling distribution:

**Definition 8.13:**
The probability distribution of a statistic is called a sampling distribution.

- Example: If $X_1, X_2, \ldots, X_n$ represents a random sample of size $n$, then the probability distribution of $\bar{X}$ is called the sampling distribution of the sample mean $\bar{X}$.

8.5 Sampling Distributions of Means:

**Result:**
If $X_1, X_2, \ldots, X_n$ is a random sample of size $n$ taken from a normal distribution with mean $\mu$ and variance $\sigma^2$, i.e. $N(\mu, \sigma)$,
then the sample mean $\overline{X}$ has a normal distribution with mean 

$$E(\overline{X}) = \mu_{\overline{X}} = \mu$$

and variance

$$Var(\overline{X}) = \sigma^2_{\overline{X}} = \frac{\sigma^2}{n}$$

- If $X_1, X_2, \ldots, X_n$ is a random sample of size $n$ from $N(\mu, \sigma)$, then $\overline{X} \sim N(\mu_{\overline{X}}, \sigma_{\overline{X}})$ or $\overline{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$.

- $\overline{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}}) \iff Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

**Theorem 8.2:** (Central Limit Theorem)

If $X_1, X_2, \ldots, X_n$ is a random sample of size $n$ from any distribution (population) with mean $\mu$ and finite variance $\sigma^2$, then, if the sample size $n$ is large, the random variable 

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

is approximately standard normal random variable, i.e., 

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

approximately.

- $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1) \iff \overline{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

- We consider $n$ large when $n \geq 30$. 

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• For large sample size \( n \), \( \bar{X} \) has approximately a normal distribution with mean \( \mu \) and variance \( \frac{\sigma^2}{n} \), i.e.,

\[
\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})
\]

approximately.

• The sampling distribution of \( \bar{X} \) is used for inferences about the population mean \( \mu \).

**Example 8.13:**
An electric firm manufactures light bulbs that have a length of life that is approximately normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have an average life of less than 775 hours.

**Solution:**
\( \bar{X} \) = the length of life

\( \mu = 800 \), \( \sigma = 40 \)

\( \bar{X} \sim N(800, 40) \)

\( n = 16 \)

\( \mu_{\bar{X}} = \mu = 800 \)

\( \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{16}} = 10 \)

\( \bar{X} \sim N(\mu_{\bar{X}}, \frac{\sigma}{\sqrt{n}}) = N(800, 10) \)

\[
\Rightarrow Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = Z = \frac{\bar{X} - 800}{10} \sim N(0, 1)
\]

\[
P(\bar{X} < 775) = P \left( \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} < \frac{775 - \mu}{\sigma / \sqrt{n}} \right)
\]

\[
= P \left( \frac{\bar{X} - 800}{10} < \frac{775 - 800}{10} \right)
\]

\[
= P \left( Z < \frac{775 - 800}{10} \right)
\]

\[
= P(Z < -2.50)
\]

\[
= 0.0062
\]
Sampling Distribution of the Difference between Two Means:
Suppose that we have two populations:
- 1-st population with mean $\mu_1$ and variance $\sigma_1^2$
- 2-nd population with mean $\mu_2$ and variance $\sigma_2^2$
- We are interested in comparing $\mu_1$ and $\mu_2$, or equivalently, making inferences about $\mu_1 - \mu_2$.
- We independently select a random sample of size $n_1$ from the 1-st population and another random sample of size $n_2$ from the 2-nd population:
  - Let $\overline{X}_1$ be the sample mean of the 1-st sample.
  - Let $\overline{X}_2$ be the sample mean of the 2-nd sample.
- The sampling distribution of $\overline{X}_1 - \overline{X}_2$ is used to make inferences about $\mu_1 - \mu_2$.

**Theorem 8.3:**
If $n_1$ and $n_2$ are large, then the sampling distribution of $\overline{X}_1 - \overline{X}_2$ is approximately normal with mean
$$E(\overline{X}_1 - \overline{X}_2) = \mu_{\overline{X}_1 - \overline{X}_2} = \mu_1 - \mu_2$$
and variance
$$Var(\overline{X}_1 - \overline{X}_2) = \sigma_{\overline{X}_1 - \overline{X}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$
that is:
\[ X_1 - X_2 \sim N(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}) \]

\[ \Leftrightarrow \]

\[ Z = \frac{(X_1 - X_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1) \]

Note:

\[ \sigma_{X_1-X_2} = \sqrt{\sigma_{X_1-X_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{\sigma_1^2}{n_1}} + \sqrt{\frac{\sigma_2^2}{n_2}} \]

Example 8.15: Reading Assignment

Example 8.16:
The television picture tubes of manufacturer \( A \) have a mean lifetime of 6.5 years and standard deviation of 0.9 year, while those of manufacturer \( B \) have a mean lifetime of 6 years and standard deviation of 0.8 year. What is the probability that a random sample of 36 tubes from manufacturer \( A \) will have a mean lifetime that is at least 1 year more than the mean lifetime of a random sample of 49 tubes from manufacturer \( B \)?

Solution:

Population \( A \) \hspace{1cm} Population \( B \)

\[ \mu_1=6.5 \hspace{1cm} \mu_2=6.0 \]

\[ \sigma_1=0.9 \hspace{1cm} \sigma_2=0.8 \]

\[ n_1=36 \hspace{0.5cm} (n_1>30) \hspace{1cm} n_2=49 \hspace{0.5cm} (n_2>30) \]

- We need to find the probability that the mean lifetime of manufacturer \( A \) is at least 1 year more than the mean lifetime of manufacturer \( B \) which is \( P(\overline{X}_1 \geq \overline{X}_2 + 1) \).
- The sampling distribution of \( \overline{X}_1 - \overline{X}_2 \) is

\[ \overline{X}_1 - \overline{X}_2 \sim N(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}) \]

- \( E(\overline{X}_1 - \overline{X}_2) = \mu_{\overline{X}_1 - \overline{X}_2} = \mu_1 - \mu_2 = 6.5 - 6.0 = 0.5 \)

- \( Var(\overline{X}_1 - \overline{X}_2) = \sigma_{\overline{X}_1 - \overline{X}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{(0.9)^2}{36} + \frac{(0.8)^2}{49} = 0.03556 \)
\[
\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{0.03556} = 0.189
\]

- \( \bar{X}_1 - \bar{X}_2 \sim N(0.5, 0.189) \)

- Recall \( Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1) \)

\[
P(\bar{X}_1 \geq \bar{X}_2 + 1) = P(\bar{X}_1 - \bar{X}_2 \geq 1)
\]

\[
= P\left( \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \geq \frac{1 - (\mu_1 - \mu_2)}{0.189} \right)
= P\left( Z \geq \frac{1 - 0.5}{0.189} \right)
= 1 - P(Z < 2.65)
= 1 - 0.9960
= 0.0040
\]

### 8.7 t-Distribution:
- Recall that, if \( X_1, X_2, \ldots, X_n \) is a random sample of size \( n \) from a normal distribution with mean \( \mu \) and variance \( \sigma^2 \), i.e. \( N(\mu, \sigma) \), then

\[
Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)
\]

- We can apply this result only when \( \sigma^2 \) is known!

- If \( \sigma^2 \) is unknown, we replace the population variance \( \sigma^2 \) with the sample variance \( S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1} \) to have the following statistic

\[
T = \frac{\bar{X} - \mu}{S / \sqrt{n}}
\]

**Result:**
If \( X_1, X_2, \ldots, X_n \) is a random sample of size \( n \) from a normal
distribution with mean $\mu$ and variance $\sigma^2$, i.e. $N(\mu, \sigma)$, then the statistic

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

has a $t$-distribution with $v = n - 1$ degrees of freedom (df), and we write $T \sim t(v)$ or $T \sim t(n-1)$.

**Note:**
- $t$-distribution is a continuous distribution.
- The shape of $t$-distribution is similar to the shape of the standard normal distribution.

**Notation:**
- $t_{\alpha}$ = The $t$-value above which we find an area equal to $\alpha$, that is $P(T > t_{\alpha}) = \alpha$
- Since the curve of the pdf of $T \sim t(v)$ is symmetric about 0, we have $t_{1 - \alpha} = - t_{\alpha}$
- Values of $t_{\alpha}$ are tabulated in Table A-4 (p.683).

**Example:**
Find the $t$-value with $v = 14$ (df) that leaves an area of:
- (a) 0.95 to the left.
- (b) 0.95 to the right.

**Solution:**
$v = 14$ (df); $T \sim t(14)$
(a) The t-value that leaves an area of 0.95 to the left is $t_{0.05} = 1.761$

(b) The t-value that leaves an area of 0.95 to the right is $t_{0.95} = -t_{1-0.95} = -t_{0.05} = -1.761$

Example:
For $\nu = 10$ degrees of freedom (df), find $t_{0.10}$ and $t_{0.85}$.

Solution:
$t_{0.10} = 1.372$
$t_{0.85} = -t_{1-0.85} = -t_{0.15} = -1.093 \quad (t_{0.15} = 1.093)$
Sampling Distribution of the Sample Proportion:
Suppose that the size of a population is \( N \). Each element of the population can be classified as type \( A \) or non-type \( A \). Let \( p \) be the proportion of elements of type \( A \) in the population. A random sample of size \( n \) is drawn from this population. Let \( \hat{p} \) be the proportion of elements of type \( A \) in the sample.

Let \( X = \) no. of elements of type \( A \) in the sample

\[ p = \text{Population Proportion} = \frac{\text{no. of elements of type } A \text{ in the population}}{N} \]

\[ \hat{p} = \text{Sample Proportion} = \frac{\text{no. of elements of type } A \text{ in the sample}}{n} = \frac{X}{n} \]

Result:
(1) \( X \sim \text{Binomial} (n, p) \) \{E(\(X\))=np , Var(\(X\))=npq\} 
(2) \( E(\hat{p}) = E(\frac{X}{n}) = p \)
(3) \( \text{Var}(\hat{p}) = \text{Var}(\frac{X}{n}) = \frac{pq}{n} ; q = 1 - p \)
(4) For large \( n \), we have
   \[ \hat{p} \sim N(p, \sqrt{\frac{pq}{n}}) \] (Approximately)
   \[ Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim N(0,1) \] (Approximately)
Sampling Distribution of the Difference between Two Proportions:

Suppose that we have two populations:

- $p_1$ = proportion of the 1-st population.
- $p_2$ = proportion of the 2-nd population.
- We are interested in comparing $p_1$ and $p_2$, or equivalently, making inferences about $p_1 - p_2$.
- We independently select a random sample of size $n_1$ from the 1-st population and another random sample of size $n_2$ from the 2-nd population:
  - Let $X_1$ = no. of elements of type $A$ in the 1-st sample.
  - Let $X_2$ = no. of elements of type $A$ in the 2-nd sample.
  - $\hat{p}_1 = \frac{X_1}{n_1}$ = proportion of the 1-st sample
  - $\hat{p}_2 = \frac{X_2}{n_2}$ = proportion of the 2-nd sample
  - The sampling distribution of $\hat{p}_1 - \hat{p}_2$ is used to make inferences about $p_1 - p_2$.

Result:

1. $E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2$
2. $Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}$ ; $q_1 = 1 - p_1$, $q_2 = 1 - p_2$
(3) For large $n_1$ and $n_2$, we have

$$
\hat{p}_1 - \hat{p}_2 \sim N(p_1 - p_2, \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}) \quad \text{(Approximately)}
$$

$$
Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \sim N(0, 1) \quad \text{(Approximately)}
$$
### Critical Values of the t-distribution ($t_\alpha$)

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<th>$\nu$</th>
<th>$0.40$</th>
<th>$0.30$</th>
<th>$0.20$</th>
<th>$0.15$</th>
<th>$0.10$</th>
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