

Let H be a separable infinite dimensional complex Hilbert space, and let $\mathbf{B}(H)$ denote the algebra of all bounded linear operators on H . Let A, B be operators in $\mathbf{B}(H)$. We define the generalized derivation $\delta_{A,B}: \mathbf{B}(H) \rightarrow \mathbf{B}(H)$ by $\delta_{A,B}(X) = AX - XB$. In this paper we consider the question posed by Turnsek in 2003, when $\overline{\text{ran}(\delta_{A,B}|_{C_p})}^{c_p} = \overline{\text{ran}(\delta_{A,B}|_{C_p \cap C_p})}^{c_p}$? We prove that this holds in the case where A and B satisfy the Fuglede-Putnam theorem. Finally, we apply the obtained results to double operator integrals.

Keywords: Generalized derivation, Fuglede-Putnam theorem, Hilbert-Schmidt class, double operator integrals.

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