Two-Factor Analysis of Variance

14.1 An experiment was conducted to study the effects of temperature and type of oven on the life of a particular component. Four types of ovens and 3 temperature levels were used in the experiment. Twenty-four pieces were assigned randomly, two to each combination of treatments, and the following results recorded.

	Oven			
Temperature (°F)	O_1	O_2	O_3	O_4
500	227	214	225	260
	221	259	236	229
550	187	181	232	246
	208	179	198	273
600	174	198	178	206
	202	194	213	219

Using a 0.05 level of significance, test the hypothesis that:
(a) different temperatures have no effect on the life of the component;
(b) different ovens have no effect on the life of the component;
(c) the type of oven and temperature do not interact.

$$a = 3\,, b = 4\,, n = 2\,, \propto = 0.05$$

$\bar{\boldsymbol{y}}_{\boldsymbol{ij}.}$: $\bar{y}_{11.} = 224$ $\bar{y}_{12.} = 236.6$ $\bar{y}_{13.} = 230.5$ $\bar{y}_{14.} = 244.5$

$\bar{y}_{21.} = 197.5$ $\bar{y}_{22.} = 180$ $\bar{y}_{23.} = 215$ $\bar{y}_{24.} = 215$

$\bar{y}_{31.} = 188$ $\bar{y}_{32.} = 196$ $\bar{y}_{33.} = 195.5$ $\bar{y}_{34.} = 212.5$

$\bar{\boldsymbol{y}}_{\boldsymbol{i}..}$: $\bar{y}_{1..} = 233.875$ $\bar{y}_{2..} = 213$ $\bar{y}_{3..} = 198.$

$\bar{\boldsymbol{y}}_{.\boldsymbol{j}.}$: $\bar{y}_{.1.} = 203.1667\,, \bar{y}_{.2.} = 204.11, \bar{y}_{.3.} = 213.667\,, \bar{y}_{.4.} = 238.833$

$\bar{\boldsymbol{y}}_{...} = 214.958$

c)

1. $\boldsymbol{H_0}$: *There is no interaction bettween the diffferent tempreatures and the different types of ovens.*
$\boldsymbol{H_1}$: *There is interaction bettween the diffferent tempreatures and the different types of ovens.*

2. *Test Statistic* :

$$SSAB = n \sum_{i=1}^{a} \sum_{j=1}^{b} \left(\bar{\boldsymbol{y}}_{\boldsymbol{ij}.} - \bar{\boldsymbol{y}}_{\boldsymbol{i}..} - \bar{\boldsymbol{y}}_{.\boldsymbol{j}.} + \bar{\boldsymbol{y}}_{...} \right)^2$$

$$= 2\,[(224 - 233.875\ - 203.1667\ +)^2 + \cdots + (212.5 - 198\ - 238.833\ + 214.958)^2] = 3126$$

$$MSAB = \frac{3126}{6} = 521$$

$$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk} - \bar{y}_{ij.}\right)^2 = (227-224)^2 + \cdots + (219-212.5)^2 = 3833$$

$$MSE = \frac{SSE}{ab(n-1)} = \frac{3833}{12} = 319.5$$

$$F_3 = \frac{MSAB}{MSE} = \frac{521}{319.5} = 1.63$$

4.Decision:

$F_3 = 1.63 \ngtr 2.996 = f_{0.05,6,12}$

We cannot reject $\boldsymbol{H_0}$ i.e ther is no interaction.

a) **1.** $\boldsymbol{H_0}$: $There\ is\ no\ difference\ in\ the\ mean\ of\ the\ comonent\ life\ of\ the\ 3\ tempreatures.$

$\boldsymbol{H_1}$: $At\ least\ one\ mean\ different.$

2.Test Statistic:

$$SSA = b\,n\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2$$

$$= 8[(233.875 - 214.958)^2 + \cdots + (198 - 214.958)^2] = 5194.$$

$$MSA = \frac{519}{2} = 2597$$

$$F = \frac{MSA}{MSE} = \frac{2597}{312.5} = 8.128$$

3.Desision:

Since $F = 8.128 > 3.9 = f_{0.05,2,12}$ we reject $\boldsymbol{H_0}$.

There is a significant difference in the mean of component life of the 3 temperatures.

b) **1.** $\boldsymbol{H_0}$: $There\ is\ no\ difference\ in\ the\ mean\ of\ the\ comonent\ life\ of\ the\ 3\ tempreatures.$

$\boldsymbol{H_1}$: $At\ least\ one\ mean\ different.$

2.Test Statistic:

$$SSB = a\,n\sum_{i=1}^{b}\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$$

$$= (203.1667 - 214.958)^2 + \cdots + (238.833 - 214.958)^2 = 4963$$

$$MSB = \frac{MSB}{MSE} = \frac{1654.4}{319.5} = 5.18$$

3.Decision:

Since $F = 5.18 > 3.49 = f_{0.05,3,12}$ we reject $\boldsymbol{H_0}$.

There is a significant difference in the mean of component life of the 4 ovens types.

H.W 14.3 Three strains of rats were studied under 2 environmental conditions for their performance in a maze test. The error scores for the 48 rats were recorded.

Environment	Strain: Bright		Mixed		Dull	
Free	28	12	33	83	101	94
	22	23	36	14	33	56
	25	10	41	76	122	83
	36	86	22	58	35	23
Restricted	72	32	60	89	136	120
	48	93	35	126	38	153
	25	31	83	110	64	128
	91	19	99	118	87	140

Use a 0.01 level of significance to test the hypothesis that:
(a) there is no difference in error scores for different environments;
(b) there is no difference in error scores for different strains;
(c) the environments and strains of rats do not interact.

c) **1.** $\boldsymbol{H_0}$: $There\ is\ no\ interaction\ bettween\ the\ diffferent\ environments\ and\ diffferent\ \ strains\ of\ rats\ .$

$\boldsymbol{H_1}$: $There\ is\ interaction\ bettween\ the\ diffferent\ the\ diffferent\ environments\ and\ diffferent\ \ strains\ of\ rats\ .$

2.Test Statistic:

$$MSAB = 617.6, \qquad MSE = 1004.6, \qquad F_3 = 0.61$$

3.Decision:

$F_3 = 0.61 \ \ngtr f_{0.01,2,42}$

We cannot reject $\boldsymbol{H_0}$ i.e ther is no interaction.

a) **1.** $\boldsymbol{H_0}$: $There\ is\ no\ difference\ in\ the\ mean\ of\ error\ scores\ for\ different\ environments$

$\boldsymbol{H_1}$: $At\ least\ one\ mean\ different.$

2.Test Statistic:

$$SSA = b\,n \sum_{i=1}^{a} (\bar{y}_{i..} - \bar{y}_{...})^2$$

$$MSA = 14875.5$$

$$F = \frac{MSA}{MSE} = 14.81$$

3.Desision:

Since $F = 14.81 > f_{0.01,1,42}$ we reject $\boldsymbol{H_0}$.

There is a significant difference in the error scores mean for different environments.

b) **1.** $\boldsymbol{H_0}: There\ is\ no\ difference\ in\ the\ mean\ of\ error\ scores\ for\ different\ strains\ of\ rats.$

$\boldsymbol{H_1}: At\ least\ one\ mean\ different.$

2.Test Statistic:

$$SSB = a\,n \sum_{i=1}^{b} \left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$$

$$MSB = \frac{MSB}{MSE} = 9077.1$$

$$F_2 = 9.04$$

3.Decision:

Since $F_2 = 9.04 > f_{0.01,2,42}$ we reject $\boldsymbol{H_0}$.

Thus, there is a significant difference in the error scores mean for different strains of rats.

Chapter 9

16.1 The following data represent the time, in minutes, that a patient has to wait during 12 visits to a doctor's office before being seen by the doctor:

17 15 20 20 32 28 12 26 25 25 35 24

Use the **sign test** at the 0.05 level of significance to test the doctor's claim that the median waiting time for her patients is not more than 20 minutes.

$$n = 12, \alpha = 0.05$$

1. $H_0: \tilde{\mu} = 20$ *vs* $H_1: \tilde{\mu} > 20$

2.Calculat x & p-value.

17	15	20	20	32	28	12	26	25	25	35	24
-	-	.	.	+	+	-	+	+	+	+	+

Thus, x=7

$$p - value = p\left(X \geq 7, p = \frac{1}{2}\right) = \sum_{x=7}^{10} \binom{10}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = \left(\frac{1}{2}\right)^{10} \sum_{x=7}^{10} \binom{10}{x} = 0.1719$$

3.DECISION:

Since P-value = 0.1719 > 0.05 = α

We cannot reject H_0. i.e the median is not greater than 20.

16.8 Analyze the data of Exercise 16.1 by using the **signed-rank** test.

$$n = 12, \alpha = 0.05$$

1. $H_0: \tilde{\mu} = 20$ *vs* $H_1: \tilde{\mu} > 20$

2.Calculat w_

time	17	15	20	20	32	28	12	26	25	25	35	24
di-d0	-3	-5	0	0	12	8	-8	6	5	5	15	4
Rank	1	4	-	-	9	7.5	7.5	6	4	4	10	2

$$W_- = 1 + 4 + 7.5 = 12.5; \quad W_+ = 9 + 7.5 + 6 + 4 + 4 + 10 + 2 = 42.5$$

3.Decision:

Since $W_- = 12.5 > 11 = W_{10,0.05}$ we cannot reject H_0 . "as in EX 16.1"

16.5 It is claimed that a new diet will reduce a person's weight by 4.5 kilograms, on average, in a period of 2 weeks. The weights of 10 women were recorded before and after a 2-week period during which they followed this diet, yielding the following data:

Woman	Weight Before	Weight After
1	58.5	60.0
2	60.3	54.9
3	61.7	58.1
4	69.0	62.1
5	64.0	58.5
6	62.6	59.9
7	56.7	54.4
8	63.6	60.2
9	68.2	62.3
10	59.4	58.7

Use the sign test at the 0.05 level of significance to test the hypothesis that the diet reduces the median weight by 4.5 kilograms against the alternative hypothesis that the median weight loss is less than 4.5 kilograms.

$$n = 10, \alpha = 0.05$$

1. $H_0: \tilde{\mu}_1 - \tilde{\mu}_2 = 4.5 \; vs \;\; H_1: \tilde{\mu}_1 - \tilde{\mu}_2 < 4.5$

2.Calculat x & p-value.

Woman	Weight Before	Weight After	di	di-d0	sign
1	58.5	60.0	-1.5	-6	-
2	60.3	54.9	5.4	0.9	+
3	61.7	58.1	3.6	-0.9	-
4	69.0	62.1	6.9	2.4	+
5	64.0	58.5	5.5	1	+
6	62.6	59.9	2.7	-1.8	-
7	56.7	54.4	2.3	-2.2	-
8	63.6	60.2	3.4	-1.1	-
9	68.2	62.3	5.9	1.4	+
10	59.4	58.7	0.7	-3.8	-

Thus, x= 4

$$p - value = p\left(X \leq 4, p = \frac{1}{2}\right) = \sum_{x=0}^{4} \binom{10}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = \left(\frac{1}{2}\right)^{10} \sum_{x=0}^{4} \binom{10}{x} = 0.377$$

3.DECISION:

Since P-value = 0.377 > 0.05 = α we cannot reject H_0. i.e the diet reduces median weight by 4.5.

16.10 The weights of **5** people before they stopped smoking and **5** weeks after they stopped smoking, in kilograms, are as follows:

	Individual				
	1	**2**	**3**	**4**	**5**
Before	66	80	69	52	75
After	71	82	68	56	73

Use the **signed-rank test** for paired observations to test the hypothesis, at the 0.05 level of significance, that giving up smoking has no effect on a person's weight against the alternative that one's weight increases if he or she quits smoking.

$$n = 5, \alpha = 0.05$$

1. $H_0: \tilde{\mu}_1 - \tilde{\mu}_2 = 0 \ vs \ \ H_1: \tilde{\mu}_1 - \tilde{\mu}_2 < 0$

2.Test statistic

Woman	Weight Before	Weight After	di	di-d0	Ranks
1	66	71	-5	-5	5
2	80	82	-2	-2	2.5
3	69	68	1	1	1
4	52	56	-4	-4	4
5	75	73	2	2	2.5

$$W_+ = 1 + 2.5 = 3.5$$

3.Decision:

Since $W_+ = 3.5 > 1 = W_{5,0.05}$

We cannot reject H_0, i.e quitting smoking doesn't effect on a person's weight.

16.12 The following are the numbers of prescriptions filled by two pharmacies over a 20 day period:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Pharmacy A	19	21	15	17	24	12	19	14	20	18	23	21	17	12	16	15	20	18	14	22
Pharmacy B	17	15	12	12	16	15	11	13	14	21	19	15	11	10	20	12	13	17	16	18

Use the signed-rank test at the **0.01** level of significance to determine whether the two pharmacies, on average, fill the same number of prescriptions against the alternative that pharmacy *A* fills more prescriptions than pharmacy *B*.

$$n = 20, \alpha = 0.01$$

1. $H_0: \tilde{\mu}_1 - \tilde{\mu}_2 = 0 \ vs \ \ H_1: \tilde{\mu}_1 - \tilde{\mu}_2 > 0$

2. Test statistic

Day	A	B	**di**		Rank
1	19	17	**2**	**3**	**4**
2	21	15	**6**	**14**	**15.5**
3	15	12	**3**	**6**	**7.5**
4	17	12	**5**	**13**	**13**
5	24	16	**8**	**20**	**19.5**
6	12	15	**-3**	**7**	**7.5**
7	19	11	**8**	**19**	**19.5**
8	14	13	**1**	**2**	**1.5**
9	20	14	**6**	**15**	**15.5**
10	18	21	**-3**	**8**	**7.5**
11	23	19	**4**	**12**	**11**
12	21	15	**6**	**16**	**15.5**
13	17	11	**6**	**17**	**15.5**
14	12	10	**2**	**4**	**4**
15	16	20	**-4**	**11**	**11**
16	15	12	**3**	**9**	**7.5**
17	20	13	**7**	**18**	**18**
18	18	17	**1**	**1**	**1.5**
19	14	16	**-2**	**5**	**4**
20	22	18	**4**	**10**	**11**

$$W_{-} = 7.5 + 7.5 + 11 + 4 = 30$$

3.Decision:

Since $W_{-} = 30 < 43 = W_{5,0.01}$

We reject H_0, i.e the two pharmacies fill the same numbers of prescriptions.

16.32 The following table gives the recorded grades for 10 students on a midterm test and the final examination in a calculus course:

Student	Midterm Test	Final Examination
1	84	73
2	98	63
3	91	87
4	72	66
5	86	78
6	93	78
7	80	91
8	0	0
9	92	88
10	87	77

(a) Calculate the rank correlation coefficient.

$$(a) \quad r_s = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$$

Student	Midterm Test	Rank1	Final Examination	Rank2	di=R1-R2
1	84	4	73	4	0
2	98	10	63	2	8
3	91	7	87	8	-1
4	72	2	66	3	-1
5	86	5	78	6.5	-1.5
6	93	9	78	6.5	2.5
7	80	3	91	10	7
8	0	1	0	1	0
9	92	8	88	9	-1
10	87	6	77	5	1

Thus, $\sum d_i^2 = 125.5$

$$r_s = 1 - \frac{6(125.5)}{10(99)} = 0.239$$

16.37 Two judges at a college homecoming parade rank eight floats in the following order:

	Float							
	1	2	3	4	5	6	7	8
Judge A	5	8	4	3	6	2	7	1
Judge B	7	5	4	2	8	1	6	3

(a) Calculate the rank correlation coefficient.

	Float							
	1	2	3	4	5	6	7	8
Judge A	5	8	4	3	6	2	7	1
Judge B	7	5	4	2	8	1	6	3
di	-2	3	0	1	2	1	1	-2

$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$$

Thus, $\sum d_i^2 = 24$

$$r_s = 1 - \frac{6(24)}{10(63)} = 0.7143$$