Operators of arbitrary large norms that are bounded by 1 on a given basis of a separable infinite dimensional Hilbert space H.

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Let (ξ_n) be an orthonormal basis for H. For $k \in \mathcal{N}$, define T_k on H by $T_k\eta =< \eta, \xi_1 + \xi_2 + \cdots + \xi_k > \xi_1$. Then

$$T_k\xi_n = \begin{cases} \xi_1 & n \leq k \\ 0 & n > k \end{cases}$$

Hence $\| T_k\xi_n \| \leq 1 (n \in \mathcal{N})$. On the other hand $T_k^*\eta =< \eta, \xi_1 > (\xi_1 + \cdots + \xi_k) \quad (\eta \in H)$; therefore $\| T_k \| = \| T_k^* \| \geq \| T_k^*\xi_1 \| = \| \xi_1 + \cdots + \xi_k \| = \sqrt{k}$.