

Tutorial 3 Multiple Random Variables

Exercise 1

A joint sample space for two R.V X and Y has four elements (1,1), (2,2), (3,3), and (4,4) with probabilities 0.1, 0.35, 0.05 and 0.5 respectively.

1. Determine through logic and sketch the distribution function $F_{X,Y}(x, y)$.
2. Find the probability of the event $\{X \leq 2.5, Y \leq 6\}$.
3. Find the probability of the event $\{X \leq 3\}$
4. Find and sketch the marginal distribution functions.

Exercise 2

The joint distribution function for two R.V X and Y is

$$F_{X,Y}(x, y) = u(x)u(y)[1 - e^{-ax} - e^{-ay} + e^{-a(x+y)}].$$

Where $u(\cdot)$ is the unit-step function and $a > 0$. For $a = 0.5$ find:

1. The probability $P\{X \leq 1, Y \leq 2\}$.
2. The probability $P\{0.5 < X < 1.5\}$.
3. The probability $P\{-1.5 < X \leq 2, 1 < Y \leq 3\}$.

Exercise 3

The joint distribution function for two R.V X and Y is

$$F_{X,Y}(x, y) = u(x)u(y)[1 - e^{-ax} - e^{-ay} + e^{-a(x+y)}].$$

Find and sketch the marginal distribution functions.

Exercise 4

A fair coin is tossed twice. Define random variables: X="number of heads on the first toss" and Y=" number of heads on the second toss" (note that X and Y can have only the values 0 or 1).

- a. Find and sketch the joint density function of X and Y.
- b. Find and sketch the joint distribution function.

Exercise 5

A joint probability density function is

$$f_{X,Y}(x, y) = \begin{cases} 1/ab & 0 < x < a \text{ and } 0 < y < b \\ 0 & \text{elsewhere} \end{cases}$$

Find and sketch the joint distribution function $F_{X,Y}(x, y)$.

Exercise 6

A joint probability density function is

$$f_{X,Y}(x, y) = u(x) u(y) x e^{-x(y+1)}$$

Find the marginal density functions $f_X(x)$ and $f_Y(y)$.

Exercise 7

Two random variables X and Y have the joint density function:

$$f_{X,Y}(x, y) = \begin{cases} (x+y)^2/40 & -1 < x < 1, \text{ and } -3 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

1. Find all the second-order moments of X and Y
2. What are the variances of X and Y
3. What is the correlation coefficient?
4. Find all the third-order moments for X and Y

Exercise 8

For the two random variables X and Y:

$$f_{X,Y}(x, y) = 0.15 \delta(x+1) \delta(y) + 0.1 \delta(x) \delta(y) + 0.1 \delta(x) \delta(y-2) + 0.4 \delta(x-1) \delta(y+2) \\ + 0.2 \delta(x-1) \delta(y-1) + 0.05 \delta(x-1) \delta(y-3)$$

Find: (a) the correlation; (b) the covariance; (c) the correlation coefficient of X and Y. (d) Are X and Y either uncorrelated or orthogonal.