

# **A Comparison of Coordinated Ordering Policies in a Three-Echelon Supply Chain Network**

**Tamer F. Abdelmaguid**

**Department of Industrial Engineering, College of Engineering  
King Saud University, PO Box 800, Riyadh 11421, Saudi Arabia**

**Mohamed M. Marzouk**

**Department of Mechanical Design and Production, Faculty of Engineering  
Cairo University, Giza 12613, Egypt**

## **Abstract**

This paper addresses the integrated production-inventory-distribution decisions in a three-echelon supply chain (SC) network. The studied SC network is composed of a raw material processor that supplies different types of processed raw materials to a set of manufacturing facilities which in turn deliver their end products to a set of retailers facing external demand from end customers. A mixed integer linear programming model is provided with the objective of minimizing the overall production, inventory and distribution costs. Balanced and synchronized ordering policies are applied to the studied problem and their impacts on the different cost elements are assessed.

## **Keywords**

Coordinated ordering policies, three-echelon supply chain networks, integrated production-inventory-distribution

## **1. Introduction**

The problem addressed in this paper is frequently encountered in home appliances industries. It is concerned with the production-inventory-distribution (PID) decisions in a three-echelon supply chain (SC) network. The studied SC network is composed of a Raw Material Processor (RMP) that produces different types of processed raw materials (PRMs) and supplies them to a set of manufacturing facilities. The PRMs are used to produce a set of end products; where, a specific combination of the PRMs is used as required by each product's bill of material. The end products are then to be delivered to a set of retailers selling them to end customers. It is required to plan for the coordinated PID decisions between the three levels (RMP, facilities and retailers) through a limited planning horizon with the objective of minimizing the overall production, inventory and transportation costs.

The consideration of the integrated and coordinated PID decisions in supply chains is relatively new. Thomas and Griffin [1] provide a literature review on coordinated policies of supply chains of two or more levels. They emphasize the importance of coordinated decisions as opposed to independent management which can result in very poor overall performance. Blumenfeld et al. [2-3] investigated the significance of such coordinated decisions. They studied a very specific scenario in which a parts producer supplies parts to a final assembly manufacturer, and there is only one destination per part type. They analyzed the tradeoffs between production setup, freight transportation, and inventory costs, and showed that coordinating production and distribution decisions can result in sufficiently large savings.

Pyke [4] studied the integrated production/distribution decisions for a simple three-node system (factory, finished goods stockpile and single retailer) under stochastic demand conditions. The studied model examines the properties of the cost functions arising for a single product case without taking into consideration transportation costs. Haq et al. [5] developed a mixed integer programming model for studying the integrated PID decisions with the objective of minimizing the total transportation, production cost, setup cost and inventory carrying cost. Pyke and Cohen [6-7] studied a three-level SC consisting of a factory, a finished goods stockpile and a retailer. They developed a Markov chain model for both the single product and multi-product cases. The results of their research indicated that small replenishment batch sizes effectively reduce production capacity due to more setups, which consequently increases production lead times and forces greater downstream inventories.

Chandra and Fisher [8] considered several products produced over several time periods in a single plant and distributed to a number of retail outlets by a fleet of fixed capacity vehicles stationed at the plant. The problem is to schedule production and distribution so as to minimize the total cost of production setups, transportation and retailers' inventory. The demand for each product in a period at each retail outlet is deterministic. Mark and Wong [9] proposed a simplified model for the integrated PID problem in a three-echelon supply chain which contains several raw material suppliers, one manufacturing plant and several retailers. The objective is to minimize the total inventory, production and distribution costs through a specific planning horizon. Different from Chandra and Fisher [8], their model does not include vehicle routing constraints and consider transportation costs as a function of the amount transported.

Fumero and Vercellis [10] studied the problem of synchronized development of production, inventory, and distribution schedules over a specific planning horizon. Their work is an extension to Chandra and Fisher [8], as they study the same problem conditions but with detailed transportation costs. In their model the transportation costs include a fixed usage cost for a vehicle in a period, variable shipping cost for each unit of product, and the cost of traveling with an empty vehicle to the production facility. They developed a mixed integer linear programming (MILP) model and used Lagrangian relaxation to decompose the problem into four smaller subproblems. A similar approach for studying the integrated PID problem is conducted by Lei et al. [11] where they develop a heuristic for solving the problem and compare it with the upper bound obtained by a commercial MILP solver.

In this paper, we investigate the effect of recently developed coordinated ordering policies on the different PID cost elements of a three-echelon supply chain. Recently developed coordinated ordering policies are classified into balanced and synchronized [12-15]. In the context of a single supplier-multi retailer SC, balanced ordering (BO) policies operate with an ordering cycle of  $m$  periods where  $m$  is the number of retailers. In every period, only one retailer is allowed to order. In synchronized ordering (SO) policies, all retailers order only once in the same predetermined period of a cycle. For both BO and SO, the supplier orders only once every  $m$  periods.

The rest of this paper is organized as follows. In section 2, a description of the studied problem is provided along with a mixed integer linear programming (MILP) model. In section 3, we discuss how the coordinated ordering policies are applied to the studied problem. In section 4, we present the developed test problems along with the experimental results. Finally, the conclusion is presented in section 5.

## 2. Problem Description and Mathematical Model

In the studied problem we consider a set  $T$  of consecutive indexes for time periods starting at 1, where a *period* refers to either a day or a week. We denote by  $N_f$  the number of facilities, and by  $N_r$  the number of retailers. The term *production center* is used to refer to either the RMP or a facility, and the term *consumption center* refers to either a facility or a retailer. The set  $F$  is the index set for the production centers, where the index 0 is reserved to the RMP and the facilities are indexed from 1 to  $N_f$  which constitute the subset  $F' = \{1, 2, \dots, N_f\}$ . The set  $R = \{N_f + 1, N_f + 2, \dots, N_f + N_r\}$  is the index set for the retailers, and  $W = F \cup R$ . The set  $A$  is the set of arcs representing direct links between production centers and consumption centers in the supply chain, i.e.  $A = \{0\} \times F' \cup F' \times R$ .

Each production center has a set of production lines which are used to manufacture different types of items (PRMs for the RMP or end products for the facilities). We denote by  $L_i$  the set of production lines in production center  $i \in F$ .  $P_i$  is the set of items that can be produced by a production center  $i \in F$ , and  $P = \bigcup_{i \in F} P_i$ . Each end product  $k$  requires quantity  $\theta_{km}$  from PRM item  $m$ . Each production line is set up at the beginning of each period to produce a specific item, and the production line will be dedicated for the production of that item throughout that period. No additional setup is required between two consecutive periods if the same item is to be produced on the same production line. Each production line  $l$  in production center  $i$  when set up has a specific production capacity per period for each item  $k \in P_i$ , denoted  $K_{ilk}$ . The production centers incur both fixed setup and variable production costs during the production operation, denoted  $s_{ilk}$  and  $p_{ilk}$  respectively.

We denote by  $V$  the set of vehicle types that can be rented to deliver the produced items to the required destinations, by  $C_v$  the capacity of vehicle type  $v$ , and by  $c_k$  the load consumed by one unit of item  $k \in P$  from a vehicle capacity. There are two types of vehicle renting costs. A fixed transportation cost for transporting items from location  $i$  to  $j$  for vehicle type  $v$ , denoted  $\tau_{jv}$ , and a variable transportation cost that depends on the quantity transported, denoted  $\rho_{kv}$ .

We study a case in which the production and consumption centers are closely located in nearby cities such that the transportation from a production to a consumption center is to be conducted at the end of a given period and will be available at the consumption center by the beginning of the next period.

There are four different types of inventories in the studied SC: 1) an inventory for the PRMs at the RMP, 2) an inventory for the PRMs at each facility, 3) an inventory for the end products at each facility, and 4) an inventory for the end products at the retailers. The set  $\Psi_i$  denotes the set of items for which a storage space is allocated in location  $i \in W$ , that is  $\Psi_i = P_0$  when  $i = 0$ ,  $P_i \cup P_0$  when  $i \in F'$ , and  $\bigcup_{j \in F'} P_j$  when  $i \in R$ . There is an inventory holding cost  $h_{ik}$  per unit of item  $k \in \Psi_i$  at location  $i \in W$  per period.

We study a case in which each facility produces a set of distinct end products that are not produced by the other facilities in the SC; therefore there is no competition between facilities. Furthermore, it is assumed that each retailer serves a distinct set of customers which are not served by any of the other retailers. The demand forecasted for end product  $k$  at retailer  $j$  is provided for each period in the planning horizon and denoted  $d_{jkt}$ . The main objective is to minimize the summation of all production, transportation and inventory costs for all entities in the SC network, which can be achieved by coordinating ordering decisions.

The decision variables include the inventory level of item  $k$  at location  $i$  at the end of period  $t$ , denoted  $I_{ikt}$ .  $I_{ik0}$  is a given parameter representing the initial inventory level at location  $i$  for item  $k$ . The number of vehicles of type  $v$  rented to deliver items from location  $i$  to location  $j$  in period  $t$  is denoted  $r_{ijvt}$ .  $x_{ilkt}$  represents the quantity of item  $k$  produced at production center  $i$  by production line  $l$  in period  $t$ . The binary variable  $y_{ilkt}$  is used to represent the condition whether production center  $i$  uses the production line  $l$  to produce item  $k$  in period  $t$  or not; while another binary variable,  $z_{ilkt}$ , is used to represent the decision whether production center  $i$  sets-up production line  $l$  to produce item  $k$  in period  $t$  or not. A third binary variable, denoted  $u_{ilkt}$ , is used to represent setup conditions of two consecutive periods  $t-1$  and  $t$ . Finally,  $q_{ijkt}$  represents the quantity of item  $k$  transported from production center  $i$  to consumption center  $j$  at the end of period  $t$ . The given parameter  $q_{ijk0}$  represents the quantity scheduled to be transported at the beginning of the planning horizon. The following is the developed MILP model.

$$\min \sum_{t \in T} \left[ \sum_{i \in F} \sum_{l \in L_i} \sum_{k \in P_i} (p_{ilk} x_{ilkt} + s_{ilk} z_{ilkt}) + \sum_{i \in W} \sum_{k \in \Psi_i} h_{ik} I_{ikt} + \sum_{(i,j) \in A} \sum_{v \in V} \sum_{k \in \Psi_i \cap \Psi_j} (\rho_{kv} q_{ijkt} + \tau_{jv} r_{ijvt}) \right] \quad (1)$$

Subject to:

$$x_{ilkt} \leq K_{ilk} y_{ilkt}, \text{ and } y_{ilkt} \leq x_{ilkt} \quad \forall t \in T, \forall i \in F, \forall k \in P_i, \forall l \in L_i \quad (2)$$

$$I_{ikt-1} + \sum_{l \in L_i} x_{ilkt} = I_{ikt} + \sum_{(i,j) \in A} q_{ijkt} \quad \forall t \in T, \forall i \in F, \forall k \in P_i \quad (3)$$

$$I_{jkt-1} + q_{ijkt-1} = I_{jkt} + d_{jkt} \quad \forall t \in T, \forall j \in R, \forall i \in F', \forall k \in P_i \quad (4)$$

$$I_{imt-1} + q_{0imt-1} = \sum_{k \in P_i} \sum_{l \in L_i} \theta_{km} x_{ilkt} + I_{imt} \quad \forall t \in T, \forall i \in F', \forall m \in P_0 \quad (5)$$

$$\sum_{k \in P_i} c_k q_{ijkt} \leq \sum_{v \in V} C_v r_{ijvt} \quad \forall t \in T, \forall (i, j) \in A \quad (6)$$

$$\sum_{k \in P_i} y_{ilkt} \leq 1 \quad \forall t \in T, \forall i \in F, \forall l \in L_i \quad (7)$$

$$z_{ilkt} = y_{ilkt} - u_{ilkt}, \quad u_{ilkt} \leq M y_{ilkt}, \quad u_{ilkt} \leq M y_{ilkt-1}, \text{ and } y_{ilkt-1} + y_{ilkt} - 1 \leq u_{ilkt} \quad \forall t \in T \setminus \{1\}, \forall i \in F, \forall l \in L_i, \forall k \in P_i \quad (8.a)$$

$$z_{ilk1} = y_{ilk1} \quad \forall i \in F, \forall l \in L_i, \forall k \in P_i \quad (8.b)$$

$$z_{ilkt}, y_{ilkt}, u_{ilkt} \in \{0,1\} \quad \forall t \in T, \forall i \in F, \forall k \in P_i, \forall l \in L_i \quad (9)$$

In addition to non-negativity constraints for all remaining decision variables, and the integrality constraint for the decision variables  $r_{ijvt}$

The objective function (1) is comprised of the minimization of three main parts, namely the production costs, the inventory holding costs incurred at all levels of the SC, and the transportation costs throughout the planning horizon. Constraints (2) perform two tasks. They limit the production quantity from exceeding the production line capacity, and they link the production quantity decision variable  $x_{ilkt}$  with its corresponding binary variable  $y_{ilkt}$ . Constraints (3) are the inventory balance equations for the produced items at each production center. Constraints (4) are the inventory balance equations of the end products at the retailers. Constraints (5) are the inventory balance equations

for the PRMs at the manufacturing facilities. They also relate the number of items produced in the RMP to the products produced in the manufacturing facilities.

constraints (6) are the vehicle capacity constraints which relate the quantity transported to the number of rented vehicles and the capacity associated with each vehicle type rented. Constraints (7) are necessary to make sure that no more than one item is to be produced on a given production line in the same period. Constraints (8 a and b) are logical constraints that define the condition that if a production line has been set up to produce a specific item in period  $t-1$ , there will be no setup required in period  $t$  if the production line is used again for producing the same item. Constraints (9) are the domain constraints. We note here that the problem represented by (1) to (9) is NP-hard since it can be reduced to multi-item capacitated lot-sizing problems.

### **3. Applied Coordinated Ordering Policies**

In this section, the mechanism of applying the two coordinated ordering policies to the studied three-echelon supply chain network is briefly described. In the case of balanced ordering policy, each consumption center adheres to an ordering plan enforced upon by the supplying production center. Each consumption center places its order depending on its place in the production center's ordering cycle schedule. Any orders placed in any other period besides the agreed upon review period is prohibited. In synchronized ordering policy all the consumption centers in a given stage order all at once based on a predetermined and fixed period in the ordering cycle which is enforced by the supplying production center. Knowing that, ordering in any other period is prohibited.

The application of the two mentioned ordering policies on the studied problem is handled within six modules. First, the demand is consumed at each retailer. The fulfillment of this demand is achieved by the inventory level and the quantity delivered at the end of the preceding period. Second, based on the ordering policy (Balanced or synchronized) the retailer which is designated to order, places an order up to level at the manufacturing facility for the specified product. Third, the manufacturing facilities which have incoming orders start to produce the orders. A manufacturing lead time is incorporated in the delivery schedule and the quantity ordered is delivered in a timely manner to the retailers. Fourth, the products manufactured at the manufacturing facilities consume known portions of the RMP items, creating the demand which will be fulfilled by the RMP. Fifth, based on the ordering policy (Balanced or synchronized) the manufacturing facility which is designated to order, places an order up to level at RMP for the specified RMP item. Sixth, the RMP starts to produce the orders. Moreover, a manufacturing lead time is incorporated in the delivery schedule and the quantity ordered is delivered in a timely manner to the manufacturing facilities. This process is repeated every time period through the planning horizon.

The production process that takes place in the manufacturing facilities and the RMP adheres to the minimum work remaining dispatching rule. Thus, the production schedule is created and then the products are assigned to the production line that will keep the order the least number of periods within the facility. The production line assigned for the specific product is to produce its full capacity, so as to benefit from the economies of scale. The product processing time for each production line is computed using the production capacities and the total product demand incoming from all the consumption centers in the subsequent stage. Knowing the processing times and which production lines are in use before the product assignment, the manufacturing lead time can be computed. The production center's on-hand inventory and on-order inventory levels are balanced every period and their summations are used to compute the inventory positions for each product. The manufactured products are delivered to the demanding consumption center after the manufacturing lead time has elapsed.

The selection of the vehicle type to deliver the shipment from a production center to a consumption center is based on a simple rule that selects the vehicle type with the size that can suit the entire shipment first, and similarly the other vehicle types are determined for the remaining part of the shipment. Based on that rule, the number of vehicles and their types are determined.

### **4. Experimentation and Results**

The purpose of the experimentation conducted in this research is to investigate the impacts of the two coordinated ordering policies on the three cost elements, namely production costs, inventory costs and transportation costs. We first present the structure of the designed experiments and then present the experimental results consecutively in the following two subsections.

### 4.1 Experimental Design

In the designed experiments, we follow the literature in setting the values for the parameters, specifically those presented in [10] and [16]. The designed experiments are based on a specific scenario in which there are two production facilities, three retailers, three types of transportation vehicles and a planning horizon of 60 periods. The first production facility has two production lines and produces three different end products; while, the second production facility has one production line and produces two different end products. The RMP has one production line and two types of PRMs. Cost elements for different locations in the designed experiments has been randomly generated using uniform distribution. The lower and upper limits for the uniform distributions used for the different cost elements are provided in table 1. Similarly, vehicle parameters have been randomly generated using uniform distributions with their limits provided in table 2.

Table 1: Selected lower and upper limits for cost parameters at the locations

Parameter	Location		RMP		Facility 1		Facility 2		Retailer 1		Retailer 2		Retailer 3	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Holding cost ( $h_{ik}$ )	3	4	5	8	5	8	5	10	5	10	5	10	5	10
Production cost ( $p_{ik}$ )	2	3	5	10	10	15	--	--	--	--	--	--	--	--
Setup cost ( $s_{ik}$ )	250	300	800	900	700	800	--	--	--	--	--	--	--	--

Table 2: Lower and upper limits for the vehicles' parameters

Parameter	Vehicle		Vehicle 1		Vehicle 2		Vehicle 3	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Fixed transportation cost ( $\rho$ )	50	60	120	160	200	220		
Variable transportation cost per unit ( $\tau$ )	7	9	5	7	2	4		
Capacity ( $C_v$ )			450		900		1400	

Table 3: Mean demand limits

Retailer #	Upper limit	Lower limit
1	1000	1100
2	1400	1500
3	1200	1300

The designed experiments have been generated by controlling two factors which are believed to have major impact on the performance of the coordinated ordering policies. These factors are the coefficient of variation (CV) for the demand of end products and the production capacity factor (PCF). The demand values for end products are generated using normal distribution, where the mean value of the demand is generated using a uniform distribution with lower and upper limits provided in table 3, and its standard deviation equals the mean demand multiplied by the CV. The chosen levels for the CV are 0.001, 0.01, 0.1 and 0.3. The production capacity of each production line for a given item is determined by multiplying the mean demand value of that item by the PCF. The values of the PCF are generated using uniform distribution and the chosen levels for the lower and upper limits are [3,4], [4,5], [5,6] and [7,8]. The values of  $c_k$  which represents the load consumed by one unit of item  $k \in P$  from a vehicle capacity is randomly generated using uniform distribution with lower and upper limits of 0.25 and 0.85 respectively. The values of  $\theta_{km}$  which represent the quantity required from PRM item  $m$  by end product  $k$  are generated using uniform distribution with a minimum value of 1 and a maximum of 3.

### 4.2 Results

For four levels of both CV and PCF, we have 16 different combinations. For each combination 10 random replicates are generated, resulting in a total number of 160 test cases. Both coordinated ordering policies are applied as described in section 3 to each test case and the production, inventory, transportation and total costs are evaluated. The percentage cost difference calculated as the difference between the cost obtained by the SO policy and the cost obtained by the BO policy divided by the latter is used as a measure. A negative value of that measure indicates lower cost obtained by the SO policy. Figures 1 through 4 show the main effects plots of the calculated measure for each cost element.

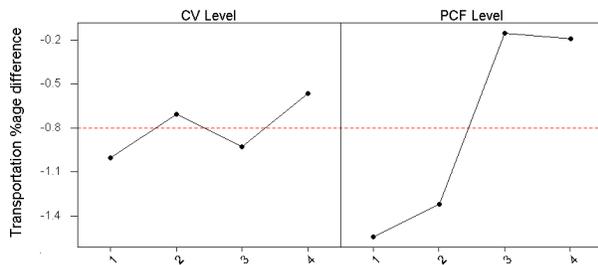


Figure 1. Main effects for transportation cost %age diff.

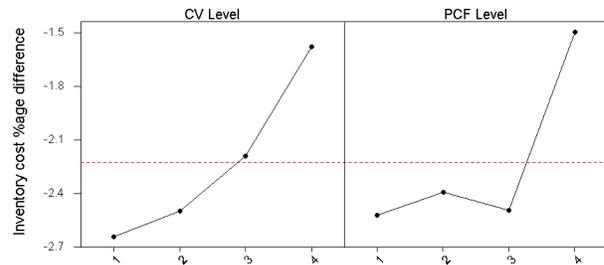


Figure 2. Main effects for inventory cost %age diff.

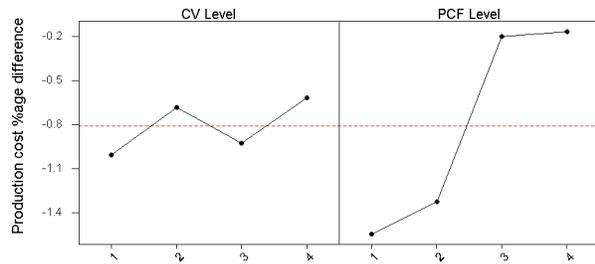


Figure 3. Main effects for production cost %age diff.

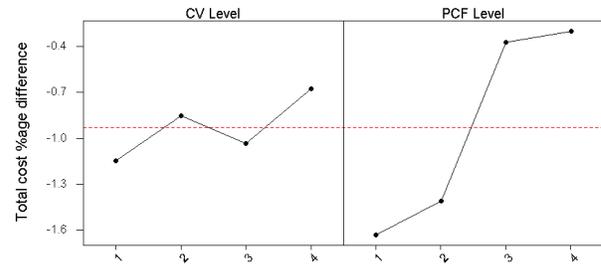


Figure 4. Main effects for total cost %age diff.

## 5. Conclusion

In this paper, the integrated PID decisions problem has been addressed in a three-echelon supply chain network. The synchronized ordering (SO) and balanced ordering (BO) policies are applied. The results indicate that on average, SO policy results in less costs for each cost element studied. However, with the increase of the production capacity, there is an evident trend towards a better performance by a BO policy. On the other hand, variability in demand does not show any preference regarding which policy is better, except for the case of inventory cost, as there is also an evident trend towards a better performance by the BO policy.

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