MATH203 Calculus

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Area and Volume

Volume:

In the previous study, we saw that if $f(x, y) \geqslant 0$ and f is continuous, then the double integral

$$\iint\limits_R f(x, y)\mathrm{d}A \tag{1}$$

gives the volume of the solid that lies under the graph of $z = f(x, y)$ and over a region R in the xy-plane.

Area:

The double integral (1) can be used to find the area of the region R if $f(x, y) = 1$ which becomes

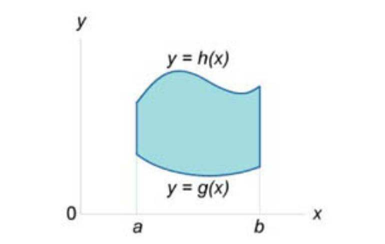
$$\iint\limits_R \mathrm{d}A \tag{2}$$

Area and Volume

Double Integral for finding area:

Formula 1 If a region R_x is defined by $a \leqslant x \leqslant b$ and $g(x) \leqslant y \leqslant h(x)$, where $g(x), h(x)$ are continuous on $[a, b]$, then the area A of R_x is given by

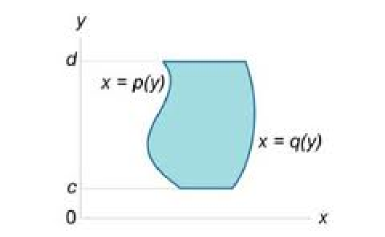
$$A = \int_a^b \int_{g(x)}^{h(x)} \mathrm{d}y \mathrm{d}x$$



Area and Volume

Formula 2 If a region R_y is defined by $c \leqslant y \leqslant d$ and $p(y) \leqslant x \leqslant q(y)$, where $p(y), q(y)$ are continuous on $[c, d]$, then the area A of R_y is given by

$$\int_c^d \int_{p(y)}^{q(y)} \mathrm{d}x\mathrm{d}y$$



Double Integrals

Examples

Sketch the region bounded by the graphs of :

(1) $y = x^2$ and $y = 2x$. Evaluate $\iint_R (x^3 + 4y)dA$ using R_x region and R_y region.

(2) $y = \sqrt{x}$ and $y = \sqrt{3x - 18}$ and $y = 0$ using R_x region and R_y region.

(3) reverse the order of the integration and evaluate

$$\int_0^4 \int_{\sqrt{y}}^2 y \cos x^5 dxdy.$$

(4) Find the area A of the region in the $xy-$plane bounded by the graph of $x = y^3$, $x + y = 2$ and $y = 0$

(4) Find the volume of the solid that lies under the graph of $z = 4x^2 + y^2$ and over the region in the xy-plane bounded by $x + y = 2$, $x = 0$, $y = 0$ and $z = 0$.