Chapter 5

Two Samples: Tests on Two Means

10.30 A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\bar{x}_1 = 81$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3.4$, has a mean $\bar{x}_2 = 76$. **Test the hypothesis that $\boldsymbol{\mu_1 = \mu_2}$ against the alternative, $\boldsymbol{\mu_1 = \mu_2}$.**

~~Quote a *P*-value in your conclusion.~~

$n_1 = 25$, $\bar{x}_1 = 81$, $\sigma_1 = 5.2$, $n_2 = 36$, $\bar{x}_2 = 76$, $\sigma_2 = 3.4$, $\alpha = 0.05$ ($\sigma's\ known$)

1. $H_0: \mu_1 - \mu_2 = 0$ against $H_1: \mu_1 - \mu_2 \neq 0$
2. Test statistic:

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{81 - 76 - 0}{\sqrt{\frac{5.2^2}{25} + \frac{3.4^2}{36}}} = 4.22, \quad d = \mu_1 - \mu_2$$

3. Decision:

We reject H_0 $if\ Z_c > Z_{1-\frac{\alpha}{2}}$ $or\ Z_c < -Z_{1-\frac{\alpha}{2}}$ $\quad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

Since $Z_c = 4.22 > Z_{1-\frac{\alpha}{2}} = 1.96$, we reject H_0 at $\alpha = 0.05$. Thus, $\mu_1 \neq \mu_2$

10.33 A study was conducted to see if increasing the substrate concentration has an appreciable effect on the velocity of a chemical reaction. With a substrate concentration of 1.5 moles per liter, the reaction was run **15** times, with an average velocity of **7.5** micromoles per 30 minutes and a standard deviation of **1.5.** With a substrate concentration of 2.0 moles per liter, **12** runs were made, yielding an average velocity of **8.8** micromoles per 30 minutes and a sample standard deviation of **1.2.** Is there any reason to believe that this increase in substrate concentration causes an increase in the mean velocity of the reaction of more than 0.5 micromole per 30 minutes**?**

Use a **0.01** level of significance and assume the populations to be approximately normally distributed with equal variances.

Solution:

substrate concentration of 1.5 : $n_1 = 15$, $\bar{X}_1 = 7.5$, $S_1 = 1.5$

substrate concentration of 2: $n_2 = 12$, $\bar{X}_2 = 8.8$, $S_2 = 1.2$, $\quad \alpha = 0.01$

(note $\sigma_1 = \sigma_2$ $and\ unknown$)

1. $H_0: \mu_2 - \mu_1 = 0.5$ against $H_1: \mu_2 - \mu_1 > 0.5$
2. Test statistic :

$$T = \frac{(\bar{X}_2 - \bar{X}_1) - d}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(8.8 - 7.5) - 0.5}{\sqrt{1.8936}\sqrt{\frac{1}{15} + \frac{1}{12}}} = 1.5011 \ , \ d = \mu_1 - \mu_2$$

Where, $S_P = \sqrt{\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}} = \sqrt{\frac{(1.5)^2(15-1)+(1.2)^2(12-1)}{15+12-2}} = 1.3761$

3. Decision:

Rejection Region$\left(T > T_{n_1+n_2-2,\alpha}\right)$

$T_{n_1+n_2-2,\alpha} = T_{25,0.01} = 2.485$

Reject H_0 $if\ T > 2.485 \implies we\ can't\ Reject\ H_0$ at $\alpha = 0.01$.

Paired t-test

10.45 A taxi company manager is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Twelve cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars

were then equipped with regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, was recorded as follows:

	Kilometers per Liter	
Car	Radial Tires	Belted Tires
1	4.2	4.1
2	4.7	4.9
3	6.6	6.2
4	7.0	6.9
5	6.7	6.8
6	4.5	4.4
7	5.7	5.7
8	6.0	5.8
9	7.4	6.9
10	4.9	4.7
11	6.1	6.0
12	5.2	4.9

Can we conclude that cars equipped with radial tires give better fuel economy than those equipped with belted tires? Assume the populations to be normally distributed. ~~Use a P-value in your conclusion.~~

$$\bar{d} = 0.1417 \, , S_d = 0.1975$$

1. $H_0: \mu_d = 0$ against $H_1: \mu_d > 0$

2. Test statistic :

$$T = \frac{\bar{d} - d_0}{\frac{S_d}{\sqrt{n}}} = \frac{0.1417}{0.1975/\sqrt{12}} = 2.485$$

3. Decision:

Rejection Region$\left(T > T_{n_1-1,\alpha}\right)$

$T_{n_1-1,\alpha} = T_{11,0.05} = 1.796$

Reject H_0 $if\ T = 2.485 > 1.796 \implies we\ \ Reject\ H_0$ at $\alpha = 0.05$.

i.e the cars equipped with radial tires give better fuel economy than those equipped with belted tires

One Sample: Test on a Single Proportion.

10.55 A marketing expert for a pasta-making company believes that 40% of pasta lovers prefer lasagna. If 9 out of 20 pasta lovers choose lasagna over other pastas, what can be concluded about the expert's claim? Use a 0.05 level of significance.

n=20 , X=9 , $\alpha = 0.05$, $\hat{P} = \frac{9}{20} = 0.45$

1. $H_0: P = 0.4$ against $H_1: P \neq 0.4$
2. Test statistic :

$$Z_C = \frac{\hat{P} - P_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.45 - 0.4}{\sqrt{\frac{0.4(0.6)}{20}}} = 0.46$$

3. Decision:

We reject H_0 $if\ Z_c > Z_{1-\frac{\alpha}{2}}\ \ or\ \ Z_c < -Z_{1-\frac{\alpha}{2}}$ $\qquad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

Since $Z_c = 0.46 \ngtr Z_{1-\frac{\alpha}{2}} = 1.96$, we can't reject H_0 at $\alpha = 0.05$. Thus, $\mu_1 \neq \mu_2$

Thus, his claim is correct.

10.57 A new radar device is being considered for a certain missile defense system. The system is checked by experimenting with aircraft in which a kill or a no kill is simulated. If, in 300 trials, 250 kills occur, accept or reject, at the **0.04** level of significance, the claim that the probability of a kill with the new system does not exceed the 0.8 probability of the existing device.

n=300 , X=250 , $\alpha = 0.04$, $\hat{P} = \frac{250}{300} = 0.833$

1. $H_0: P = 0.8$ against $H_1: P > 0.8$

H_0: the claim that the probability of a kill with the new system does not exceed the 0.8 probability of the existing device.

H_1: the claim that the probability of a kill with the new system exceed the 0.8 probability of the existing device.

2. Test statistic :

$$Z_C = \frac{\hat{P} - P_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.833 - 0.8}{\sqrt{\frac{0.8(0.2)}{300}}} = 1.44$$

3. Decision:

We reject H_0 $if\ Z_c > Z_{1-\alpha}$ $\qquad Z_{1-\alpha} = Z_{0.96} = 1.75$

Since $Z_c = 1.44 \ngtr Z_{1-\alpha} = 1.75$, we can't reject H_0 at $\alpha = 0.04$.

Thus, the claim is correct.

Two Samples: Tests on Two Proportions

H.W 10.61 In a winter of an epidemic flu, the parents of 2000 babies were surveyed by researchers at a wellknown pharmaceutical company to determine if the company's new medicine was effective after two days.
Among 120 babies who had the flu and were given the medicine, 29 were cured within two days. Among 280 babies who had the flu but were not given the medicine, 56 recovered within two days. Is there any significant indication that supports the company's claim of the effectiveness of the medicine?

10.63 In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportions of urban and suburban residents who favor construction of the nuclear plant? ~~Make use of a *P*-value.~~

$n_1 = 100,\ x_1 = 63,\ \widehat{P}_1 = \frac{63}{100} = 0.63$

$$n_2 = 125,\ X_2 = 59,\ \widehat{P}_2 = \frac{59}{125} = 0.472, \qquad \alpha = 0.05$$

1. $H_0: p_1 = p_2$ against $H_1: p_1 \neq p_2$
2. Test statistic :

$$Z_C = \frac{\widehat{P}_1 - \widehat{P}_2}{\sqrt{\hat{p}\hat{q}(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{(0.63) - 0.472}{\sqrt{(0.542)(0.458)\frac{1}{100} + \frac{1}{125}}} = 2.36$$

Where, $\hat{p} = \frac{X_1 - X_2}{n_1 + n_2} = \frac{63+59}{100+125} = 0.542$

3. Decision:

We reject $H_0\ if\ Z_c > Z_{1-\frac{\alpha}{2}}\ \ or\ \ Z_c < -Z_{1-\frac{\alpha}{2}} \qquad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

Since $Z_c = 2.36 > Z_{1-\frac{\alpha}{2}} = 1.96$, we reject H_0 at $\alpha = 0.05$.

If we take $\alpha = 0.01$

$Z_c = 2.36 \ngtr Z_{0.995} = 2.575$. Thus, we can't reject H_0 at $\alpha = 0.01$. (week reject)

One- and Two-Sample Tests Concerning Variances

10.67 The content of containers of a particular lubricant is known to be normally distributed with a variance of 0.03 liter. Test the hypothesis that $\sigma^2 = 0.03$ against the alternative that $\sigma^2 \neq 0.03$ for the random sample of 10 containers in Exercise 10.23 on page 356. Use a *P*-value in your conclusion.

10.23 Test the hypothesis that the **average** content of containers of a particular lubricant زيوت التشحيم is 10 liters if the contents of a random sample of **10** containers are **10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3**, and **9.8** liters. Use a **0.01** level of significance and assume that the distribution of contents is normal.

$\sigma_0^2 = 0.03\ , n = 10\ , S = 0.2459\ (from\ exercise\ 10.23), \alpha = 0.01$

1. $H_0: \sigma^2 = 0.03$ against $H_1: \sigma^2 \neq 0.03$
2. Test statistic :

$$\chi^2 = \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} = \frac{(9)(0.2459)^2}{0.03} = 18.14$$

3. Decision:

We reject $H_0\ if\ \chi^2 > {\chi^2}_{\frac{\alpha}{2}}\ \ or\ \ \chi^2 < {\chi^2}_{1-\frac{\alpha}{2}}$

$$\chi^2{}_{1-\frac{\alpha}{2},\ n-1} = \chi^2{}_{0.995,9} = 1.735 \qquad ; \chi^2{}_{\frac{\alpha}{2}} = \chi^2{}_{0.005,9} = 23.589$$

Since $\chi^2 = 18.14 \ngtr 23.589$ & Since $\chi^2 = 18.14 \nless 1.735$, we can't reject H_0.

10.73 A study is conducted to compare the lengths of time required by men and women to assemble a certain product. Past experience indicates that the distribution of times for both men and women is approximately normal but the variance of the times for women is less than that for men. A random sample of times for 11 men and 14 women produced the following data:

Men	women
$n_1 = 11$	$n_2 = 14$
$s_1 = 6.1$	$s_2 = 5.3$

Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 > \sigma_2^2$. Use a *P*-value in your conclusion.

1. $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 > \sigma_2^2$
2. Test statistic :

$$F = \frac{S_1^2}{S_2^2} = \frac{(6.1)^2}{(5.3)^2} = 1.32$$

3. Decision:

We reject H_0 if $F > F_{\propto,n_1-1,n_2-1}$; $\quad F_{\propto,n_1-1,n_2-1} = f_{0.05\,,10\,,13} = 2.67$

Since $F = 1.32 \ngtr 2.67$, we can't reject H_0.

i.e the variance for men is equal to the variance for women.